INTERACTION OF LIGHT WITH SPIN WAVES IN A FERRODIELECTRIC

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Expressions are obtained which describe the interaction of electromagnetic and of coherent spin waves in ferrodielectrics, on the assumption that the frequencies and wave vectors of the electromagnetic waves are much larger than the frequencies and wave vectors of the spin waves.

THE interaction of electromagnetic waves with oscillations of the magnetic-moment density in ferromagnetic media has been studied in a number of papers.[1-4] The theoretical investigation of the scattering of light on thermodynamic fluctuations of the magnetic-moment density was first carried out by Bass and Kaganov[1] and was subsequently developed in papers of Elliott and Loudon,[2] Akhiezer and Bolotin,[3] and L'vov.[4] In [1], account was taken only of the magneto-dipole interaction of the electromagnetic waves with the magnetic moment. Elliott and Loudon pointed out another mechanism of interaction, arising from spin-orbit interaction. These two mechanisms lead to the appearance of gyrotropic components in, respectively, the magnetic susceptibility tensor μ_{ik} and the dielectric permittivity tensor ϵ_{ik}; these components depend on the vector magnetization **M**. The size of the terms caused by spin-orbit interaction depends on the shape of the ferromagnet and on the frequency of the incident light and can vary by several orders of magnitude.

The results of [1,3,4] enable us to describe the process of scattering of light on magnetic-moment fluctuations in the approximation of single-stage scattering (the Born approximation). This approximation is applicable if the dimensions of the ferromagnetic specimen and the correlation radius of the fluctuations of magnetic-moment density are small enough so that the effects of multiple and coherent scattering of the light may be neglected.

Dillon et al.[5] and Anderson[6] experimentally observed a modulation of the rotation of the plane of polarization of electromagnetic waves in ferromagnetic crystals with excited oscillations of the magnetic moment. The theoretical estimates used here[6] were obtained in the "quasistatic" approximation, with neglect of the rate of change of the magnetic moment; they are very rough and are applicable only in the case of a uniform precession in sufficiently thin specimens.

The present paper considers the scattering of electromagnetic waves on coherent spin waves (of fixed phase), on the assumption that the frequency ω_0 and the wave vector k_0 of the electromagnetic wave are much larger than the frequency Ω and the wave vector κ of the spin wave: $\omega_0 \gg \Omega$, $k_0 \gg \kappa$. Account is taken of the magneto-dipole and spin-orbit mechanisms of interaction of electromagnetic waves with spin waves. The dependence of the amplitudes of the scattered waves on the propagation distance is obtained for the case of a spin wave ($\kappa \neq 0$) and of uniform precession ($\kappa = 0$).

1. On the surface of the half space $x > 0$, which is filled with an isotropic ferrodielectric, let there be excited an electromagnetic wave with frequency ω_0 and a spin wave with frequency Ω. As a result of the interaction of the waves there appear, according to the degree of propagation, waves at the combination frequencies $\omega_n = \omega_0 + n\Omega$ (n = arbitrary integer), and the amplitudes of the original waves change. The problem is to find the amplitudes of the waves as functions of the distance of propagation.

Maxwell's equations in a ferrodielectric have the following form:

$$\partial \mathbf{d} / \partial t = c \operatorname{rot} \mathbf{h}, \qquad \partial \mathbf{b} / \partial t = -c \operatorname{rot} \mathbf{e},$$
$$\operatorname{div} \mathbf{b} = \operatorname{div} \mathbf{d} = 0, \qquad \mathbf{b} = \mathbf{h} + 4\pi \mathbf{M},$$
$$\mathbf{d} = \int_{-\infty}^{\infty} \{\varepsilon(t-t')\mathbf{e}(t') + i\zeta(t-t')[\mathbf{e}(t')\mathbf{B}(t')]\}\, dt',$$
$$\mathbf{B} = \mathbf{H}_0 + 4\pi\mathbf{M}, \quad \varepsilon = \varepsilon' + i\varepsilon'', \tag{1}$$

and the equation for the magnetic-moment density is

$$\frac{\partial \mathbf{M}}{\partial t} = g[\mathbf{M}\mathbf{H}^e] - \frac{1}{\tau}(\mathbf{M} - \mathbf{M}_0). \tag{2*}$$

In (1) and (2), $\zeta(t)$ is the specific Faraday rotation, caused by the spin-orbit interaction; $\mathbf{H}^e = \mathbf{H}_0 + \alpha\Delta\mathbf{M} + \tilde{\mathbf{h}}$, g is the gyromagnetic ratio, $\mathbf{H}_0$ is the effective constant magnetic field including the anisotropy field, α is the exchange-interaction constant, τ is the relaxation time of a spin wave, $\tilde{\mathbf{h}}$ is the alternating magnetic field, and $\mathbf{M}_0$ is the component of the magnetic moment along $\mathbf{H}_0$. In the case in which a uniform precession is excited in the ferrodielectric, the alternating field $\tilde{\mathbf{h}}$ includes, besides the high-frequency field of the light wave, also the external uniform alternating field that is exciting the precession:

$$\tilde{\mathbf{h}}(t) = \mathbf{h}(t) + \mathbf{h}_\Omega e^{-i\Omega t}. \tag{3}$$

The boundary conditions corresponding to excitation on the surface (x = 0) of an electromagnetic wave with frequency ω_0 and of a spin wave with frequency Ω can be expressed in the following form:[1)]

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*$[\mathbf{M}\mathbf{H}^e] \equiv \mathbf{M} \times \mathbf{H}^e$

[1)] Since in the medium the oscillations of all physical quantities are coupled, the high-frequency oscillations of the magnetic field are accompanied by weak (of order $O(\Omega/\omega)$) oscillations of the magnetic moment, and, inversely, the low-frequency oscillations of the magnetic moment are accompanied by weak oscillations of the magnetic field. In the case under consideration, $\Omega/\omega \ll 1$, and the coupling of the oscillations of the magnetic field and of the magnetic moment may be neglected.

$$\mathbf{h}(x=0,t)=\mathbf{h}_0e^{-i\omega_0t}+\text{c.c.},$$
$$\mathbf{m}(x=0,t)=\mathbf{m}_0e^{-i\Omega t}+\text{c.c.},\qquad(4)$$

where $\mathbf{m}=\mathbf{M}-\mathbf{M}_0$.

For simplicity we shall suppose that the wave vector of the electromagnetic wave is directed along the x axis. From equations (1) and (2), the following equation for the electromagnetic field can be obtained:

$$c^2\,\mathrm{rot\,rot}\,\mathbf{h}+\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\Big\{\varepsilon(t-t')\left(\frac{\partial}{\partial t'}\mathbf{h}(t')+4\pi g[\mathbf{M}_0\mathbf{h}(t')]\right)+i\zeta(t-t')\left[\frac{\partial}{\partial t'}\mathbf{h}(t')\mathbf{B}_0\right]\Big\}dt'$$
$$=-4\pi\frac{\partial}{\partial t}\int_{-\infty}^{\infty}\Big\{g\varepsilon(t-t')[\mathbf{m}(t')\mathbf{h}(t')]+i\zeta(t-t')\left[\frac{\partial}{\partial t'}\mathbf{h}(t')\mathbf{m}(t')\right]\Big\}dt',\qquad(5)$$

where $\mathbf{B}_0=\mathbf{H}_0+4\pi\mathbf{M}_0$. In the derivation of this equation, terms have been omitted that are quadratic in the magneto-dipole and spin-orbit interaction constants.

We shall seek solutions of equations (2) and (5), with the boundary conditions (4), in the form

$$h^{\pm}(\mathbf{r},t)=\mathrm{Re}\sum_n a_n^{\pm}\exp\{-i\omega_nt+i\mathbf{k}_n^{\pm}\mathbf{r}\}.\qquad(6)$$

Here the circular polarization has been introduced: $h^{\pm}=h_y\pm ih_z$;

$$\mathbf{m}(\mathbf{r},t)=\mathrm{Re}\,\mathbf{b}(x)e^{-i\Omega t+i\varkappa\mathbf{r}}.\qquad(6')$$

The amplitudes $a_n^{\pm}$ and $\mathbf{b}$ depend only on x, since the problem is uniform in y and z. The wave vectors $k_n^{\pm}$ and κ, as is clear from equations (2) and (5), are related to ω_n and Ω by the known dispersion laws:

$$k^{\pm}+i\gamma^{\pm}=\frac{\omega}{c}\left[\varepsilon'(\omega)\pm\left(\zeta(\omega)\frac{\mathbf{B}_0\mathbf{k}}{k}+\frac{4\pi g\varepsilon'(\omega)}{\omega k}\mathbf{M}_0\mathbf{k}\right)\right]^{1/2},\qquad(7)$$
$$\gamma^{\pm}=\frac{\omega^2}{c^2k^{\pm}}\varepsilon''(\omega)$$

and

$$\Omega=\Omega_0+\alpha gM_0\varkappa^2,\quad\Omega_0=gH_0\qquad(8)$$

while

$$k_{n,y}^{\pm}=n\varkappa_y,\quad k_{n,z}^{\pm}=n\varkappa_z.\qquad(9)$$

On substituting (6′) into (2) and (5) and taking account of (8) and (9), we get the following equations for the amplitudes:

$$-2ik_n^{\pm}\cos\theta_n(\partial/\partial x+\gamma^{\pm}(\omega_n))a_n^{\pm}$$
$$=v(n-1,n)b_xa_{n-1}^{\pm}e^{-i\Delta x}+v(n,n+1)b_x^*a_{n+1}^{\pm}e^{i\Delta x},\qquad(10)$$

where

$$v(n-1,n)=\frac{4\pi\omega_n}{c^2k_n}[g\varepsilon'(\omega_n)+\omega_{n-1}\zeta(\omega_n)],$$
$$v(n,n+1)=\frac{4\pi\omega_n}{c^2k_n}[g\varepsilon'(\omega_n)+\omega_{n+1}\zeta(\omega_n)],$$
$$\cos\theta_n\approx1+O(\varkappa^2/k^2),\quad\Delta=\Omega/c-\varkappa_x+O(\varkappa^2/k^2),\qquad(11)$$

and

$$i\left(\frac{\partial}{\partial x}+\Gamma\right)b_x=\frac{g^2M_{0x}}{2\alpha M_0\varkappa\Omega}\sum_n[(a_{n-1}^{+})^*a_n^{+}+(a_{n-1}^{-})^*a_n^{-}]\qquad(12)$$

where $\Gamma=(\alpha gM_0\kappa\tau)^{-1}$.

We recall that in our case the length of the electromagnetic waves is much smaller than the length of the spin wave. Furthermore, the interaction of the waves is weak enough, and the amplitudes change so little over distances of the order of the wavelengths, that their second derivatives can be omitted in (10) and (12).

In the case of a uniform precession of the magnetic-moment density ($\kappa=0$), Eq. (12) ceases to be valid. In this case, $\mathbf{b}$ must be determined as the amplitude of a stationary solution of Eq. (2):

$$\mathbf{b}=\frac{g}{(i\Omega+\tau^{-1})^2+\Omega_0^2}\Big\{\Omega_0M_0\mathbf{h}_\Omega+(i\Omega+\tau^{-1})[\mathbf{M}_0\mathbf{h}_\Omega]+\frac{g}{2\pi}\int_{-\infty}^{\infty}[[\mathbf{M}_0\mathbf{h}(t')]\mathbf{h}(t')]e^{i\Omega t'}dt'\Big\}.\qquad(13)$$

2. The solution of the differential-difference equation (10), correct to terms quadratic in Ω/ω and κ/k, can be found without difficulty by applying the WKB method in conjunction with the Laplace transformation:

$$a_n^{\pm}=a_0^{\pm}(\pm i)^n\exp\{-\gamma^{\pm}x\pm in\beta_1\}J_n(\bar v_n|B_1|);\qquad(14)$$

here

$$\bar v_n=(v_0v_n)^{1/2},\quad v_n=\tfrac{1}{2}[v(n-1,n)+v(n,n+1)],$$
$$B_1(x)=\int_0^x b_x(y)e^{i\Delta y}dy,\quad\beta_1(x)=\arg B_1(x),\qquad(15)$$

and $J_n(x)$ is a Bessel function.

As was to be expected, the extinction of the light is determined by the imaginary part ϵ'' of the dielectric permittivity. As is well known, Bessel functions are negligibly small if the value of the index exceeds the value of the argument. Therefore the number of electromagnetic waves excited whose amplitudes differ appreciably from zero is proportional to $|B_1|$. Since $B_1(x)$, and consequently also $|B_1|$, is a periodic function, the whole process of transformation of electromagnetic waves is periodic in x with a period equal to the period of the function $|B_1|$. At the beginning of the process, waves appear at the combination frequencies; the number of appreciably excited waves reaches a maximum along with $|B_1|$, and then it begins to diminish. At the place where $|B_1|$ vanishes, there is present only the wave at the fundamental frequency ω_0. The maximum number of appreciably excited waves is approximately determined by the following relation:

$$N_{max}\approx2\max v_0|B_1|.$$

Since the coefficients v_n depend only slightly on n:

$$v_n=v_0+O\left(\frac{\Omega}{\omega}\right),\quad v_0=\frac{4\pi\omega_0}{c^2k_0}[g\varepsilon(\omega_0)+\zeta(\omega_0)\omega_0],$$

therefore, with neglect of this dependence,

$$a_n^{\pm}=a_0^{\pm}(\pm i)^n\exp\{-\gamma^{\pm}x\pm in\beta_1\}J_n(v_0|B_1|).\qquad(16)$$

On substituting (16) into (6) and taking account of (9), we get

$$h^{\pm}(\mathbf{r},t)=a_0^{\pm}\exp\{-i\omega_0t+i\mathbf{k}^{\pm}\mathbf{r}\}$$
$$\times\exp\{\pm iv_0|B_1|\cos(\Omega t-\varkappa_yy-\varkappa_zz-\Omega x/c+\beta_1)\}.\qquad(17)$$

From this formula it is clear that the interaction of the light with the spin wave leads to modulation of the light, in particular to modulation of the rotation of the plane of polarization of linearly polarized light. In fact,

$$\varphi(x)=\mathrm{arctg}\frac{h_y}{h_z}=\varphi_0(x)+v_0|B_1|\cos\left(\Omega t-\varkappa_yy-\varkappa_zz-\frac{\Omega}{c}x+\beta_1\right),$$

where

$$\varphi_0(x)=\frac{\omega}{c\sqrt{\varepsilon'}}\left[\zeta(\omega)\frac{\mathbf{B}_0\mathbf{k}}{k}+4\pi g\,\varepsilon'(\omega)\frac{\mathbf{M}_0\mathbf{k}}{\omega k}\right].$$

In the limit of small x ($x \ll 2\pi c/\Omega$), these formulas agree with the result of the "quasistatic" approximation.[6]

3. In order to clarify in more detail the peculiarities of the process of interaction of the waves, we turn to the calculation of $B_1(x)$.

In the case of the uniform precession, as is clear from (13) and (11),

$$b_x=\frac{g}{(i\Omega+\tau^{-1})^2+\Omega_0^2}\Big\{\Omega_0M_0h_{\Omega,x}+(i\Omega+\tau^{-1})(M_{0y}h_{\Omega,z}-M_{0z}h_{\Omega,y})-\frac{gM_{0x}}{2\pi}\int_{-\infty}^{\infty}h^2(t')e^{i\Omega t'}dt'\Big\};$$

$$\Delta=\Omega/c. \qquad (18)$$

On noting that the last term in (18) is of order $\Omega h^2/\omega$ and therefore makes an unimportant contribution, we get

$$B_1(x)=\int_0^x b_x(y)e^{i\Delta y}dy$$

$$=\frac{gc\tau}{i\Omega}(e^{-i\Omega x/c}-1)\frac{\Omega_0M_0h_{\Omega,x}+(i\Omega+\tau^{-1})(M_{0y}h_{\Omega,z}-M_{0z}h_{\Omega,y})}{(i\Omega+\tau^{-1})^2+\Omega_0^2},$$

$$|B_1|=\frac{2gc\tau}{\Omega}\left|\frac{\Omega_0M_0h_{\Omega,x}+(i\Omega+\tau^{-1})(M_{0y}h_{\Omega,z}-M_{0z}h_{\Omega,y})}{(i\Omega+\tau^{-1})^2+\Omega_0^2}\sin\frac{\Omega}{2c}x\right|. \qquad (19)$$

Thus in the case of a uniform precession of the magnetic moment, the process of transformation of light waves is periodic in x with period $2\pi c/\Omega$; the maximum number of waves excited at combination frequencies is

$$N_{max}\approx v_0\frac{4gc\tau}{\Omega}\left|\frac{\Omega_0M_0h_{\Omega,x}+(i\Omega+\tau^{-1})(M_{0y}h_{\Omega,z}-M_{0z}h_{\Omega,y})}{(i\Omega+\tau^{-1})^2+\Omega_0^2}\right|$$

If the wave vector of the spin wave is different from zero ($\kappa \neq 0$), then in order to find b_x it is necessary to solve Eq. (12). On substituting (16) into (12), to terms of order $O(\Omega h^2/\omega)$, we get

$$(\partial/\partial x+\Gamma)b_x=0,$$

that is,

$$b_x=b_{0x}e^{-\Gamma x},$$

and consequently

$$B_1(x)=\frac{b_{0x}}{i\Delta-\Gamma}(e^{(i\Delta-\Gamma)x}-1). \qquad (20)$$

When $x \ll \Gamma^{-1}$ and $\Delta \neq 0$, the expression (20) coincides in form with (19). In this case, $B_1(x)$ and the process of transformation of the waves are periodic with period $2\pi/\Delta$. When $\Delta = 0$, the magnitude $|B_1|$ is an increasing function, and consequently the number of appreciably excited waves at combination frequencies grows with increase of the scattering distance x. When $x \gg \Gamma^{-1}$,

$$|B_1|=|b_{0x}|(\Delta^2+\Gamma^2)^{-1/2}.$$

Thus in this range, the number of appreciably excited electromagnetic waves does not change with increase of x:

$$N\approx v_0|b_{0x}|(\Delta^2+\Gamma^2)^{-1/2}.$$

4. In closing, we shall consider a specific example of scattering in which the formulas obtained take the simplest possible form and the effect of interaction of the waves is greatest. Let $\mathbf{M}_0$ be directed along the z axis, $\boldsymbol{\kappa}$ and $\mathbf{k}_0$ along the x axis; $\kappa = \Omega/c$. In this case $\Delta = 0$, and the effect of the light wave on the spin wave in this approximation is absent. Since the waves are connected in resonance fashion, $B_1(x)$ is a monotonically increasing function:

$$|B_1|=\frac{b_{0x}}{\Gamma}(1-e^{-\Gamma x}).$$

In the initial stage, the number of combination electromagnetic waves increases, as $v_0|b_{0x}|x$. When $x \gg \Gamma^{-1}$, the number of excited waves approaches the limit $N = v_0|b_{0x}|\Gamma^{-1}$.

The results can be easily generalized to the case of several sublattices.

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