SOVIET PHYSICS JETP VOLUME 29, NUMBER 2 AUGUST, 1969

CYCLOTRON WAVES IN SEMIMETALS

N. V. BROVTSYNA and V. G. SKOBOV

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

Submitted September 11, 1968

Zh. Eksp. Teor. Fiz. 56, 694–708 (February, 1969)

We describe a theoretical investigation of the electromagnetic properties of semimetals in the vicinity of cyclotron resonance. It is shown that the spatial inhomogeneity of the high-frequency field can play a very important role even in the frequency region corresponding to the normal skin effect in the absence of a magnetic field. Non-local effects near resonance cause the dielectric constant of the electron-hole plasma to become positive, and an "ordinary" cyclotron wave can exist in the crystal. The properties of such waves are investigated. It is shown that their presence leads to singularities in the semimetal.

1. INTRODUCTION

CYCLOTRON resonance in metals was theoretically predicted by Azbel' and Kaner[1] and first observed experimentally by Fawcett[2]. It is presently one of the main methods of investigating the properties of conduction electrons in metal. As is well known, resonance takes place when a constant magnetic field **H** is parallel to the surface of the sample, and the frequency of the high frequency field ω is a multiple of the cyclotron frequency of the carriers Ω (for metals with a nonquadratic dispersion law—to the extremal value of the cyclotron frequency Ω_{ext}). The increase of the high-frequency conductivity as $\Omega \to \omega/n$ (n—integer) leads to resonant maxima of the reflection coefficients and to minima of the high-frequency impedance (surface impedance). In view of the high conductivity of the metals, it is usually assumed that the characteristic distance over which the electromagnetic field attenuaates is much smaller than the dimensions of the electron orbits, i.e., that the cyclotron resonance takes place under the conditions of the anomalous skin effect.

Kaner and one of the authors[3] called attention to the fact that in the vicinity of the resonance the dielectric constant of the electron gas becomes real and positive, and cyclotron waves may propagate in a metal, with a wavelength much smaller than the Larmor radius of the electron orbit $R = v/\Omega$ (v—characteristic velocity on the Fermi surface).

Subsequently Walsh and Platzman[4] have shown that beside the short-wave branch ($kR \gg 1$, where k is the wave number), there exists also a long-wave branch ($kR \ll 1$), which has anomalous dispersion. The transfer of the dispersion extracted from the waves changes in the region $kR > 1$, where the wave length becomes smaller than the Larmor radius R. The long-wave branch is excited much more weakly than the short-wave branch responsible for the cyclotron-resonance line. However, the attenuation length of the long-wave branch turns out to be so large that standing waves are produced in thin metal plates and lead to a fine structure of the resonance line.

In bismuth, the carrier density is smaller by five orders of magnitude than in typical metals. Accordingly, the characteristic distance over which electromagnetic field changes appreciably is much larger. In the microwave region of frequencies, the effective range $l^* = v/\omega$ (the high frequency field accelerates the carriers within a time on the order of $1/\omega$) becomes smaller than the skin depth of the plasma

$$\delta_0 = c/\omega_0, \tag{1.1}$$

where

$$\omega_0^2 = \sum_j \omega_{0j}^2, \quad \omega_{0j}^2 = \frac{4\pi e^2 N_j}{m_j}, \tag{1.2}$$

ω_{0j} is the plasma frequency of the carriers of group j, and N_j and m_j are their densities and effective masses.

If the inhomogeneity of the high frequency field over distances on the order of l^* is small, then the nonlocal effects do not play any role and the skin effect is normal (classical). In the absence of a constant magnetic field, the dielectric constant of an electron-hole plasma is negative:

$$\varepsilon = -\omega_0^2/\omega^2 \tag{1.3}$$

and the field actually attenuates over a distance δ_0 from the surface. In other words, the condition of the normal skin effect at $H = 0$ is given by

$$l^* \ll \delta_0. \tag{1.4}$$

In the region of cyclotron resonance, where $\Omega \sim \omega$, the characteristic radius of the carrier orbits is of the order of l^*:

$$R \sim l^*. \tag{1.5}$$

It can therefore be assumed that in the limiting case (1.4) the spatial inhomogeneity of the high frequency field on the trajectory of the electron is small, and consequently, local theory is applicable. Cyclotron resonance in bismuth under the conditions of the normal skin effect was investigated by Smith, Hebel, and Buchsbaum[5]. They reached the conclusion that in the local limit cyclotron resonance of one definite group of carriers is impossible. The resonance is screened by a longitudinal depolarizing field[8], which couples strongly the longitudinal and transverse degrees of freedom of the plasma and shifts the resonant frequencies away from the cyclotron frequencies of the electrons and the holes. The resonant frequencies are hybrid and include the cyclotron masses of all the carrier groups. On the other hand, in the region of true cyclotron resonance, $\Omega \to \omega$, the intrinsic polarization of the electromagnetic field changes in such a manner that it becomes strictly orthogonal to that necessary for the excitation of the resonance. As a result, the effective dielectric constant for the intrinsic polarizations has no singularities at $\Omega \to \omega$. Resonance at $\Omega \to \omega$ is possible only at such directions of the magnetic field **H**, for which the cyclotron masses of two or three non-equivalent electron groups are the same, but their orbits are inclined at different angles to the magnetic field. Such a resonance is called "cyclotron resonance on inclined orbits"[5].

Actually, besides the hybrid resonances and resonances on inclined orbits, the experimental curves of[5] reveal the presence of maxima corresponding to cyclotron resonance of individual carrier groups. This fact offers evidence of the nonapplicability of the local theory to the analysis of cyclotron resonance. Subsequently Hebel has shown[7] that a small but finite spatial inhomogeneity of the field gives rise to a resonant term in the dielectric constant and in the absorption coefficient. Hebel considered cyclotron resonance of holes for the polarization of the electric field **E** along a constant magnetic field **H** ("ordinary wave"). He confined himself to the case of relatively small hole free path l, when the resonant part of the dielectric constant due to the nonlocal effect is a small resonant addition to the main local term. In this case the propagation of the wave in the crystal is impossible, as before, since the dielectric constant remains negative.

It is shown in the present paper that nonlocal effects can play a very important role if the carrier mean free path is large. In the direct vicinity of the cyclotron resonances, the dielectric constant along the constant magnetic field becomes positive. As a result, an "ordinary" cyclotron wave can propagate in the semimetal. We shall also show below that the existence of a wave in narrow frequency intervals leads to singularities in the reflection and in the absorption coefficients.

2. DISPERSION EQUATION AND CONDUCTIVITY TENSOR

We are interested in the properties of electromagnetic waves that can propagate in the volume of a semimetal. The influence of the surface of the sample on the field distribution near the surface will not be considered. We therefore consider first the propagation of waves in an unbounded electron-hole plasma, and then study the singularities of the surface impedance of a semimetal, due to the presence of these waves.

The electromagnetic field in a crystal is described by Maxwell's equations, which can be written, after eliminating the high-frequency magnetic field, in the form

$$-\operatorname{rot}\operatorname{rot}\mathbf{E}(\mathbf{r},t)=\frac{4\pi}{c}\frac{\partial\mathbf{j}(\mathbf{r},t)}{\partial t}+\frac{\varepsilon_0}{c^2}\frac{\partial^2\mathbf{E}(\mathbf{r},t)}{\partial t^2}, \qquad (2.1)$$

where $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{j}(\mathbf{r}, t)$ are the electric field and the conduction-current density at the point with radius vector **r** at the instant t, ϵ_0 is the dielectric constant of the lattice. The second term in the right side of (2.1) is connected with the displacement current. At frequencies ω much smaller than the plasma frequency ω_0, this current is small compared with the conduction current. Therefore the displacement current can be neglected.

The conduction current density is proportional to the electric field. However, in the presence of nonlocal effects, the connection between them is integral: the value of the current density at the point **r** at the instant t is determined by the value of the electric field in a certain region of states in the preceding instants of time. If the medium is homogeneous, then the kernel of the integral operator is of the difference type. It follows therefore that in the Fourier representation the connection between the current density and the field becomes algebraic:

$$j_\alpha(\mathbf{k},\omega)=\sigma_{\alpha\beta}(\mathbf{k},\omega)E_\beta(\mathbf{k},\omega), \qquad (2.2)$$

where **k** is the wave vector and ω the frequency; $j_\alpha(\mathbf{k}, \omega)$, $E_\beta(\mathbf{k}, \omega)$, and $\sigma_{\alpha\beta}(\mathbf{k}, \omega)$ are the Fourier components of the current density, the electric field, and the conduction-tensor elements; summation over the repeated vector indices β is implied.

The dependence of the elements of the tensor $\sigma_{\alpha\beta}$ on the frequency is a consequence of the retardation effects. Nonlocal effects become manifest in the dependence of $\sigma_{\alpha\beta}$ on the wave vector **k**. We note that in the local limit $\sigma_{\alpha\beta}$ does not depend on **k**.

For an unbounded medium, the field equation (2.1) now takes the form

$$[\mathbf{k}[\mathbf{kE}]]+\frac{\omega^2}{c^2}\hat{\varepsilon}(\mathbf{k},\omega)\mathbf{E}=0, \qquad (2.3)^*$$

where the elements of the dielectric tensor are

$$\varepsilon_{\alpha\beta}(\mathbf{k},\omega)=\varepsilon_0\delta_{\alpha\beta}+\frac{4\pi i}{\omega}\sigma_{\alpha\beta}(\mathbf{k},\omega) \qquad (\alpha,\beta=x,y,z). \qquad (2.4)$$

By equating to zero the determinant of the system of the three homogeneous equations (2.3) we obtain the dispersion equation that determines the properties of the electromagnetic waves.

We are interested in frequencies ω much lower than the plasma frequency ω_0. At such frequencies, the displacement current is small compared with the conduction current, and the dielectric permeability of the lattice ϵ_0 in (2.4) can be neglected. Consequently, the properties of the electromagnetic waves are determined completely by the elements of the tensor $\sigma_{\alpha\beta}$. An explicit form can be obtained with the aid of the kinetic Boltzmann equation for the distribution function of the carriers in a constant magnetic field and in the field of a plane monochromatic wave with wave vector **k** at frequency ω. The general expression for $\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H})$ in the case of a complicated prior energy spectrum was obtained by Azbel' and Kaner[1]. Before we present it, we shall discuss briefly the dynamics of the carriers in a constant magnetic field[8].

We consider a conduction electron with an arbitrary dispersion law $\epsilon = \epsilon(\mathbf{p})$, where ϵ is its energy and **p** the momentum. The electron velocity is

$$\mathbf{v}(\mathbf{p})=\partial\varepsilon(\mathbf{p})/\partial\mathbf{p}. \qquad (2.5)$$

*$[\mathbf{kE}] \equiv \mathbf{k}\times\mathbf{E}$

In the presence of a magnetic field **H**, the electron executes a very complicated motion, and the transverse components of its momentum are not conserved. Therefore it is convenient to describe the state of the electron with the aid of two integrals of motion ϵ and p_z ($z \parallel \mathbf{H}$), and the phase of the periodic motion φ. The phase φ determines the position of the electron on its orbits, and varies within one revolution from zero to 2π.

Using these variables, we can write the conduction tensor in the form

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H}) = \sum \frac{2e^2}{(2\pi\hbar)^3} \int \delta(\varepsilon - \varepsilon_F) d\varepsilon \int \frac{m}{\Omega} dp_z \times \int_0^{2\pi} v_\alpha(\varepsilon, p_z, \varphi) d\varphi \int_{-\infty}^{\varphi} v_\beta(\varepsilon, p_z, \varphi') d\varphi' \times \exp\left\{\frac{1}{\Omega} \int_{\varphi}^{\varphi'} [\nu - i\omega + i\mathbf{k}\mathbf{v}(\varepsilon\ p_z, \varphi'')] d\varphi''\right\} \quad (2.6)$$

where e is the absolute value of the electron charge, $2\pi\hbar$ is Planck's constant, ϵ_F is the Fermi energy

$$\Omega(\varepsilon, p_z) = eH / m(\varepsilon, p_z) c \quad (2.7)$$

is the cyclotron frequency,

$$m(\varepsilon, p_z) = \frac{1}{2\pi} \frac{\partial S(\varepsilon, p_z)}{\partial \varepsilon} \quad (2.8)$$

is the cyclotron effective mass, $S(\epsilon, p_z)$ is the area of the intersection of the constant-energy surface $\epsilon(\mathbf{p}) = \epsilon$ with the plane p_z = const, and $v_\alpha(\epsilon, p_z, \varphi)$ are projections of the velocity vector (2.5), in which p_x and p_y are expressed in terms of ϵ, p_z, and φ. The factor 2 preceding (2.6) takes into account two possible spin projections, the sign of the sum denotes summation of the contributions from different carrier groups.

In the present paper we consider cyclotron waves in bismuth in which the carrier dispersion law is well known. We used the Shoenberg ellipsoid model[9]. In this model, the electronic zone consists of three ellipsoids, slightly inclined to the basal plane (the plane perpendicular to the trigonal axis). The energy of the electrons of one of the ellipsoids is given by the formula

$$\varepsilon_a(\mathbf{p}) = \frac{1}{2m_0} (\alpha_1 p_1^2 + \alpha_2 p_2^2 + \alpha_3 p_3^2 + 2\alpha_4 p_2 p_3), \quad (2.9)$$

where m_0 is the mass of the free electron; p_1, p_2, and p_3 are the momentum components along the binary, bisector, and trigonal axes;

$$\alpha_1 = 197, \quad \alpha_2 = 1.64, \quad \alpha_3 = 81.1, \quad \alpha_4 = 9.41 \quad (2.10)$$

are the values of the elements of the reciprocal-mass tensor taken from[10]. This ellipsoid will be designated (a). The other two ellipsoids, (b) and (c), are obtained by rotating ellipsoid (a) through ±120° around the trigonal axis.

The hole band is an ellipsoid of revolution, the axis of which is parallel to the trigonal axis. The hole energy is

$$\varepsilon(\mathbf{p}) = \frac{p_1^2}{2M_1} + \frac{p_2^2}{2M_1} + \frac{p_3^2}{2M_3}, \quad (2.11)$$

where, according to[10], $M_1 = 0.06m_0$ and $M_3 = 0.55m_0$.

Let us consider first the cyclotron resonance of the holes in the case when the field **H** is directed along the bisector axis and the vector **k** along the binary axis (y axis). The x axis in this coordinate system is parallel to the trigonal axis. In this case the cyclotron hole mass

$$m = \sqrt{M_1 M_3} \quad (2.12)$$

is approximately one fifth of m_0, and the cyclotron electron masses are

$$m_a = m_0 / \sqrt{\alpha_1 \alpha_2} \sim 0.008 m_0, \quad (2.13)$$

$$m_b = m_c = m_a / \left|\cos\frac{2\pi}{3}\right| = 2m_a. \quad (2.14)$$

In view of the smallness of m_a, m_b, and m_c compared with m the diameters of the electron orbits are much smaller than those of the hole orbits, and the electrons feel the spatial inhomogeneity of the wave field much more weakly than the holes. Therefore, near the hole cyclotron resonance, the electronic part of the conductivity is practically independent of the wave number k. To the contrary, the nonlocal effects play an important role in the hole part of the conductivity $\sigma^{(h)}_{\alpha\beta}$. Let us examine $\sigma^{(h)}_{\alpha\beta}$ in greater detail.

It follows from (2.11) and (2.5) that the hole velocity components v_α can be written in the form

$$v_x(\varepsilon, p_z, \varphi) = \left[\frac{2}{M_3}\left(\varepsilon - \frac{p_z^2}{2M_1}\right)\right]^{1/2} \sin\varphi,$$

$$v_y(\varepsilon, p_z, \varphi) = \left[\frac{2}{M_1}\left(\varepsilon - \frac{p_z^2}{2M_1}\right)\right]^{1/2} \cos\varphi,$$

$$v_z(\varepsilon, p_z, \varphi) = p_z / M_1. \quad (2.15)$$

We substitute these expresions in (2.6) and integrate with respect to φ'':

$$\frac{1}{\Omega} \int_{\varphi}^{\varphi'} \mathbf{kv}(\varphi'') d\varphi'' = \frac{k}{\Omega} \int_{\varphi}^{\varphi'} v_y(\varphi'') d\varphi'' = \varkappa(\sin\varphi' - \sin\varphi), \quad (2.16)$$

where

$$\varkappa = \frac{kc}{eH}\left[2M_3\left(\varepsilon - \frac{p_z^2}{2M_1}\right)\right]^{1/2}. \quad (2.17)$$

The product of the velocity $v_\alpha(\varphi)$ by the phase factor $\exp(-i\kappa \sin\varphi)$, describing the variation of the wave field along the carrier trajectory, is a periodic function of φ with period 2π. Consequently, it can be expanded in a Fourier series

$$v_\alpha(\varphi) \exp(-i\varkappa \sin\varphi) = \sum_{n=-\infty}^{\infty} w_{\alpha n} e^{-in\varphi}, \quad (2.18)$$

$$w_{\alpha n} = \frac{1}{2\pi} \int_0^{2\pi} \exp\{i(n\varphi - \varkappa \sin\varphi)\} d\varphi. \quad (2.19)$$

We now substitute the expansion (2.18) in the expression for the conductivity tensor and integrate with respect to φ and φ'. Noting that the integration with respect to ϵ in (2.6) reduces to a substitution of ϵ by ϵ_F, and introducing in lieu of p_z the dimensionless variable

$$\mu = p_z / \sqrt{2M_1 \varepsilon}, \quad (2.20)$$

we obtain

$$\sigma_{\alpha\beta}^{(h)} = \frac{3Ne^2}{4\varepsilon_F} \int_{-1}^{+1} d\mu \sum_{n=-\infty}^{+\infty} \frac{w_{\alpha n}(\mu) w_{\beta n}^*(\mu)}{\nu + i(n\Omega - \omega)}, \quad (2.21)$$

where N is the hole density

$$N = \frac{2M_1 \varepsilon_F}{3\pi^2 \hbar^3} (2M_3 \varepsilon_F)^{1/2}, \quad (2.22)$$

and the components of the complex vector w depend on n and μ in accordance with formulas (2.19) and (2.17). To obtain the explicit form of $w_{\alpha n}(\mu)$ it is necessary to substitute (2.15) in (2.19). This yields

$$w_{xn}(\mu) = \frac{i}{2}\{J_{n-1}[\varkappa(\mu)] - J_{n+1}[\varkappa(\mu)]\}\left[\frac{2\varepsilon_F}{M_3}(1-\mu^2)\right]^{1/2}, \quad (2.23a)$$

$$w_{yn}(\mu) = \frac{1}{2}\{J_{n-1}[\varkappa(\mu)] - J_{n+1}[\varkappa(\mu)]\}\left[\frac{2\varepsilon_F}{M_1}(1-\mu^2)\right]^{1/2}, \quad (2.23b)$$

$$w_{zn}(\mu) = J_n[\varkappa(\mu)]\left(\frac{2\varepsilon_F}{M_1}\right)^{1/2}\mu.$$

Obviously, w_z is an odd function of μ, while w_x and w_y are even. Consequently, the non-diagonal elements are

$$\sigma_{xz}^{(h)} = \sigma_{zx}^{(h)} = \sigma_{yz}^{(h)} = \sigma_{zy}^{(h)} = 0. \quad (2.24)$$

The remaining elements of the conductivity tensor differ from zero. Their explicit dependence on k, ω, and H is quite complicated and cannot be obtained in the general case. In the direct vicinity of the cyclotron resonance, where $\Omega \to \omega/n$, the principal role is played by the resonant term. This greatly simplifies the expression for $\sigma_{\alpha\beta}^{(h)}$, but the integrals

$$I_{\alpha\beta}{}^n(kR) = \int_{-1}^{+1} d\mu w_{\alpha n}(\mu, kR) w_{\beta n}{}^*(\mu, kR), \quad (2.25)$$

$$R = (c/eH)\sqrt{2M_3\varepsilon_F}, \quad (2.26)$$

are expressed in terms of elementary functions only in the limiting cases of small or large values of kR. In the intermediate region, these integrals are represented in the form of a series in powers of $(kR)^2$, and the dispersion equation must be solved numerically.

We now turn to consider the electronic part of the conductivity. It was noted above that the dimensions of the electron orbits are small compared with the dimensions of the hole orbits. Therefore in the region of the hole resonance the electrons practically do not feel the spatial inhomogeneity of the wave field and there are no nonlocal effects in the electronic part of the conductivity. In other words, in the calculation of $\sigma_{\alpha\beta}^{(h)}$ we confine ourselves to the local limit $k \to 0$.

The contribution of the electrons of ellipsoid (a) to the conductivity along the magnetic field H is

$$\sigma_{zz}^{(a)} = \frac{N_a e^2}{m_{zz}^{(a)}(\nu_e - i\omega)}, \quad (2.27)$$

where the longitudinal effective mass is

$$m_{zz}^{(a)} = m_0(\alpha_2 - \alpha_4^2/\alpha_3)^{-1}. \quad (2.28)$$

The expressions for $\sigma_{zz}^{(b)}$ and $\sigma_{zz}^{(c)}$ are similar. The masses $m_{zz}^{(b)}$ as $m_{zz}^{(c)}$ can be obtained from the tensor $m_{\alpha\beta}^{(a)}$ by rotation through $\pm 120°$ around the trigonal axis (the x axis). It is easy to show that

$$m_{zz}^{(b)} = m_{zz}^{(c)} = m_{zz}^{(a)}\cos^2\frac{\pi}{3} + m_{yy}^{(a)}\sin^2\frac{\pi}{3}. \quad (2.29)$$

$m_{zz}^{(h)}$ is of the order of m_0, while $m_{yy}^{(h)} = m_0/\alpha_1$ is smaller by two orders of magnitude. We can therefore neglect the second term in (2.29) and we obtain

$$m_{zz}^{(b)} = m_{zz}^{(c)} \approx {}^1/_4\, m_{zz}^{(a)}. \quad (2.30)$$

Thus, the contribution of the electrons of all the ellipsoids to the longitudinal conductivity is

$$\sigma_{zz}^{(e)} \approx \frac{3Ne^2}{m_e(\nu_e - i\omega)} \qquad m_e = \frac{1}{3}m_{zz}^{(a)}, \quad N = 3N_a. \quad (2.31)$$

It should be noted that owing to a certain inclination of the electron ellipsoids ($\alpha_4 \neq 0$), the nondiagonal elements $\sigma_{zy}^{(e)} = -\sigma_{yz}^{(e)}$ differ from zero. They cause a coupling of the ordinary wave ($\mathbf{E} \parallel z$) with the extraordinary and longitudinal waves. This coupling, however, is quite weak (the coupling coefficient is proportional to α_4^2/α_3^2), and will not be taken into account. It then follows from (2.3) and (2.4) that the dispersion equation of the ordinary wave ($\mathbf{E} \parallel z$) is of the form

$$k^2c^2 = \omega^2\varepsilon_{zz} = 4\pi i\omega(\sigma_{zz}^{(e)} + \sigma_{zz}^{(h)}), \quad (2.32)$$

where $\sigma_{zz}^{(e)}$ and $\sigma_{zz}^{(h)}$ are determined by expression (2.31), (2.21), and (2.23c).

3. PROPERTIES OF ORDINARY WAVES

As already mentioned, we are considering in this paper hole cyclotron resonance in bismuth under conditions when the maximum radius of the hole orbit is smaller than the plasma depth of the skin layer: $R < \delta_0$. It is therefore natural to begin the analysis of the wave properties with the long-wave limit

$$kR \ll 1. \quad (3.1)$$

In this wavelength region, the integrals $I_{zz}^{(n)}$ can be calculated, and the expressions for $\sigma_{zz}^{(h)}$ becomes much simpler. In fact, the function w_{zn} can be expanded in powers of κ^2 and only the first few terms retained. Even in the form of the Bessel functions at small values of the argument, we can easily verify that the integrals $I_{zz}^{(n)}$ behave asymptotically like $(kR)^{2n}$. This means that the largest amplitude is that of the zero harmonic

$$I_{zz}^{(0)} = \frac{4\varepsilon_F}{3M_1}\left[1 - \frac{1}{5}(kR)^2 + \ldots\right]. \quad (3.2)$$

The first term in the right side corresponds to the local limit $k \to 0$, and the second represents a small nonlocal correction. The existence of the remaining terms is due completely to nonlocal effects, and their amplitudes $I_n^{(n)}$ are small compared with (3.2). Nevertheless, near the corresponding resonances they can play a very important role. Thus, the amplitudes of the fundamental resonance

$$I_{zz}^{(1)} = \frac{4\varepsilon_F}{3M_1}\left[\frac{1}{10}(kR)^2 - \frac{1}{70}(kR)^4 + \ldots\right] \quad (3.3)$$

contains the small factor $(kR)^2$. At a large hole mean free path ($\nu \ll \omega$), however, this moment can be compensated for by the resonant factor

$$\left(\frac{\Omega}{\omega} - 1 - i\frac{\nu}{\omega}\right)^{-1}, \quad (3.4)$$

the magnitude of which in the direct vicinity of the resonance becomes very large.

Thus, the dielectric constant ϵ_{zz} near the fundamental hole resonance can be written in the form

$$\varepsilon_{zz} = -\frac{\omega_0^2}{\omega^2}\left\{1 - \frac{\eta}{\Delta - i\gamma}\left[\frac{(kR)^2}{10} - \frac{(kR)^4}{70}\right]\right\}, \quad (3.5)$$

where

$$\omega_0^2 = 4\pi Ne^2\left(\frac{1}{m_e} + \frac{1}{M_1}\right), \quad \eta = \frac{m_e}{m_e + M_1}, \quad (3.6)$$

$$\Delta = \frac{\Omega}{\omega} - 1, \quad \gamma = \frac{\nu}{\omega}. \tag{3.7}$$

We note that for the considered orientation of the magnetic field, the effective electron mass m_e is larger by one order of magnitude than the hole mass M_1. Therefore the coefficient η is close to unity and the dielectric constant ϵ_{zz} is determined almost completely by the holes.

We now substitute (3.5) into the dispersion equation (2.32) and introduce a new variable

$$u = (kR)^2. \tag{3.8}$$

Then the dispersion equation takes the form

$$\frac{u}{\xi} = -1 + \frac{\eta}{\Delta - i\gamma}\left(\frac{u}{10} - \frac{u^2}{70}\right), \tag{3.9}$$

where

$$\xi = (R/\delta_0)^2, \quad \delta_0 = c/\omega_0. \tag{3.10}$$

The unity in the right side of (3.9) corresponds to the local limit, while the second term describes the cyclotron resonance. It is obvious that it is completely due to the spatial inhomogeneity of the wave field (the resonant term vanishes when $u \to 0$). This has a simple physical meaning. In fact, the homogeneous high-frequency field **E**, which is parallel to the constant magnetic field, on the average does not accelerate the carriers, and their motion in this direction does not depend on the value of **H**. Therefore in the local limit there is no cyclotron resonance. The situation is different when the high-frequency electric field **E** is inhomogeneous in a direction perpendicular to the vector **H**. As a result of this inhomogeneity, the phase of the alternating field, within the limits of the cyclotron orbit, changes by an amount equal to 2π. Now the actions of the electric field on the particle in opposite sections of its orbit no longer cancel each other completely, and when the cyclotron frequency of the carriers equals the frequency of the wave field, resonant acceleration of the carriers is possible.

Let us investigate the solutions of the dispersion equation (3.9) in various cases.

1. The hole mean free path is not very large and γ satisfies the inequalities

$$\xi < \gamma \ll 1. \tag{3.11}$$

In this case the resonant term in the right side of the inequality (3.9) is small compared with unity, and we obtain Hebel's result[7]:

$$u \approx -\xi\left(1 + \frac{\xi}{10}\frac{\eta}{\Delta - i\gamma}\right). \tag{3.12}$$

The real part of the right side of (3.12) is negative and wave propagation is impossible. The cyclotron resonance consists of the appearance of a small resonant term, which slightly changes the depth of the skin layer.

2. Of greatest interest is the limiting case of large mean free paths:

$$\gamma \ll \xi^{3/2}. \tag{3.13}$$

At small positive Δ, the resonant term in (3.9) now becomes larger than unity, and the dielectric constant is positive. A weakly damped cyclotron wave can propagate in the frequency interval defined by the condition $\Delta < \eta\xi/10$.

The experiment is usually performed with a fixed field frequency ω, so that the dispersion equation should be solved with respect to the complex wave number, assuming ω to be real. The solutions of the quadratic equation (3.9) are

$$u_{1,2} = \frac{7}{2}\left\{1 - \frac{\Delta}{\Delta_0} + i\frac{\gamma}{\Delta_0} \mp \left[\left(1 - \frac{\Delta}{\Delta_0} + i\frac{\gamma}{\Delta_0}\right)^2 - \frac{40}{7\eta}(\Delta - i\gamma)\right]^{1/2}\right\}, \tag{3.14}$$

where

$$\Delta_0 = \eta\xi/10. \tag{3.15}$$

In the vicinity of the resonance, where

$$\Delta \gg \gamma, \quad 1 - \Delta/\Delta_0 \gg \sqrt{\Delta_0}, \tag{3.16}$$

the second term in the square brackets in (3.14) is much smaller than the first, and the solutions (3.14) can be approximately represented in the form

$$u_1 \approx \frac{10}{\eta}\left(\frac{1}{\Delta} - \frac{1}{\Delta_0} + i\frac{\gamma}{\Delta^2}\right)^{-1}, \tag{3.17}$$

$$u_2 \approx 7\left(1 - \frac{\Delta}{\Delta_0} + i\frac{\gamma}{\Delta_0}\right). \tag{3.18}$$

The imaginary part of u_1 is negative, and therefore, when finding the wave number k_1, it is necessary to take the square root with the minus sign, i.e.,

$$k_1 \equiv k_1' + ik_1'' \approx -\frac{1}{R}\left(\frac{10}{\eta}\right)^{1/2}\left(\frac{1}{\Delta} - \frac{1}{\Delta_0}\right)^{-1/2}\left[1 - i\frac{\gamma}{2\Delta}\left(1 - \frac{\Delta}{\Delta_0}\right)^{-1}\right]. \tag{3.19}$$

We note that by virtue of the inequalities (3.13) and (3.16), the relative attenuation of this solution is small: $k_1'' \ll |k_1'|$.

Since $k_1' < 0$, the phase and group velocities of the wave have opposite directions. This means that the solution k_1 is a branch with anomalous dispersion. To the contrary, the second branch has normal dispersion:

$$k_2 \equiv k_2' + ik_2'' \approx \frac{\sqrt{7}}{R}\left\{\left(1 - \frac{\Delta}{\Delta_0}\right)^{1/2} + i\frac{\gamma}{2\Delta_0}\left(1 - \frac{\Delta}{\Delta_0}\right)^{-1/2}\right\}. \tag{3.20}$$

It must be emphasized that expressions (3.18) and (3.20) for u_2 and k_2 are applicable only at those values of Δ, for which $u_2 < 1$. This is connected with the fact that the expression used by us for the dielectric constant is valid only in the long-wave region (3.1). To obtain the true form of the second branch in the short-wave region it is necessary to know the behavior of ϵ_{zz} at $u \gg 1$.

It follows from (3.19)–(3.20) that when $\Delta \ll \Delta_0$ the solution u_1 is small, while u_2 is large. With increasing Δ, u_1 increases and u_2 decreases. As Δ approaches Δ_0, the radicand in (3.14) decreases and both solutions come close together. The damping of both branches increase, particularly that of the first branch. When

$$\Delta \to \Delta_m \equiv \Delta_0\left[1 - \left(\frac{40}{7\eta}\Delta_0\right)^{1/2}\right] \tag{3.21}$$

the solutions u_1 and u_2 differ only in the sign of the imaginary part:

$$u_{1,2} \approx \sqrt{7\xi}\left[1 \mp i\left(\frac{5\gamma}{\eta}\right)^{1/2}\left(\frac{7}{\xi^3}\right)^{1/4}\right]. \tag{3.22}$$

This means that at this point both branches merge and the dispersion curve of the cyclotron wave has a minimum. The damping of the wave reaches here its maximum value, but by virtue of the inequality (3.13) the

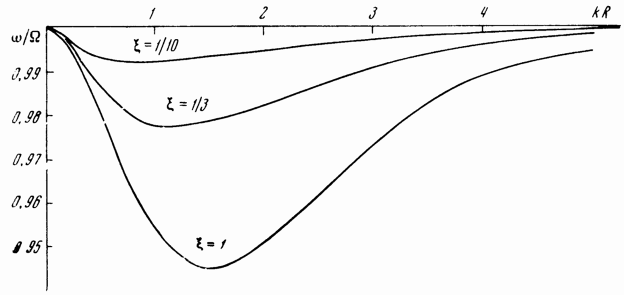

FIG. 1. Spectral curves of the ordinary cyclotron wave for values of ξ equal to 1/10, 1/3, and $l \to \infty$.

relative damping k''/k' still remains small. The corresponding values of the damping length L, over which the wave amplitude changes by a factor e, is proportional to the square root of the hole mean free path $l = v/\nu$:

$$L_{\min} \equiv \frac{1}{k''_{\max}} = R\left(\frac{4\eta}{35}\frac{Rl}{\delta_0^2}\right)^{1/2}. \tag{3.23}$$

The spectrum of the ordinary wave is shown in Fig. 1. The curves were obtained by numerically solving the equation

$$u = \xi\left\{-F_0(u) + \left(\frac{1}{\Delta} - \frac{1}{2}\right)F_1(u)\right\}, \tag{3.24}$$

where

$$F_n(u) = 3\int_0^1 J_n^2[\sqrt{u(1-\mu^2)}]\,\mu^2\,d\mu. \tag{3.25}$$

When $\Delta > \Delta_0$, the solutions of the dispersion equation (3.14) are negative, and the corresponding wave numbers $k_{1,2}$ are imaginary. The propagation of the extraordinary waves in this region of frequencies turns out to be impossible (the dispersion curve lies in the interval $0 < \Delta \le \Delta_m$).

The dependence of the wave damping length L on Δ is shown in Fig. 2. L decreases in the vicinity of the points $\Delta = 0$ and $\Delta = \Delta_m$, when the character of the dispersion changes.

We shall now stop to discuss the question of the limits of applicability of the obtained results.

The most stringent requirement is that the relative damping be small at the minimum of the spectral curve. This requirement can be represented in the form of the inequalities

$$1 \gg R/\delta_0 \gg 2\pi(\delta_0/l)^{1/2}. \tag{3.26}$$

The first of them denotes that the characteristic kR are small, justifying by the same token the long-wave approximation (3.1). The second inequality represents the condition of smallness of the relative damping of the wave. These inequalities limit the region of values of the magnetic field at which the properties of the waves are described by the expressions given above. If the second inequality is violated, then the damping becomes large when $\Delta \to \Delta_m$ and the wave vanishes.

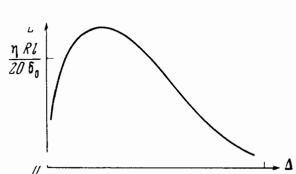

FIG. 2. Dependence of the damping length of the long-wave branch of the cyclotron wave on the value of Δ.

Nonetheless, it remains weakly damping in the region $\gamma \ll \Delta < \Delta_m$, if the less stringent conditions

$$1 \gg R/\delta_0 \gg 2\pi\delta_0/l. \tag{3.27}$$

are satisfied. The spectrum and damping of the waves in this region are described by expressions (3.19) and (3.20).

We have seen above that in the case $R < \delta_0$ the spectrum of the cyclotron wave exists both in the region of long waves (3.1) and in the region of short waves $kR > 1$. On the other hand, Walsh and Platzman[11] have shown that in alkali metals, which usually have a very large ratio R/δ_0, the dispersion curve of the cyclotron wave also begins with $k = 0$ and goes on the region of large k. It is quite clear that the weakly damped waves exist also in the intermediate region, where

$$R \gtrsim \delta_0, \quad R \ll l. \tag{3.28}$$

It is apparently easiest to experimentally observe the cyclotron waves in bismuth precisely in this region of magnetic fields. Unfortunately, it is impossible here to exchange analytic expressions for the spectrum and for the damping of the wave, inasmuch as the long-wave approximation is less applicable. The wave spectrum should be obtained by numerically solving the dispersion equation. It is then necessary to take into account in the expression for the dielectric constant not only terms with $n = 0$ and $n = 1$, but also terms with large values of n. The dispersion equation at $\eta = 1$ and $\nu \to 0$ is of the form

$$\frac{u}{\xi} = -F_0(u) + 2\sum_{n=1} F_n(u)\left[\left(\frac{n\Omega}{\omega}\right)^2 - 1\right]^{-1}. \tag{3.29}$$

The results of the numerical solution of this equation for three different values of ξ are shown in Fig. 3.

4. WAVE EXCITATION. SURFACE IMPEDANCE

So far we have considered the properties of the cyclotron wave in an unbounded crystal. We now investigate the excitation of a wave by a high frequency external field in a semi-infinite semimetal. We confine ourselves to the limiting case (3.26) and disregard the changes of the conductivity operator resulting from collisions of the carrier with the surface. This can be done, since only a relatively small fraction of the car-

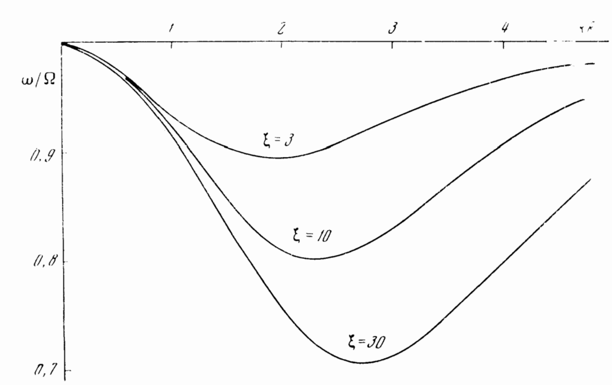

FIG. 3. Spectral curves of the cyclotron wave for ξ equal to 3, 10, and 30 as $l \to \infty$.

riers collides with the surface when $kR \ll 1$. Neglecting the contribution of such carriers, the dielectric constant ϵ_{zz} remains the same as in an unbounded crystal.

The distribution of the electromagnetic field in the semimetal is determined by the function

$$Z(y) = -4i\omega \int_{-\infty}^{+\infty} \frac{e^{iky}\,dk}{k^2c^2 - \omega^2\varepsilon_{zz}(k)}, \qquad (4.1)$$

the value of which on the surface $y = 0$ constitutes the high-frequency surface impedance $Z \equiv Z(0)$.

We substitute expression (3.5) for ϵ_{zz} in the integral (4.1), introduce a new integration variable $x = kR$, and transform the integral into

$$Z(y) = -\frac{4i\omega R}{c^2}\,\frac{70(\Delta - i\gamma)}{\eta\xi} \int_{-\infty}^{+\infty} \frac{dx}{(x^2 - u_1)(x^2 - u_2)} \exp\left(i\frac{y}{R}x\right), \qquad (4.2)$$

where the complex quantities $u_{1,2}$ are defined by (3.14).

We close the integration contour in the upper half of the complex plane and take the residues of the integrand of (4.2). As a result we obtain

$$Z(y) = \frac{4\pi\omega}{c^2}\,\frac{\Delta - i\gamma}{\Delta_0}\left[\left(1 - \frac{\Delta - i\gamma}{\Delta_0}\right)^2 - \frac{40}{7\eta}(\Delta - i\gamma)\right]^{-1/2} \times \left\{-\frac{1}{k_1}e^{ik_1y} + \frac{1}{k_2}e^{ik_2y}\right\} \quad (\mathrm{Re}\,k_1 < 0). \qquad (4.3)$$

The first term describes the field of the long-wave branch, which has anomalous dispersion, and the second the field of the short-wave branch. In the vicinity of resonance in the region (3.16), the wave numbers and attenuations of both branches are determined by expressions 3.19 and (3.20). At the corresponding values of the magnetic field, the first branch is excited much more strongly than the second ($|k_1| \ll |k_2|$). The field distribution has here the form

$$Z(y) \approx \frac{4\pi\omega\delta_0^2}{c^2R}\left(\frac{10}{\eta}\right)^{1/2}\left(\frac{1}{\Delta - i\gamma} - \frac{1}{\Delta_0}\right)^{-1/2} \times \exp\left\{-i\frac{y}{R}\left(\frac{10}{\eta}\right)^{1/2}\left(\frac{1}{\Delta - i\gamma} - \frac{1}{\Delta_0}\right)^{-1/2} - k_1''y\right\}. \qquad (4.4)$$

It follows from this formula that when $\Delta = 0$ the damping length is $L_0 \sim (Rl)^{1/2}$. In the region $\gamma \ll \Delta \ll \Delta_0$ the wave number and the amplitude of the wave increase like $\sqrt{\Delta}$, and the damping length is $L \sim l\sqrt{\Delta}$.

With further increase of Δ, the wave number of the first branch increases and that of the second decreases. The damping increases in both branches.

Finally, as $\Delta \to \Delta_m$ the wave numbers and the dampings of both waves, and also the corresponding amplitudes of the field, become equal. The field distribution takes on the form of a standing wave

$$Z_{max}(y) \approx (1 - i)\frac{4\pi\omega}{c^2}(R\delta_0 l^2)^{1/4}\left(\frac{\eta}{20}\right)^{1/2} \times \cos\left[\frac{7^{-1/4}y}{(R\delta_0)^{1/2}}\right]\exp\left(-\frac{y}{L_{min}}\right), \qquad (4.5)$$

where the damping length L_{min} is given by formula (3.23).

The field amplitude turns out to be proportional to the square root of the free path l. This means that in the limit as $l \to \infty$ the amplitude increases without limit. In other words, the presence of a minimum in the spectral curve of the cyclotron wave leads to singularities in the surface impedance of the semimetal. The dependence of the impedance Z' on the value of Δ is shown in Fig. 4.

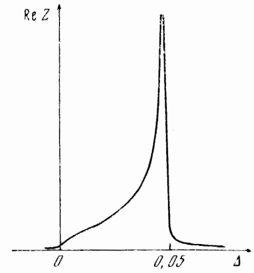

FIG. 4. Behavior of the real part of the surface impedance in the direct vicinity of the cyclotron resonance for values $\xi = 1$; $\gamma = 0.001$, and $Z_0 = 4\pi\omega\delta_0 c^{-2}$.

We have considered cyclotron waves whose properties are determined completely by the holes. Similar waves can exist in bismuth in the vicinity of the electron cyclotron frequencies. Electron cyclotron waves apparently are most conveniently observed when the magnetic field H is parallel to the binary axis C_2, and the normal to the crystal surface is parallel to the trigonal axis C_3. In this case the cyclotron mass of the electrons of ellipsoid (a) is $m_a = m_0(\alpha_2\alpha_3 - \alpha_4^2)^{-1/2}$, and the plasma frequency ω_0 is determined by the longitudinal effective mass m_0/α_1 and by the electron density $N_a = N/3$. At such an orientation of the vectors **k** and **H**, the cyclotron mass is minimal, and the value of the parameter ξ is maximal. In the frequency region $\omega \approx \Omega_a$ there should exist an electromagnetic wave due to the cyclotron resonance of the electrons of ellipsoid (a). On the other hand, the contribution of the remaining carriers to the dielectric constant of the semimetal plasma turns out to be small.

[1] M. Ya. Azbel' and É. A. Kaner, Zh. Eksp. Teor. Fiz. 32, 896 (1957) [Sov. Phys.-JETP 5, 730 (1957)]; Phys. Chem. Sol. 6, 113 (1958).

[2] E. Fawcett, Phys. Rev. 103, 1582 (1956).

[3] É. A. Kaner and V. G. Skobov, Fiz. Tverd. Tela 6, 1104 (1964) [Sov. Phys.-Solid State 6, 851 (1964/65).

[4] W. M. Walsh and P. M. Platzman, Phys. Rev. Lett. 15, 784 (1965); Proc. 10th Int. Conf. on Low Temp. Phys., Moscow, VINITI, 1966.

[5] G. E. Smith, L. C. Hebel, and S. J. Buchsbaum, Phys. Rev. 129, 154 (1963).

[6] P. W. Anderson, Phys. Rev. 100, 749 (1955).

[7] L. C. Hebel, Phys. Rev. 138, A1641 (1965).

[8] I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, Zh. Eksp. Teor. Fiz. 31, 63 (1956) [Sov. Phys.-JETP 4, 41 (1957).

Translated by J. G. Adashko
85