

## SUPERCONDUCTIVITY IN LAMELLAR SEMICONDUCTING STRUCTURES

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It is shown that longitudinal collective oscillations of the free carriers (of a "surface wave" type) exist on the interface between heavily doped n- and p-type semiconductors with identical work functions (a heterojunction) but with substantially different effective masses for the conduction electrons and holes ( $m_n \ll m_p$ ), the mobility of the holes being sufficiently large. The problem of the interaction between the conduction electrons in a film and the surface oscillations of the "heavy" holes in p-semiconductors is considered for lamellar structures (of a "sandwich" type) consisting of p-semiconductors separated by thin layers of degenerate n-semiconductors (semimetals), and it is shown that under certain conditions this interaction in principle may lead to the formation of bound electron pairs (Cooper pairs) and, consequently, may lead to the appearance in such systems of two-dimensional superconductivity with rather high critical temperatures ( $T_C \sim 10^2$  to  $10^3$ °K).

## 1. INTRODUCTION

IN 1964 Ginzburg and Kirzhnits<sup>[1]</sup> conjectured the possible existence of superconductivity on the surface of a metal due to surface electron states (Tamm levels<sup>[2]</sup>) and surface phonons (Rayleigh waves).

Later on<sup>[3-5]</sup> in order to obtain such a surface (two-dimensional) superconductivity, it was proposed to use dielectric or semiconducting overlapping layers of metals and, in particular, thin metallic films (so-called "sandwiches"). In this connection, in analogy to Little's ideas for one-dimensional systems,<sup>[6]</sup> it was conjectured that a pairing of free electrons near the surface of a metal may take place as a result of their interaction with the excitations of bound electrons in the dielectric or semiconductor (the so-called "exciton" mechanism of superconductivity<sup>[5]</sup>).

At the same time, the characteristic features of the phonon and "electron"<sup>[9]</sup> mechanisms for superconductivity in very thin (quantizing) metallic and semiconducting films, in which the electron motion is actually two-dimensional, were investigated in a series of articles.<sup>[7,8]</sup>

In the present article we show that longitudinal collective oscillations of the free carriers of a "surface wave" type exist on the interface between heavily doped n- and p-type semiconductors with identical work functions (a heterojunction) but with substantially different effective masses for conduction electrons and holes ( $m_n \ll m_p$ ), the mobility of the holes being sufficiently large. In principle the interaction of the degenerate conduction electrons of an n-type semiconductor with these oscillations may lead to the formation of bound electron pairs near the interface between the crystals and, consequently, it may lead to the formation of surface superconductivity.<sup>1)</sup> This interaction is especially effective in thin n-semiconducting (or semi-

metal) films which are bounded on both sides by p-semiconducting crystals, that is, in lamellar structures of "sandwich" type. Numerical estimates indicate that the critical temperatures for the transition of such systems into the superconducting state may be rather high ( $T_C \sim 10^2$  to  $10^3$ °K).

## 2. SURFACE WAVES

Among the numerous branches of eigenvibrations in unbounded, semiconducting crystals there exist, as is well known, collective (plasma) branches of longitudinal oscillations of the free carriers (conduction electrons and holes), whose frequency in the case of isotropic semiconductors is equal to

$$\Omega^* = \sqrt{4\pi e^2 N / m^* \epsilon} \quad A$$

(where  $m^*$ ,  $e$ , and  $N$  are, respectively, the effective mass, charge, and concentration of the carriers, and  $\epsilon$  is the dielectric constant of the crystal). In finite crystals, together with such volume plasma oscillations, surface oscillations of the charge density may also exist whose amplitude falls off exponentially with increasing distance from the surface (compare with surface phonons<sup>[5]</sup>).

Let us consider the contact of semi-infinite isotropic impurity semiconductors of the p- and n-type. Let the region  $x < 0$  correspond to a p-semiconductor with effective mass  $m_p$  for the holes, and let the region  $x > 0$  correspond to an n-type semiconductor with effective mass  $m_n$  for the conduction electrons, where  $m_p \gg m_n$ .<sup>2)</sup>

Let us further assume that the donor concentration in the n-semiconductor is large enough so that the conduction electrons are degenerate up to a temperature on the order of room temperature and above,<sup>3)</sup>

<sup>1)</sup>In its nature this mechanism is the two-dimensional analogue of the "plasmon" mechanism for superconductivity which is considered in [10] (also see [16]).

<sup>2)</sup>In what follows we shall be interested in effects with characteristic lengths which appreciably exceed the lattice constant so that distortion of the band structure and, consequently, changes in the effective masses of the carriers near the crystal boundaries can be neglected.

<sup>3)</sup>We note that a semimetal with a sufficiently small electron effective mass may be chosen in place of a heavily doped n-semiconductor.

whereas the "heavy" holes remain nondegenerate even at comparatively low temperatures (see below).

Finally, we shall assume that the semiconductor work functions are identical,  $\chi_n = \chi_p$ , so that in the absence of an external electric field the thickness of the transition layer tends to zero (the so-called heterojunction), and in any case it is much smaller than the wavelength of the oscillations.

The corresponding energy band scheme for a heterojunction is shown in Fig. 1a. As we see, for a certain choice of the semiconducting parameters one can obtain the result that their forbidden bands (the valence band of the p-semiconductor and the filled conduction band of the n-semiconductor or semimetal) are contiguous to the corresponding forbidden bands so that the electrons cannot pass from one crystal into the other (in spite of the fact that  $\chi_n = \chi_p$ ).

The collective longitudinal oscillations of the "heavy" holes in a p-semiconductor may be described with the aid of the equations of motion and the equation of continuity:

$$\frac{\partial \mathbf{v}_p}{\partial t} = -\frac{e}{m_p} \nabla \varphi_p - \mathbf{v}_p / \tau_p; \quad \frac{\partial (\delta N_p)}{\partial t} + N_p \operatorname{div} \mathbf{v}_p = 0 \quad (2.1)$$

(where  $\mathbf{v}_p$  and  $\delta N_p$  are small perturbations of the velocity and concentration of the holes,  $N_p$  is the unperturbed concentration of holes, i.e., the concentration of acceptors, and  $\tau_p$  is the momentum relaxation time of the holes due to scattering by phonons and impurities) and with the aid of Poisson's equation for the potential of the self-consistent field  $\varphi_p \sim e^{i\omega t}$  of the oscillations. The latter equation, with Eqs. (2.1) taken into account, may be represented in the form

$$\tilde{\varepsilon}_p(\omega) \Delta \varphi_p = 0, \quad \varepsilon_p(\omega) = \varepsilon_p \left\{ 1 - \frac{\Omega_p^2}{\omega(\omega + i/\tau_p)} \right\}; \quad (2.2)$$

where  $\Omega_p = (4\pi e^2 N_p / m_p \varepsilon_p)^{1/2}$  is the Langmuir (plasma) frequency of the holes, and  $\varepsilon_p$  is the dielectric constant of the crystal (it is assumed that  $\varepsilon_p \cong \text{const}$  in the range of frequencies of interest to us).

Under the condition  $\omega \tau_p \gg 1$  (i.e., for sufficiently large mobility of the holes) Eq. (2.2) also has, along with solutions of the plane wave type  $\sim e^{i\mathbf{k} \cdot \mathbf{r}}$  with frequency  $\omega = \Omega_p$  (volume plasma oscillations), a solution of the following form ( $x < 0$ ) near the surface of the crystal:

$$\varphi_p(\mathbf{r}) \sim \exp \{ \kappa_p x + i(q_y y + q_z z) \}; \quad \kappa_p = q \equiv \sqrt{q^2 + q^2}, \quad (2.3)$$

corresponding to oscillations of a "surface wave" type. The frequency of these oscillations is determined by the boundary conditions on the surface of the semiconductor (see below).

On the other hand, neglecting the inertia of the light conduction electrons in the frequency region  $\omega \ll qv_{Fn}$  (where  $v_{Fn} = p_{Fn}/m_n$  is the Fermi velocity,  $p_{Fn} = \hbar(3\pi^2 N_n)^{1/3}$  is the Fermi momentum, and  $N_n$  is the concentration of conduction electrons, i.e., the concentration of donors), we bring Poisson's equation for the potential  $\varphi_n$  in an n-semiconductor to the form

$$\varepsilon_n \{ \Delta \varphi_n - \varphi_n / d_n^2 \} = 0, \quad (2.4)$$

where  $d_n = (\varepsilon_n E_{Fn} / 6\pi e^2 N_n)^{1/2}$  is the effective electronic screening radius,  $E_{Fn} = p_{Fn}^2 / 2m_n$  is the Fermi

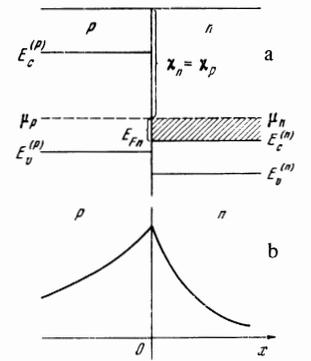


FIG. 1. Energy band scheme (a) and distribution of the surface wave potential (b) for a heterojunction. (If the forbidden band of the p-semiconductor has a sufficiently large width, the heterojunction is impenetrable to electrons.)

energy of the electrons measured from the bottom of the conduction band (see Fig. 1a), and  $\varepsilon_n$  is the dielectric constant of the n-semiconductor. Hence for oscillations of the surface wave type we obtain ( $x > 0$ )

$$\varphi_n(\mathbf{r}) \sim \exp \{ -\kappa_n x + i(q_y y + q_z z) \}; \quad \kappa_n = \sqrt{q^2 + 1/d_n^2}. \quad (2.5)$$

In the case of an ideal infinitely thin (in comparison with the Debye radius or the wavelength of the oscillations) interface between the crystals, the boundary conditions reduce to the conditions for continuity of the normal component of the electric displacement vector  $\mathbf{D}$  and of the potential  $\varphi$  describing the longitudinal oscillations.<sup>4)</sup> With the aid of these conditions it is not difficult to obtain the following dispersion equation for the collective surface oscillations of the free carriers in a heterojunction:

$$\kappa_p \tilde{\varepsilon}_p(\omega) + \kappa_n \varepsilon_n = 0. \quad (2.6)$$

Hence, with the aid of Eqs. (2.2), (2.3), and (2.5) we obtain the following expression for the frequency of the surface waves ( $\Omega_p \tau_p \gg 1$ ):

$$\operatorname{Re} \omega \equiv \omega_q = \Omega_p \left\{ 1 + \frac{\varepsilon_n}{\varepsilon_p} \sqrt{1 + \frac{1}{q^2 d_n^2}} \right\}^{-1/2}. \quad (2.7)$$

In particular, for  $\varepsilon_n = \varepsilon_p$  the frequency of oscillations in the region  $qd_n \ll 1$  is given by  $\omega_q = \Omega_p \sqrt{qd_n}$ , and for  $qd_n \gg 1$  (but  $qa \ll 1$ ) it tends to  $\Omega_p / \sqrt{2}$ .

Now let us consider the collective oscillations of "heavy" holes in a system consisting of two semi-infinite isotropic p-semiconductors separated by an n-semiconductor film of thickness  $L$  (see Fig. 2), assuming as before that  $m_p \gg m_n$  and  $\chi_p = \chi_n$  (heterojunction). Choosing the solution for the potential inside the film in the form  $\varphi_n(x) \sim \cosh \kappa_n x$ , with the aid of the boundary conditions mentioned above we obtain the following dispersion equation for the oscillations in such a system:

$$\kappa_p \tilde{\varepsilon}_p(\omega) + \kappa_n \varepsilon_n \operatorname{th}(\kappa_n L/2) = 0. \quad (2.8)$$

So we see that as  $L \rightarrow \infty$  this equation goes over into the dispersion equation (2.6) for the oscillations on the surface of semi-infinite crystals whereas as  $L \rightarrow 0$  it reduces to the condition  $\tilde{\varepsilon}_p(\omega) = 0$ , i.e., in the case of

<sup>4)</sup>The latter condition assumes that no oscillations of a "double layer" type, when  $\phi$  is discontinuous are present on the boundary.

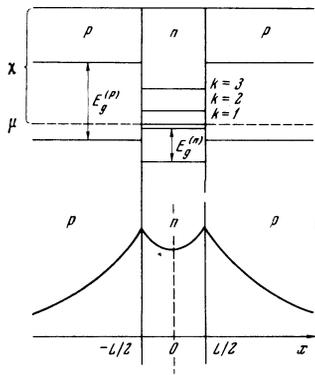


FIG. 2. Band structure (a) and distribution of the potential for the oscillations (b) in a system of the "sandwich" type. (Reflection of the electrons at the boundary of the n-semiconductor film leads to quantization of their transverse motion and to a splitting of the conduction band into discrete levels — the two-dimensional subbands).

sufficiently thin films ( $\kappa_n L \ll 1$ ) the oscillations in a lamellar structure of the "sandwich" type are little different from the ordinary plasma oscillations of the holes ( $\omega_q \approx \Omega_p$ ), and these oscillations freely penetrate through the n-semiconductor film ( $\cosh \kappa_n L \approx 1$ ).

3. SUPERCONDUCTIVITY

Here we shall demonstrate that the interaction of the conduction electrons in a thin film of degenerate n-semiconductor (or semimetal) with the above-considered collective surface oscillations of "heavy" holes in p-type semiconducting crystals, which are in contact with the film (a lamellar structure of the "sandwich" type), may lead to the formation of bound electron pairs and, consequently, to the appearance of superconductivity in such a film.<sup>5)</sup>

Singularities of the electron spectrum in thin metallic and semiconducting films were investigated in articles<sup>[7,8]</sup> where it was shown that as a consequence of the quantization of the electrons' transverse motion, the conduction band is split into a series of discrete levels (subbands) with energies  $E_k = (\hbar^2/2m^* (k\pi/L)^2$  ( $k = 1, 2, \dots$ ). In this connection, if the electron concentration is not very large ( $N \lesssim L^{-3}$ ), as occurs for example in semiconductors or semimetals, and if the temperature of the film is sufficiently low ( $T \ll |E_2 - E_1|$ ) or, what amounts to the same thing, if the thickness of the film is sufficiently small, then only the first (lowest) subband turns out to be occupied, and the electron motion in momentum space turns out to be two-dimensional (although one can regard the film as three-dimensional since  $L \gg a$  (where  $a$  is the lattice constant)).

It should be noted that for heavily doped n-semiconductors with a small effective mass of the conduction electrons, the basic upper bound on the film thickness may be not the temperature condition  $T \ll (\hbar^2/2m_n) (\pi/L)^2$  but the condition  $L \ll l_n$  which is associated with finite mobility of the carriers (here  $l_n$  is the mean free path of an electron), since the scattering by impurities makes the electrons' momentum distribution isotropic.

<sup>5)</sup>We note that in a bulk n-semiconductor the intensity of the interaction between the electrons and the surface waves falls off rapidly with increasing crystal thickness.

As mentioned in Sec. 2, the heterojunction is opaque to electrons for a specific type of semiconductor band structure (see Fig. 1a). If it is assumed that in this case the reflection of the conduction electrons from the boundary of the n-semiconductor with the p-semiconductor occurs analogously to their reflection from a boundary with vacuum,<sup>6)</sup> then everything asserted above also pertains to the case of a semiconductor (semimetallic) film "sandwich" (see Fig. 2a).

Thus, let us consider the interaction of degenerate conduction electrons in a thin n-semiconductor (semimetal) film with surface oscillations of the "heavy" holes in p-semiconductors ( $m_p \gg m_n$ ), assuming that only one subband is populated, and as  $T \rightarrow 0$  the electrons in two-dimensional momentum space fill up the circle with limiting momentum  $\hbar\kappa_0 = \hbar(2\pi N_n L)^{1/2}$ . We note that depending on the parameters of the system, in the two-dimensional case the limiting energy of the electrons,  $\mu_0 = \hbar^2\kappa_0^2/2m_n$  may be either larger or smaller than the maximum energy  $\hbar\Omega_p$  of the surface oscillations (see<sup>[7]</sup>).

Since the surface waves represent longitudinal oscillations of the charge density, the Hamiltonian describing the interaction of the electrons with these oscillations may be represented in the form

$$H_{int} = -e \int \Psi_{\sigma^+}^{\dagger}(\mathbf{r}) \Psi_{\sigma}(\mathbf{r}) \varphi_n(\mathbf{r}) d\mathbf{r}; \tag{3.1}$$

where

$$\Psi_{\sigma}(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{x}} a_{\mathbf{x}\sigma} e^{i\mathbf{x}\cdot\mathbf{r}}; \quad \mathbf{x} \equiv \mathbf{x} \{ \pi/L, x_y, z_z \}; \tag{3.2}$$

$$\varphi_n(\mathbf{r}) = \frac{1}{S} \sum_{\mathbf{q}} \{ \varphi_{\mathbf{q}}(x) e^{i\mathbf{q}\cdot\mathbf{r}} + \text{c.c.} \}; \quad \mathbf{q} \equiv \mathbf{q} \{ 0, q_y, q_z \}; \tag{3.3}$$

( $V$  and  $S$  are, respectively, the normalization volume and surface area, where  $V/S = L$ ). We note that in the case of sufficiently thin films, one can assume  $\varphi_{\mathbf{q}} \approx \text{const}$  (see Sec. 2).

Changing to a second-quantized representation for the oscillation field with the aid of the relation

$$\int \frac{d\mathbf{E}}{4\pi} d\mathbf{r} \approx \epsilon_n \frac{L}{S} \sum_{\mathbf{q}} q^2 \frac{|\varphi_{\mathbf{q}}|^2}{2\pi} = \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}}. \tag{3.4}$$

where  $b_{\mathbf{q}}^{\dagger}$  ( $b_{\mathbf{q}}$ ) is the boson creation (annihilation) operator for a quantized surface vibration ("plasmon") with frequency  $\omega_{\mathbf{q}} = \Omega_p$ , we bring the Hamiltonian (3.1) to the following form (compare with the Hamiltonian for the electron-phonon interaction<sup>[11,12]</sup>):

$$H_{int} = \sum_{\mathbf{x}, \mathbf{q}} g(\mathbf{q}) \left\{ \frac{\hbar\omega_{\mathbf{q}}}{2V} \right\}^{1/2} a_{\mathbf{x}\sigma}^{\dagger} a_{\mathbf{x}-\mathbf{q}, \sigma} b_{\mathbf{q}}^{\dagger} + \text{c.c.} \tag{3.5}$$

where

$$g(\mathbf{q}) = -\frac{2e}{|\mathbf{q}|} \sqrt{\pi/\epsilon_n} \quad (|\mathbf{q}| \leq q_{max} \approx 2\kappa_0). \tag{3.6}$$

Introducing in analogy with<sup>[13]</sup>, in second-order perturbation theory, a model Hamiltonian for the direct electron-phonon interaction due to the exchange of virtual surface "plasmons"

$$H_{red} = -\frac{1}{V} \sum_{\mathbf{x}, \mathbf{x}'} \frac{g^2(\mathbf{x} - \mathbf{x}') \hbar^2 \omega_{\mathbf{x} - \mathbf{x}'}}{\hbar^2 \omega_{\mathbf{x} - \mathbf{x}'}^2 - (\xi_{\mathbf{x}} - \xi_{\mathbf{x}'})^2} a_{\mathbf{x}\sigma}^{\dagger} a_{\mathbf{x}' - \sigma} a_{\mathbf{x}, -\sigma} a_{\mathbf{x}\sigma}, \tag{3.7}$$

where  $\xi_{\mathbf{K}} = (\hbar^2 \kappa^2/2m_n - \mu_0)$ , within the framework of

<sup>6)</sup>The question of the boundary conditions for a heterojunction requires additional investigation.

the BCS theory we attain an equation for the gap  $\Delta(\kappa)$  in the spectrum of the conduction electrons which characterizes the binding energy of the electron (Cooper) pairs:

$$\Delta(\kappa) = \frac{1}{2V} \sum_{\kappa'} g^2(\kappa - \kappa') \frac{\Delta(\kappa')}{\epsilon(\kappa')} \tanh \frac{\epsilon(\kappa')}{2T}, \quad (3.8)$$

where  $\epsilon(\kappa) = \sqrt{\xi_\kappa^2 + \Delta^2(\kappa)}$ . Replacing  $g^2(\kappa - \kappa')$  by its minimal value  $g_0^2 = \pi e^2 / \epsilon_n \kappa_0^2$ , for the condition  $\mu_0 \gtrsim \hbar\Omega_p$  we obtain from here the following approximate estimate for the critical temperature  $T_c$  for the transition of a "sandwich" type system into a superconducting state (compare with<sup>[8]</sup>):

$$T_c \approx \hbar\Omega_p \exp \left\{ -\frac{2\pi\hbar^2 L}{g_0^2 m_n} \right\} \equiv \hbar\Omega_p \exp \{-4\pi N_n a_n L^2\}, \quad (3.9)$$

where  $a_n = \epsilon_n \hbar^2 / m_n e^2$  is the effective Bohr radius of a conduction electron ( $\epsilon_n \sim 1$ ).

As we see, the critical temperature depends very strongly on the thickness of the film. Choosing  $L \sim 1/2 \kappa_0$ , i.e.,  $L \sim (8\pi N_n)^{-1/3}$  (it is precisely for such thicknesses that, on the one hand, the surface waves penetrate well into the n-semiconductor film and, on the other hand, the condition that only the first subband be populated is fulfilled), we obtain according to Eq. (3.9)

$$T_c \sim \hbar\Omega_p \exp \{-a_n (\pi N_n)^{1/3}\}. \quad (3.10)$$

As an example let us consider a "sandwich" consisting of semiconductors with free carrier effective masses  $m_p \sim m_e$  and  $m_n \sim 0.1 m_e$  (where  $m_e$  is the mass of an electron) and concentrations  $N_n \sim N_p \sim 10^{18} \text{ cm}^{-3}$ . In this case  $L \sim 3 \times 10^{-7} \text{ cm}$ ,  $a_n \sim 5 \times 10^{-8} \text{ cm}$ , and  $\Omega_p \sim 5 \times 10^{13} \text{ sec}^{-1}$ , so that  $T_c \sim \hbar\Omega_p \sim 0.03 \text{ eV} \cong 300^\circ \text{K}$  (whereas the temperature for degeneracy of the holes in p-semiconductors is  $T_p \sim 30^\circ \text{K}$ ). We note that the temperature condition<sup>[7,8]</sup> for  $T \sim T_c$  reduces to the inequality  $L \ll 10^{-4} \text{ cm}$ , but the mean free path of the conduction electrons for  $\tau_n \sim 10^{-12} \text{ sec}$  (where  $\tau_n$  is the relaxation time of the electrons with respect to momentum) equals  $l_n \sim 2 \times 10^{-5} \text{ cm}$ .

However, the estimate given above for the critical temperature may be somewhat overstated since we did not take the Coulomb repulsion between electrons into consideration. With the latter taken into account, the Hamiltonian for the electron-electron interaction is given by<sup>7)</sup>

$$\begin{aligned} \bar{H}_{red} &= H_{rel} + H_c \\ &= -\frac{1}{V} \sum_{\kappa, \kappa'} \frac{g^2(\kappa - \kappa') (\xi_\kappa - \xi_{\kappa'})^2}{\hbar^2 \Omega_p^2 - (\xi_\kappa - \xi_{\kappa'})^2} a_{\kappa'}^\dagger \sigma a_{-\kappa'}^\dagger \cdot_{-\sigma} a_{-\kappa} \cdot_{-\sigma} a_{\kappa\sigma}, \end{aligned} \quad (3.11)$$

and the equation for the gap for  $T \ll T_c$  (but  $T > T_p$ ) takes the form

$$\Delta(\kappa) = \frac{1}{2L} \int \frac{d\kappa'}{(2\pi)^2} \frac{g^2(\kappa - \kappa') (\xi_\kappa - \xi_{\kappa'})^2 \Delta(\kappa')}{\hbar^2 \Omega_p^2 - (\xi_\kappa - \xi_{\kappa'})^2 \epsilon(\kappa')}. \quad (3.12)$$

Integrating with respect to the angle between the vectors  $\kappa$  and  $\kappa'$  with Eq. (3.6) taken into account, and

<sup>7)</sup>The Hamiltonian (3.11) is analogous to the three-dimensional Hamiltonian in the so-called "jellium" model. [14] We note that in the two-dimensional case, one can neglect the effects of screening of the Coulomb interaction.

changing to new variables  $\xi \equiv \xi_\kappa$  and  $\xi' \equiv \xi_{\kappa'}$ , from here we obtain

$$\Delta(\xi) = \frac{e^2}{2\epsilon_n L} \int \frac{|\xi - \xi'|}{\hbar^2 \Omega_p^2 - (\xi - \xi')^2} \frac{\Delta(\xi') d\xi'}{\sqrt{\xi'^2 + \Delta^2(\xi')}}. \quad (3.13)$$

In what follows we shall be interested in the width of the gap  $\Delta \equiv \Delta(0)$  for  $\xi = 0$ . Assuming that  $\Delta(\xi) \cong \Delta = \text{const}$  in the region  $\xi \leq \hbar\Omega_p$  and  $\Delta(\xi) = 0$  for  $\xi > \hbar\Omega_p$ ,<sup>8)</sup> we arrive at the following equation for the determination of  $\Delta(\mu_0 > \hbar\Omega_p)$ :

$$1 = \frac{e^2}{\epsilon_n L} \int_0^{\hbar\Omega_p} \frac{\xi' d\xi'}{(\hbar^2 \Omega_p^2 - \xi'^2) \sqrt{\xi'^2 + \Delta^2}}. \quad (3.14)$$

It is not difficult to see that the integral on the right hand side of Eq. (3.14) is logarithmically divergent at the point  $\xi' = \hbar\Omega_p$ . This divergence is associated with the fact that within the framework of perturbation theory the normal state of the conduction electrons in an n-semiconductor was chosen as the zero-order approximation. For a more consistent approach with the electron pairing taken into consideration (see<sup>[10,15]</sup> for three-dimensional superconductors), this divergence vanishes and under the condition  $\Delta \ll \hbar\Omega_p$  we obtain, correct to within terms  $\sim \ln(\Delta/\hbar\Omega_p)$ , the following asymptotic formula for the energy gap:

$$\Delta \cong 2\hbar\Omega_p e^{-1/\rho}, \quad \rho = \frac{e^2}{\epsilon_n L \hbar\Omega_p}. \quad (3.15)$$

In connection with the values of the "sandwich" parameters chosen above, expression (3.15) leads, as before, to a value  $T_c \sim 300^\circ \text{K}$ . However, assuming  $N_p \sim 10^{20} \text{ cm}^{-3}$  (in this connection  $\hbar\Omega_p \gg \mu_0$  and  $\Delta = 2\hbar\Omega_p e^{-2/\rho}$ ; see<sup>[7]</sup>), we now obtain the estimate  $T_c \sim 10^3^\circ \text{K}$  corresponding to the maximum of the gap (for  $\rho \approx 2$ ).

Thus, the numerical estimates made above show that in lamellar structures of the "sandwich" type consisting of heavily doped p-semiconductors separated by sufficiently thin layers ( $L < 10^{-6} \text{ cm}$ ) of degenerate n-semiconductors (or semimetals), under certain conditions (equality of the work functions, large width of the p-semiconductors forbidden band, etc.) superconductivity in principle is possible with rather high critical temperatures  $T_c \sim (10^2 \text{ to } 10^3)^\circ \text{K}$ , owing its existence to the interaction of conduction electrons in the film with collective surface oscillations of the "heavy" holes in the p-semiconductors.

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<sup>2</sup>I. E. Tamm, Phys. Z. Soviet Union 1, 733 (1932).

<sup>8)</sup>The latter assumption is, in general, not compulsory. In particular, if the gap  $\Delta(\xi) < 0$  in the region " $\xi$ "  $> \hbar\Omega_p$  (see for example, [10]), then this only leads to an effective increase of the interaction constant in the region " $\xi$ "  $< \hbar\Omega_p$ . It should be emphasized that here (just like in the case of the "plasmon" mechanism for superconductivity [10]) no theoretical upper bound on the interaction constant is present analogous to the restriction  $g < 1/2$  for the phonon mechanism of superconductivity, [5,12] which is related to stability of the crystal lattice.

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