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SPIN WAVES IN AN ANTIFERROMAGNETIC METAL IN THE PRESENCE OF A MAGNETIC FIELD

A. Ya. BLANK and P. S. KONDRATENKO

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

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The spectrum of magnetic excitations of an antiferromagnetic metal is investigated in the long wavelength limit. As a result of the s–d exchange interaction the frequencies of the paramagnetic and antiferromagnetic resonances turn out to be related, which leads to a nonlinear dependence of the resonance frequencies on the magnetic field. In addition, the s–d exchange decreases the magnitude of the phase-transition field in a magnetic system. In particular, in metals in which the s–d exchange interaction is not small, the antiferromagnetic lattice turns out to be unstable in arbitrarily weak magnetic fields, which indicates the essential role of the conduction electrons in establishing the nature of the magnetic order in a metal.

In a broad sense of the word, the magnetic excitations in metals represent collective branches of the spectrum of an interacting system, including the conduction electrons. The latter play an essential, and frequency also decisive (as in the case of the rare-earth metals), role in the establishment of magnetic order in a metal. In a number of cases the appearance of a magnetic superstructure in the lattice is associated with the conduction electrons as, for example, in the case of a helicoidal structure in antiferromagnetic metals.[1] In addition, the s-electrons may decisively change the spectrum of the magnetic excitations. Even in the absence of magnetic order the electronic subsystem may possess proper magnetic branches of its spectrum (spin excitations of the zero-sound type), which appear as a result of the Fermi-liquid interaction between electrons. The magnetic excitations in a Fermi liquid of the conduction electrons in ferromagnetic and antiferromagnetic metals were considered in articles by one of the authors.[2,3]

In the presence of an external magnetic field the magnetic spectrum of a metal is strongly modified. This occurs both as a result of the interaction of the intrinsic magnetic moments of the electrons with the magnetic field and as a consequence of the orbital motion of the conduction electrons. The change in the spectrum of the magnetic excitations in paramagnetic metals[4] is a characteristic example. In metals of this type in an external magnetic field, instead of gapless branches of spin-wave zero-sound, spin waves with a quadratic spectrum appear, possessing a nonvanishing activation energy.

The orbital motion of the s-electrons was taken in account in[5] in connection with an investigation of the spin-wave spectrum in a ferromagnetic metal. It should be noted that the influence of a magnetic field on the magnetic spectrum of a ferromagnetic metal is comparatively small because of the large magnitude of the separation between the Fermi surfaces of electrons with oppositely-oriented spins.

In this connection the most interesting objects are antiferromagnetic metals in which, on the one hand, a magnetic structure exists, but on the other hand, a separation of the Fermi surfaces is not present.[3] We shall regard an antiferromagnet as a system consisting of two mirror magnetic sublattices with anisotropy of the "easy axis" type.[6] An external magnetic field is applied along the axis of antiferromagnetism. For simplicity we shall confine our attention to an investigation of a one-band model of s-electrons with an isotropic energy spectrum. The external magnetic field will be assumed to be smaller than the critical value H_0, which corresponds to a violation of the collinearity of the sublattice magnetic moments (in a dielectric $2\mu_0 H_0$ is of the order of the antiferromagnetic resonance frequency at $H = 0$).

Just as in a paramagnetic metal,[4] in an antiferromagnet without allowance for the interaction of the s-electrons with the magnetic sublattices, the electronic subsystem must possess in an external-magnetic field a two-parameter family of spin waves with frequencies $\omega_{lm}(\mathbf{k})$. In this connection the frequency $\omega_{00}(0)$ coincides with the frequency of the paramagnetic resonance. On the other hand, in a system of magnetic sublattices two spin-wave branches exist, whose frequencies at $\mathbf{k} = 0$ ($\mathbf{k}$ is the wave vector of a magnon) correspond to an antiferromagnetic resonance. When $\mathbf{k} = 0$, because of the exchange interaction of the conduction electrons with the sublattice magnetic moments, an "intermingling" of the antiferromagnetic resonance frequencies with the frequency ω_{00} occurs. This leads to a nonlinear dependence of the resonance frequencies on the magnitude of the magnetic field. As is well known, in an antiferromagnetic dielectric the vanishing of one of the resonance frequencies at a definite value of the magnetic field, $H = H_0$, corresponds to a phase transition in the magnetic system.[1)] In our case the critical field turns out to be associated with a parameter of the exchange interaction, characterizing the conduction electrons. Depending on the value of this parameter, both the frequency of the antiferromagnetic

[1)]Actually the field H_0 corresponds to the boundary of metastability of a state with the total magnetic moment of the sublattices equal to zero. A detailed discussion of the phase transition in an antiferromagnetic metal will be the subject of a separate article.

resonance and the frequency ω_{00} may tend to zero. In that case when the exchange parameter is sufficiently large, the phase transition in a magnetic system may occur in magnetic fields which are appreciably smaller than the field corresponding to the antiferromagnetic resonance frequency.

For nonvanishing magnon wave vectors, in the approximation quadratic in $\mathbf{k}$, an interaction of lattice spin waves and spin waves in the electronic system with frequencies $\omega_{00}(0)$ and $\omega_{1m}(0)$ occurs. Due to the nonlinear dependence of the frequency ω_{00} on the magnetic field, at a definite value of the field it is possible that the frequency $\omega_{00}(0)$ may coincide with one of the frequencies $\omega_{1m}(0)$. Near the frequency $\omega_{00}(0) = \omega_{1m}(0)$, the spin-wave dispersion law turns out to be linear.

In the considerations which follow below, we shall follow a formalism of quantum field theory previously developed in[3] for the case of an antiferromagnetic metal. The difference between the formalism in a magnetic field and that developed in[3] consists in the fact that the electron Green's functions in the presence of a magnetic field cease to depend on the difference of spatial coordinates. This leads to complicated dependences of the vertex part and of the Green's functions on the arguments in the momentum representation; however, it does not change the nature of the corresponding equations. Therefore we shall use a symbolic way of writing down the equations, omitting the notation referring to arguments and momentum integrations. This notation turns out to be all the more convenient since, as will be shown below, in the quasiclassical approximation the investigation has a great deal of similarity to the case when no magnetic field is present.

1. As is well known, the spectrum of boson excitations is determined by the singularities of the two-particle vertex part $\Gamma_{\alpha\beta\gamma\delta}(p, p'; k)$ with respect to the momentum transfer $k = (\mathbf{k}, \omega)$. Since the spin excitations represent oscillations of the transverse part of the magnetic moment, we shall only be interested in the components of the vertex part which are transverse to the spin, $\Gamma_{\uparrow\downarrow\downarrow}$ or $\Gamma_{\downarrow\uparrow\uparrow\downarrow}$, whose spin indices we omit in what follows. The total vertex part includes both the direct exchange interaction in a system of s-electrons and the interaction induced by the virtual exchange of lattice spin waves. Thus, the elements which introduce singularities into the vertex part at small momentum transfer are, on the one hand, the loops of the electron Green's functions, as this ordinarily occurs in the microscopic theory of a Fermi liquid,[7] and, on the other hand, the lattice spin-wave Green's function D(k). In accordance with these considerations, let us represent the vertex part in the following form:

$$\Gamma = \tilde{\Gamma} + gDg, \tag{1}$$

where $\tilde{\Gamma}$ is determined by the set of diagrams for Γ which do not contain any D-functions with argument k; g is the total vertex describing the interaction of the electrons with the lattice spin waves. D, in turn, is determined by the equation

$$D = D_0 + D_0 \Pi D, \tag{2}$$

where

$$\Pi = -ig_0 GGg_0 - g_0 GG\tilde{\Gamma}GGg_0 \tag{3}$$

is the polarization operator of a spin wave, g_0 is the bare coupling constant, D_0 is the spin-wave propagator. The quantity g is related to g_0 by the following equation:

$$g = g_0 - ig_0 GG\tilde{\Gamma}. \tag{4}$$

The equation for Γ has the same form as the equation for an ordinary Fermi liquid:

$$\Gamma = \Gamma^1 - i\Gamma^1 GG\Gamma, \tag{5}$$

where Γ^1 is defined as the set of diagrams for Γ, not containing any of the above-mentioned singular elements.

Following article[3], it is not difficult to obtain an explicit expression for D_0 in the presence of a magnetic field:

$$D_0(\omega, \mathbf{k}) = \frac{\omega_s^2(\mathbf{k})/2\delta\mu_0 M_0}{(\omega - 2\mu_0 H)^2 - \omega_s^2(\mathbf{k})} \tag{6}$$

where ω_s is the frequency of a lattice spin wave in the absence of any magnetic field, δ is a parameter which characterizes the exchange interaction between sublattices, M_0 is the equilibrium density of the magnetic moment of each of the sublattices, and μ_0 is the Bohr magneton.

Substituting expression (2) for D into Eq. (1) and using Eq. (6) we obtain

$$\Gamma = \tilde{\Gamma} + g\frac{\omega_s^2/2\delta\mu_0 M_0}{(\omega - 2\mu_0 H)^2 - \omega_s^2(1 + \Pi/2\delta\mu_0 M_0)}g. \tag{7}$$

As is evident from this equation, the singularities of the vertex part consist of the singularities contained in $\tilde{\Gamma}$ and the poles of the second term in (7). Let us concern ourselves with an elucidation of the nature of the singularities in $\tilde{\Gamma}$. It is obvious that these singularities are associated with spin waves in the electronic subsystem. These spin waves actually do not differ at all from the spin waves in paramagnetic metals[2)] which are considered in articles[4,8,9]. Keeping in mind that the quantities Π and g, appearing in the second term of (7), can according to Eqs. (3) and (4) be expressed in terms of $\tilde{\Gamma}$, we shall give a derivation of the expression for $\tilde{\Gamma}$ in the long wavelength limit.

2. Equation (5) is the standard equation for the theory of a Fermi liquid. As is well known, a singularity in $\tilde{\Gamma}$ arises in connection with the integration of a product of electron Green's functions near the Fermi surface. Let us represent the product GG in the form of a sum of the product of the pole parts of G and the regular part $\overline{GG}$. Then Eq. (5) may be rewritten in the form

$$\tilde{\Gamma} = \tilde{\Gamma}' - i\tilde{\Gamma}'(GG - \overline{GG})\tilde{\Gamma}. \tag{8}$$

where $\tilde{\Gamma}'$ satisfies the equation

$$\tilde{\Gamma}' = \tilde{\Gamma}^1 - i\tilde{\Gamma}^1\overline{GG}\tilde{\Gamma}' \tag{9}$$

Since $\tilde{\Gamma}'$ and $\overline{GG}$ are regular, they differ from their

[2)] We wish to take this opportunity to thank V.P. Silin for kindly giving us an opportunity to become acquainted with preprint [9].

values for H = 0 and k = 0 by quantities of order Ω/ϵ_F, kv/ϵ_F, and ω/ϵ_F (ϵ_F denotes the Fermi energy, v is the Fermi velocity, Ω is the cyclotron frequency), and one can neglect in them the dependence on k and H. Therefore one can assume that $\widetilde{\Gamma}'$ is equal to $\widetilde{\Gamma}^{\omega}$, which is the limiting value of $\widetilde{\Gamma}$ for $\mathbf{k} = 0$, $\omega \to 0$ in the absence of a magnetic field, and $\overline{GG}$ is equal to $(GG)_{\omega}$,

$$(GG)_{\omega} = \lim_{\mathbf{k}=0,\ \omega\to 0} G(p)G(p+k).$$

Thus, one can rewrite Eq. (9) as follows:

$$\widetilde{\Gamma} = \widetilde{\Gamma}^{\omega} - i\widetilde{\Gamma}^{\omega}(GG - (GG)_{\omega})\widetilde{\Gamma}. \qquad (10)$$

In order to solve Eq. (10) we shall use a pole expression for G in the quasiclassical approximation:[10]

$$G_{\pm}(p, p') = \gamma^{1/2}\delta(\varepsilon - \varepsilon')\delta(p_x - p_x')\delta(p_z - p_z') \times \sum_n \frac{a\psi_n(\gamma^{1/2}p_y)\psi_n(\gamma^{1/2}p_y')\exp\{i\gamma p_x(p_y - p_y')\}}{\varepsilon - (n\Omega + p_z^2/2m^* - p_{\pm}^2/2m^*) + i\delta \operatorname{sign}\varepsilon}, \qquad (11)$$

where m* is the effective mass of the Fermi excitations, a is a constant of the order of unity,[11] $p_{\pm}$ are the limiting Fermi momenta for electrons with oppositely-directed spins, the $\psi_n(x)$ are Hermite functions; the eigenfunctions of an electron in a magnetic field are selected from the condition for conservation of the momentum component in the direction $\mathbf{k}_{\perp} = \mathbf{k} - (\mathbf{k}\cdot\mathbf{H})\mathbf{H}/(\mathbf{H}\cdot\mathbf{H})$, which is adopted as the x axis. Since in the quasiclassical approximation the wave functions of an electron are appreciably different from zero only on the trajectory, the momenta appearing as the arguments of the vertex part turn out to be of the order of the Fermi momentum. Therefore, to within quantities of order $(p_0R)^{-1}$ (where R is the Larmor radius, p_0 is the limiting Fermi momentum), scattering processes occur with conservation of momentum. Thus, $\widetilde{\Gamma}$ in Eq. (10) has the form $\widetilde{\Gamma} = \widetilde{\Gamma}(p, p'; k)$ just like the case when no magnetic field is present. As a result Eq. (10) may be explicitly written down in the following way:

$$\widetilde{\Gamma}(p_1, p_2; k) = \widetilde{\Gamma}^{\omega}(p_1, p_2) - \frac{eH}{c}\sum_{n,\alpha}\int dp_z\, dp_y'\, dp_y''\widetilde{\Gamma}^{\omega}(p_1, p') \times a^2\Phi_{n\alpha}(p_y', k_{\perp})\Phi_{n\alpha}^*(p_y'', k_{\perp})\frac{n(\varepsilon_n^+(p_z)) - n(\varepsilon_{n+\alpha}^-(p_z + k_z))}{\omega - \varepsilon_{n+\alpha}^-(p_z + k_z) + \varepsilon_n^+(p_z) + i\delta \operatorname{sign}\omega} \cdot \widetilde{\Gamma}(p'', p_2; k). \qquad (12)$$

Here

$$\Phi_{n\alpha}(p_y, k_{\perp}) = \exp(i\gamma p_y k_{\perp})\psi_n(\gamma^{1/2}p_y)\psi_{n+\alpha}(\gamma^{1/2}p_y).$$

In connection with the transition to Eq. (12) we have carried out an integration with respect to the energy, having taken into consideration that $\widetilde{\Gamma}(p, p'; k)$ is a regular function of ϵ and ϵ'.

Let us expand $\widetilde{\Gamma}$ and $\widetilde{\Gamma}^{\omega}$ in a series with respect to the spherical functions:

$$\widetilde{\Gamma}(p, p'; k) = \sum_{\substack{l,\, l' \\ m,\, m'}} \widetilde{\Gamma}_{ll'}^{mm'}(k)\, Y_{lm}\left(\frac{\mathbf{p}}{p}\right) Y_{l'm'}^*\left(\frac{\mathbf{p}'}{p'}\right), \qquad (13)$$

$$\widetilde{\Gamma}^{\omega}(p, p') = \sum_{l,\, m}\frac{4\pi\widetilde{\Gamma}_l^{\omega}}{2l+1} Y_{lm}\left(\frac{\mathbf{p}}{p}\right) Y_{lm}^*\left(\frac{\mathbf{p}'}{p'}\right), \qquad (13a)$$

where $\widetilde{\Gamma}_l^{\omega}$ is the coefficient of an expansion of $\widetilde{\Gamma}^{\omega}$ in terms of Legendre polynomials. Since $\widetilde{\Gamma}^{\omega}$ depends only on the single angle between $\mathbf{p}$ and $\mathbf{p}'$, the sum over the products of the spherical functions has a diagonal form.

Upon substitution of the expansions (13) into Eq. (12), the following matrix elements appear:

$$M_{nl}^{m\alpha} = \int \psi_n^*(\gamma^{1/2}p_y)\, Y_{lm}\, \psi_{n+\alpha}(\gamma^{1/2}p_y)\exp\left\{i\frac{c}{eH}k_{\perp}p_y\right\}dp_y. \qquad (14)$$

Expression (14) can be converted into an integral along the classical trajectory of an electron in a magnetic field and is given by

$$M_{nl}^{m\alpha} = \oint \frac{d\varphi}{2\pi}\exp\left[i\frac{c}{eH}k_{\perp}p_n\cos\varphi - i\alpha\varphi\right]Y_{lm}(\vartheta, \varphi) = J_{\alpha-m}\left(\frac{c}{eH}k_{\perp}p_n\right)Y_{lm}(\vartheta, \varphi)e^{-im\varphi}, \qquad (15)$$

where $J_n(x)$ are Bessel functions. Here

$$p_n = \left(\frac{eH}{c}(2n+1)\right)^{1/2} = p_0\sin\vartheta -$$

is the quasiclassical value of the electron momentum component perpendicular to the direction of the magnetic field. Let us carry out an expansion with respect to α and k_z, taking into consideration that $\epsilon_n^+(p_z)$ and $\epsilon_{n+\alpha}^-(p_z + k_z)$ are close to the Fermi level. We simultaneously change from a summation over n to an integration with respect to $p_{\perp}$. As a result, taking formulas (13) and (15) into account, we obtain

$$\widetilde{\Gamma}_{l_1 l_2}^{m_1 m_2}(k) = \frac{4\pi}{a^2\nu}B_{l_1}\delta_{l_1 l_2}\delta_{m_1 m_2} + \sum_{l'm'}B_{l_1}\int do\, Y_{l_1 m_1}^*(\vartheta, \varphi) \times Y_{l'm'}(\vartheta, \varphi)\sum_{\alpha}J_{\alpha-m_1}\left(\frac{c}{eH}k_{\perp}p_0\sin\vartheta\right)J_{\alpha-m'}\left(\frac{c}{eH}k_{\perp}p_0\sin\vartheta\right) \times e^{i(m_1-m')\varphi}\frac{v(\Delta + k_{\|}\cos\vartheta) + \alpha\Omega}{\omega - v(\Delta + k_{\|}\cos\vartheta) - \alpha\Omega}\widetilde{\Gamma}_{m'm_2}^{l'l_2}(k). \qquad (16)$$

Here $k_{\|} = k\cos\Phi$, Φ is the angle between $\mathbf{k}$ and $\mathbf{H}$, $B_l = a^2\nu\widetilde{\Gamma}_l^{\omega}/(2l+1)$, $\nu = m^*p_0^2/2\pi^2$ is the density of electron states on the Fermi surfaces, $\Delta = p^+ - p^-$ is the paramagnetic separation between the Fermi surfaces. In accordance with Landau's theory of a Fermi liquid,[12] $\Delta = 2\mu_0H/v(1 + B_0)$. We set $\mathbf{k} = 0$ in Eq. (16), and we find $\widetilde{\Gamma}(\mathbf{k} = 0)$, denoting it by $\widetilde{\Gamma}_H^{\omega}$. In this connection Eq. (16) assumes diagonal form, and its solution has the form

$$(\widetilde{\Gamma}_H^{\omega})_{mm'}^{ll'} = \frac{4\pi}{a^2\nu}B_l\frac{\omega - v\Delta - m\Omega}{\omega - \omega_{lm}(0)}\delta_{ll'}\delta_{mm'}, \qquad (17)$$

where

$$\omega_{lm}(0) = \left(\frac{2\mu_0 H}{1 + B_0} + m\Omega\right)(1 + B_l) \qquad (18)$$

represents the well-known resonance frequencies for a homogeneous excitation of the electronic subsystem in the presence of a magnetic field.[4]

For small wave vectors in the approximation quadratic in k, only the components $\Gamma_{mm'}^{ll}$, associated with the values $l' = l$, $l \pm 1$ and $m' = m$, $m \pm 1$ turn out to enter into Eq. (16):

$$\widetilde{\Gamma}_{mm}^{ll} = \widetilde{\Gamma}_{mm}^{\omega ll} + \widetilde{\Gamma}_{mm}^{\omega ll}\sum_{l'm'}\frac{a^2\nu}{4\pi}X_{mm'}^{ll'}(\omega, k)\widetilde{\Gamma}_{m'm}^{l'l}, \qquad (19)$$

$$\widetilde{\Gamma}_{m'm}^{l'l} = \widetilde{\Gamma}_{m'm'}^{\omega l'l'}\frac{a^2\nu}{4\pi}X_{mm'}^{ll'}\widetilde{\Gamma}_{mm}^{ll}. \qquad (19a)$$

A summation with respect to l' and m' is carried out over the values indicated above. The diagonal elements X_{mm}^{ll} have the form

$$X_{mm}^{ll}(\omega, \mathbf{k}) = \frac{(vk_{\|})^2\omega}{[\omega - \omega_{lm}(1 + B_l)^{-1}]^3}\langle lm|\cos^2\vartheta|lm\rangle + \frac{1}{2}\frac{(vk_{\perp})^2\omega}{[(\omega - \omega_{lm}(1 + B_l)^{-1})^2 - \Omega^2][\omega - \omega_{lm}(1 + B_l)^{-1}]}\langle lm|\sin^2\vartheta|lm\rangle, \qquad (20)$$

and the nondiagonal elements have the form

$$X_{mm}^{l\,l\pm1}(\omega,\mathbf{k})=\frac{\omega v k_{\parallel}}{\omega-\omega_{lm}(1+B_l)^{-1}}\langle lm|\cos\vartheta|l\pm1,m\rangle, \tag{21}$$

$$X_{mm'}^{l\,l\pm1}(\omega,\mathbf{k})=\frac{1}{2}\frac{\omega\Omega v k_{\perp}}{[\omega-\omega_{lm'}(1+B_l)^{-1}][\omega-\omega_{lm}(1+B_l)^{-1}]}\times\langle lm|\sin\vartheta\, e^{i(m-m')\varphi}|l\pm1,m'\rangle, \tag{22}$$

m′ takes the values m ± 1. We have introduced the notation

$$\langle lm|f(\vartheta,\varphi)|l'm'\rangle=\int do\, Y_{lm}^{*}f(\vartheta,\varphi)Y_{l'm'}.$$

Solving the system of equations (19) with respect to $\tilde{\Gamma}_{mm}^{ll}$, we obtain

$$\tilde{\Gamma}_{mm}^{l\,l}=\frac{4\pi}{a^2\nu}B_l\frac{\omega-v\Delta-m\Omega}{\omega-\omega_{lm}(\mathbf{k})}, \tag{23}$$

where $\omega_{lm}(\mathbf{k})$ is the long wavelength limit of the spectrum for spin waves in the electronic system:

$$\omega_{lm}(\mathbf{k})=\omega_{lm}(0)+P_{lm}(\omega_{lm}(0),\mathbf{k}), \tag{24}$$

where $\omega_{lm}(0)$ is determined by formula (18),

$$P_{lm}(\omega,\mathbf{k})=B_l(\omega-\omega_{lm}(0)(1+B_l)^{-1})\Big[X_{mm}^{ll}(\omega,\mathbf{k})+\sum_{l'm'}{}'|X_{mm'}^{ll'}|^2\frac{B_{l'}(\omega-\omega_{lm'}(0)(1+B_l)^{-1})}{\omega-\omega_{l'm'}(0)}\Big]. \tag{25}$$

Using formulas (20)–(22), we obtain an explicit expression for $P_{lm}(\omega,\mathbf{k})$

$$P_{lm}(\omega,\mathbf{k})=\frac{B_l\omega(kv)^2}{\omega-\omega_{lm}(0)(1+B_l)^{-1}}\sum_{\substack{l'=l\pm1\\ m'=m,\,m\pm1}}\frac{1+B_{l'}}{\omega-\omega_{l'm'}(0)}\times\Big\{|\langle lm|\cos\vartheta|l'm'\rangle|^2\cos^2\Phi+|\langle lm|\sin\vartheta\, e^{i(m-m')\varphi}|l'm'\rangle|^2\frac{\sin^2\Phi}{4}\Big\} \tag{26}$$

Finally, the spin-wave spectrum in a system of electrons in a magnetic field has the following form near the frequencies $\omega_{lm}(0)$:

$$\omega_{lm}(\mathbf{k})=\omega_{lm}(0)+\frac{(kv)^2}{\omega_{lm}}(\alpha_{lm}\cos^2\Phi+\beta_{lm}\sin^2\Phi);$$

$$\alpha_{lm}=(1+B_l)^2\sum_{l'=l\pm1}\frac{1+B_{l'}}{B_l-B_{l'}}\frac{[\max(l,l')]^2-m^2}{(2l+1)(2l'+1)}$$

$$\beta_{lm}=\frac{1}{4}(1+B_l)^2\sum_{\substack{l'=l\pm1\\ m'=m\pm1}}\frac{1+B_{l'}}{B_l-B_{l'}-(m-m')(1+B_l)(1+B_{l'})\Omega/\omega_{lm}}\times\frac{(l'm'-lm)^2+(m'-m)(l'm'-lm)}{(2l+1)(2l'+1)}. \tag{27}$$

Formula (27) agrees with the results of article[9] and describes, as already mentioned above, spin waves in paramagnetic metals, which have recently been observed experimentally.[13]

In actual fact the spin waves obtained above are not, generally speaking, excitations peculiar to an antiferromagnet, since the actual excitations in an antiferromagnetic metal must be the result of collective interactions in the system consisting of "electrons plus magnetic sublattices." As one can show, for $\mathbf{k}=0$ the pole of $\tilde{\Gamma}$ corresponding to $\omega=\omega_{00}$ is not a singularity of the total vertex part Γ (as $\omega\to\omega_{00}(0)$ the same pole term with opposite sign is contributed by the second term in expression (7)). As $\omega\to\omega_{lm}(0)$ $(l\neq0)$ the remaining poles of $\tilde{\Gamma}$ as before determine the spectrum of eigenfrequencies of the system for $\mathbf{k}=0$. In the approximation quadratic in $\mathbf{k}$, $\omega=\omega_{lm}(\mathbf{k})$ ceases to lead to singularities of the total vertex part Γ, which is a result of the interaction of these harmonics with oscillations of the magnetic sublattices. In analogous fashion, in the higher-order approximations in $\mathbf{k}$, the poles associated with the higher harmonics $\omega_{lm}(\mathbf{k})$ $(l>1)$ are cancelled. Thus, in order to obtain the spectrum of the actual excitations in an antiferromagnetic metal, one must consider the singularities of the second term in formula (6).

3. Since the total vertex g describing the interaction of an electron with a lattice spin wave has only the same poles as $\tilde{\Gamma}$, the other singularities in Γ correspond to the vanishing of the denominator of the second term in expression (7):

$$(\omega-2\mu_0H)^2-\omega_s^2(1+\Pi(\omega,\mathbf{k})/2\delta\mu_0M_0)=0. \tag{28}$$

Setting $\mathbf{k}$ equal to zero, we first find the frequency of the homogeneous resonance. For this, we need to know the value of $\Pi(\omega,0)$. In order to determine its value, we transform Eq. (3) to a form in which the integration is carried out in the neighborhood of the Fermi surface. Treating the problem in analogy to the way in which Eq. (10) was derived, we find

$$\Pi=\Pi^{\omega}-ig_{\omega}(GG-\overline{GG})g=\Pi^{\omega}-ig_{\omega}(GG-\overline{GG})g_{\omega}-g_{\omega}(GG-\overline{GG})\tilde{\Gamma}(GG-\overline{GG})g_{\omega}; \tag{29}$$

Π^{ω} and g_{ω} are obtained from the definitions of Π and g by replacing GG by $\overline{GG}$ and $\tilde{\Gamma}$ by $\tilde{\Gamma}^{\omega}$ and, to within terms of order Ω/ϵ_F, agree with their values in the limit $\mathbf{k}=0$, $\omega\to0$ in the absence of a magnetic field. The quantity Π^{ω} determines the renormalization of the antiferromagnetic resonance frequency in the absence of a magnetic field.[3] The renormalized frequency is given by

$$\tilde{\omega}_s^2=\omega_s^2(1+\Pi^{\omega}/2\delta\mu_0M_0). \tag{30}$$

For $\mathbf{k}=0$, carrying out the same calculations as in the evaluation of $\tilde{\Gamma}(\omega,0)$ and taking into consideration that g_{ω} is an isotropic function of the momentum, we obtain

$$\Pi(\omega,0)=\Pi^{\omega}+g_{\omega}^2a^2\nu\frac{v\Delta}{\omega-\omega_{00}(0)}. \tag{31}$$

Thus, Eq. (28) takes the following form:

$$(\omega-\omega_{00}(0))[(\omega-2\mu_0H)^2-\tilde{\omega}_s^2]=\xi\tilde{\omega}_s^2 2\mu_0H, \tag{32}$$

where

$$\xi=\frac{a^2\nu g_{\omega}^2}{1+B_0}\frac{\omega_s^2}{\tilde{\omega}_s^2 2\delta\mu_0M_0}=\frac{\tilde{\omega}_s^2-\omega_0^2}{\tilde{\omega}_s^2}. \tag{33}$$

Here ω_0 is the frequency associated with the gapless branch of spin waves for $H=0$.[3]

For $\mathbf{k}=0$ only the frequency $\omega_{00}(0)$ of the zeroth harmonic, which is equal to the paramagnetic resonance frequency, enters into Eq. (31). As is evident from Eq. (31) the paramagnetic frequency and the antiferromagnetic resonance frequencies turn out to be related among themselves, where the coupling constant is, generally speaking, the not-so-small parameter ξ of the exchange interaction. This leads to a nonlinear dependence of the resonance frequencies on the magnitude of the magnetic field. For small magnetic fields $(2\mu_0H/\tilde{\omega}_s\ll1)$ the resonance frequencies have the form

$$\omega_1=(1-\xi)2\mu_0H-\xi2\mu_0H(\xi2\mu_0H/\tilde{\omega}_s)^2,$$

$$\omega_{2,3}=\pm\tilde{\omega}_s+\Big(1+\frac{\xi}{2}\Big)2\mu_0H\mp\frac{3}{8}\xi^2 2\mu_0H\frac{2\mu_0H}{\tilde{\omega}_s}. \tag{34}$$

For $\xi = 0$ the frequency ω_1 coincides with the usual paramagnetic resonance frequency; therefore, in what follows we shall also use this name for nonvanishing values of ξ. Correspondingly, we shall call ω_2 and ω_3 the antiferromagnetic resonance frequencies.

As is well known, at a certain critical value of the magnetic field, $H = H_c$, in an antiferromagnet a phase transition may occur into a state with noncollinear sublattice magnetic moments. For $H = H_c$ the transverse part of the static magnetic susceptibility tends to infinity, which corresponds to the vanishing of one of the resonance frequencies. Having set ω equal to zero in Eq. (32), we find the following expression for the magnetic field associated with the phase transition in an antiferromagnetic metal:

$$H_c = \frac{\omega_s}{2\mu_0}\sqrt{1-\xi} = \frac{\omega_0}{2\mu_0}. \qquad (35)$$

This means that in the system under consideration, an antiferromagnetic ordering can occur only for $\xi < 1$.

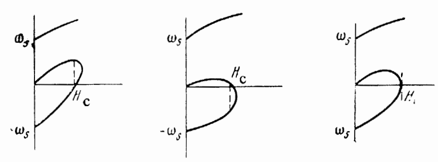
We note that $H = H_c$ may correspond to both the vanishing of the antiferromagnetic resonance frequencies, as occurs in an antiferromagnetic dielectric, and to the vanishing of the paramagnetic resonance frequency. In order to find out which of the indicated cases occurs, let us solve Eq. (32) near $H = H_c$ with respect to the frequency. Let us assume $H = H_c + h$; then for the frequency which tends to zero when $H = H_c$ we obtain

$$\omega = 2\mu_0 h \frac{2(\xi - 1)}{2 - 3\xi}. \qquad (36)$$

From here it is seen that for $\xi < 2/3$ we have $\omega < 0$, and the frequency ω_3 tends to zero. For $\xi > 2/3$ the paramagnetic resonance frequency tends to zero. For $\xi = 2/3$ this formula loses its meaning; however, from Eq. (32) one can see that here the paramagnetic and one of the antiferromagnetic resonance frequencies simulataneously tend to zero:

$$\omega = \pm 2\mu_0 H_c \left(\frac{2}{3}\frac{h}{H_c}\right)^{1/2}. \qquad (37)$$

The latter case is, of course, an exotic situation since ξ is an intrinsic parameter of the metal. The magnetic-field dependence of the resonance frequencies is shown schematically in the accompanying figure.



The case when ξ is close to unity is of special interest, that is, $1 - \xi \ll 1$. In this case it follows from formula (35) that the magnetic fields over the entire range of their variation are small in comparison with the antiferromagnetic frequency ω_s. Therefore, formulae (34) are valid over the entire admissible range, $H < H_c$. The case when ξ is close to unity corresponds to an anomalously small critical field whereas the antiferromagnetic resonance frequencies, which are determined by the exchange interaction between sublattices, are large as before. Thus, in this case we have to deal with a peculiar "hidden ferromagnetism" antiferromagnetic magnetic system. As is evident from Eq. (33), the value of ξ is to a significant degree determined by the factor $(1 + B_0)^{-1}$, whose tending to infinity is the condition for the appearance of ferromagnetism in the electronic subsystem.[3] In other words, the exchange interaction in the conduction-electron subsystem may play a dominant role in changing the nature of the magnetic structure of the lattice.

4. Let us determine the spin-wave frequencies for the system under consideration by solving Eq. (28) in a quadratic approximation with respect to the **k**. From expression (29), omitting the intermediate calculations which are entirely analogous to those carried out in order to determine $\tilde{\Gamma}(\omega, \mathbf{k})$, we obtain

$$\Pi(\omega, \mathbf{k}) = \Pi(\omega, 0) + \Delta\Pi,$$

$$\Delta\Pi = a^2 \nu g_\omega^2 \left(\frac{\omega - v\Delta}{\omega - 2\mu_0 H}\right)^2 \left\{X_{00}^{00} + \sum_{m=\pm1,0} \tilde{\Gamma}_{mm}^{11}(\omega, \mathbf{k}) \frac{a^2\nu}{4\pi} |X_{0m}^{01}|^2\right\}. \qquad (38)$$

With the aid of formulas (20)–(22) we can write $\Delta\Pi$ explicitly:

$$\Delta\Pi = \frac{1}{3} a^2 \nu g_\omega^2 \frac{\omega(\mathbf{k}v)^2}{(\omega - 2\mu_0 H)^2}(1 + B_1)\left\{\frac{\cos^2\Phi}{\omega - \omega_{10}(\mathbf{k})} + \frac{1}{2}\sin^2\Phi\left(\frac{1}{\omega - \omega_{11}(\mathbf{k})} + \frac{1}{\omega - \omega_{1,-1}(\mathbf{k})}\right)\right\}. \qquad (39)$$

As is evident from expressions (39) and (28), in the approximation quadratic in **k** not only the zeroth harmonic, $\omega_{00}(\mathbf{k})$, but also the first harmonics, $\omega_{1m}(\mathbf{k})$, intermix with the antiferromagnetic frequencies. Substituting expression (39) into the dispersion equation (28) it is not difficult to find the spin-wave spectrum near the eigenfrequencies $\omega_i (i = 1, 2, 3)$ and ω_{1m}. In order to avoid cumbersome expressions, we do not begin to write out the corresponding formulas, but present only, as an example, the expression for the spin-wave spectrum near the "paramagnetic" frequency ω_1 for small values of $2\mu_0 H/\omega_s$:

$$\omega_1(\mathbf{k}) = (1 - \xi)\left\{2\mu_0 H + \frac{1}{3}\frac{(1 + B_0)^2(1 + B_1)(kv)^2}{2\mu_0 H} \times \left[\frac{\cos^2\Phi}{B_0 - B_1 + (1 + B_0)\xi} + \sin^2\Phi \frac{B_0 - B_1 - (1 + B_0)\xi}{[B_0 - B_1 - (1 + B_0)\xi]^2 - q^2}\right]\right\}, \qquad (40)$$

where $q = m/m^*$ is the ratio of the free electron mass to its effective mass. For $\xi = 0$ the spectrum goes over into the expression given in article[8].

The possibility that one of the eigenfrequencies ω_{1m} coincides with one of the frequencies $\omega_i(0)$ represents an interesting case. This may occur at a definite value of the magnetic field, due to the fact that ω_{1m}, according to Eq. (18), is proportional to H whereas $\omega_i(0)$ as mentioned depends on the magnetic field in a nonlinear fashion. The spin-wave spectrum near the coincidence frequency turns out to be linear over practically the entire range of angles. For example, in the case of coincidence of the frequencies $\omega_1(0)$ and $\omega_{10}(0)$ the spin-wave spectrum has the form

$$\omega = \omega_1(0) \pm \left[\frac{(1 - \xi)(1 + B_0)(1 + B_1)}{3}\right]^{1/2} kv\cos\Phi + O(k^2). \qquad (41)$$

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