

*EFFECT OF THE INHOMOGENEITY OF THE RADIATION FIELD IN THE ACTIVE ROD  
OF THE DYNAMICS OF A RUBY LASER*

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Variations of the spectrum and of the angular and transverse energy distributions of the radiation from a ruby laser operating under free generation conditions in resonators with plane or spherical mirrors were investigated experimentally, and it was concluded that the inhomogeneities of the spatial distribution of the standing wave fields of the modes affected the parameters of the laser emission. It was found that the spectrum width integrated over the whole emission time was governed mainly by the combined effect of the random selection of ruby elements and a transverse inhomogeneity. Longitudinal and transverse inhomogeneities reduced the width of the spectrum generated in each emission spike. The origin of undamped spikes was explained qualitatively. A strong effect of an inhomogeneity, due to excitation of different modes in different regions, on the damping rate of regular spiking was detected. A method for increasing the efficiency of mode selection was suggested and realized experimentally.

## INTRODUCTION

TANG et al.<sup>[1]</sup> were the first to note that, because of a slight spatial diffusion of the population inversion in a ruby laser during emission, the distribution of the inversion in an active rod became inhomogeneous. Assuming that a resonator of a real laser is equivalent to a resonator with spherical mirrors even if these mirrors are plane,<sup>[2]</sup> we can distinguish three types of such inhomogeneity.

1. An inhomogeneity due to the caustic surface bounding the volume where a mode is excited. The cross section of this surface, regarded as a function of the length of a resonator  $L$  with square-apertured spherical mirrors of radius of curvature  $r$ , can be written in the following form:<sup>[2]</sup>

$$D_m = \chi_m \left[ \frac{(2rL - L^2)^{1/2} \lambda}{\pi} \left( 1 + \frac{4z^2}{2rL - L^2} \right) \right]^{1/2}. \quad (1)$$

Here,  $\chi_m$  represents coefficients which increase with increasing transverse index of a mode  $m$ , which have been calculated by Leontovich and Veduta<sup>[2]</sup> for  $m \leq 14$ ; the coefficients for higher values of  $m$  can be found from the formula<sup>[3]</sup>

$$\chi_m = (2.09m + 5.9)^{1/2}. \quad (2)$$

It is evident from Eq. (1) that  $D_m$  is a function of  $z$  (which is the coordinate along the resonator axis), which gives rise to an inhomogeneous spatial distribution of the population inversion during generation of a mode with a transverse index  $m$ .

2. A transverse inhomogeneity, due to a number of maxima  $m$  in the distribution of the mode field along a direction perpendicular to  $z$ . The characteristic linear dimension of this inhomogeneity for modes usually excited in a laser with plane mirrors amounts to about 0.1 mm, as demonstrated in several experimental investigations.

3. A longitudinal inhomogeneity due to a distribution

of the fields of the axial modes. The characteristic linear dimension of this inhomogeneity is equal to half the wavelength of the emitted radiation.

A large number of theoretical and experimental papers have been published on the influence of the inhomogeneity of the third type on laser dynamics.<sup>[1, 4-8]</sup> The main results obtained so far can be summarized as follows: the longitudinal inversion inhomogeneity is the cause of the multimode emission of ruby; the experimental width of the integrated spectrum of the emitted radiation agrees, to within an order of magnitude, with the theoretical value obtained from the solution of the rate equations, taking into account the longitudinal inhomogeneity on the assumption of the existence of steady-state conditions; the numerical solution of these equations under transient conditions in the case of generation of not more than three modes gives rise to, as in the case of an inhomogeneous field distribution, a regular spiking kinetics with damping to some level of the constant component;<sup>1)</sup> when the longitudinal inhomogeneity is eliminated, it is possible to obtain single-mode emission conditions.

However, it has been established in a large number of investigations that in the most widely used lasers with plane mirrors, as well as in lasers with spherical mirrors when the resonator is far from the concentric or confocal shape, one usually observes transient conditions of random spikes with sudden changes of frequencies of the emitted modes from one spike to another. Hence, it follows that the agreement between the theoretical and experimental values of the integrated spectrum, reported in<sup>[1, 4]</sup>, is—to a considerable degree—accidental. Irregular undamped spikes are observed almost throughout the whole range of possible laser parameters encountered in practice and not only

<sup>1)</sup> An irregular spiking kinetics, reported in [4] for the case of generation of two modes, is obtained assuming such laser parameters which are not encountered in practice (for example, a  $Q$  of  $10^3$ ).

under the conditions assumed in [4]. Moreover, when the longitudinal inhomogeneity is smoothed out [9] at pumping powers exceeding the laser threshold by 20%, it is found that the number of the generated modes increases, which supports the suggestion that the longitudinal inhomogeneity is probably not the main cause of the multimode emission of a ruby laser. The elimination of the longitudinal inhomogeneity, achieved in [5, 7], narrows the spectrum but no rigorous proof of the single-mode emission has been provided.

Statz and Tang [4] derived the rate equations taking into account inhomogeneities of the second and third types but they have not solved these equations because of their complexity. Korobkin, Leontovich, and Smirnova [3] demonstrated that, in the case of an almost concentric resonator, the difference between the excitation volumes of modes with different transverse indices gives rise to the mode competition at the beginning of the emission, and that simultaneous excitation of a large number of modes (which should have similar values of  $Q$ ) eliminates the transverse and longitudinal inhomogeneities and this produces regular kinetics of spiking. In this case, <sup>2)</sup> the inversion homogeneity in the active medium is the consequence of the excitation of a large number of modes, while the interest lies in effective methods of constructing single-mode and single-frequency lasers and the influence of these inhomogeneities in the case of excitation of a small number of modes, which is true of lasers with resonators far from the concentric shape, particularly in the presence of selective elements in the resonators.

With this view in mind we carried out several experiments and the results are reported in the present paper.

## EXPERIMENTS AND DISCUSSION

We used a suitably prepared ruby rod, 7 mm in diameter and 120 mm long. To eliminate the possibility of obtaining accidental results, the experiments were repeated using a different ruby rod of the same dimensions.

1. Figure 1 shows the time-resolved slit distributions of the intensity of radiation in the far zone (a) and interferograms of the emission (b) during one spike. Figures 1c and 1d show similar time-resolved distributions of the radiation at a mirror and interferograms for the same spike. The laser resonator consisted of two plane mirrors with the reflectivities  $R_1 = R_2 = 0.95$ ; these mirrors were separated by a distance of 60 cm. The dispersion region of the Fabry-Perot etalon was  $0.25 \text{ cm}^{-1}$  and the pumping power exceeded the threshold value by 50%. An analysis of Fig. 1 shows the following features:

A. The spectrum of a spike usually consists of one packet (rarely two or three packets) of closely spaced frequencies, whose width is  $0.01\text{--}0.05 \text{ cm}^{-1}$  and whose position relative to the center of the line varies randomly from spike to spike.

B. The nature of the spatial distribution of the field in the near and far zones is similar, in its general

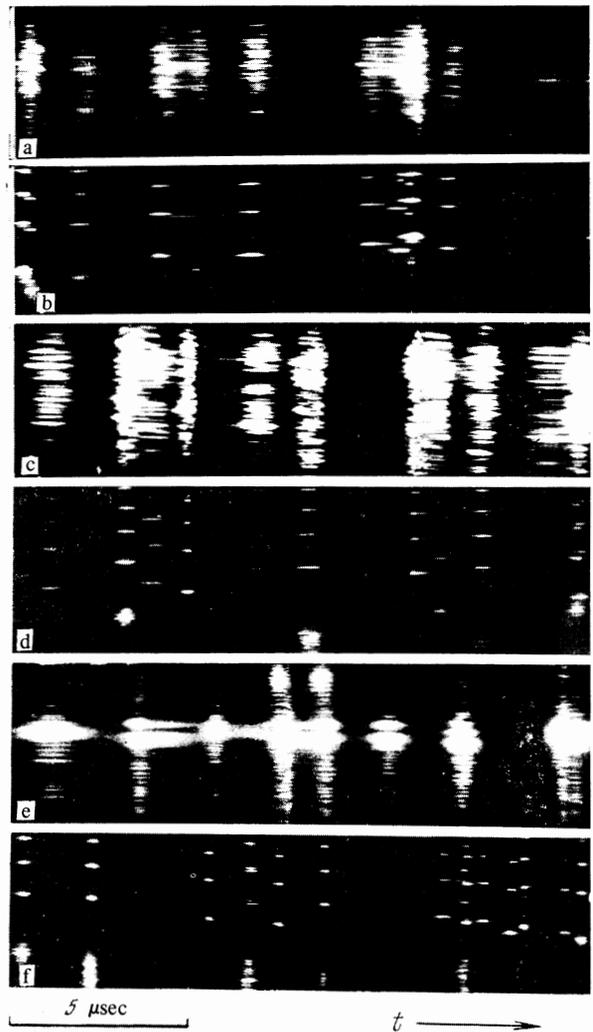


FIG. 1

form, to the distribution of modes in a spherical resonator. We can see that some parts of the rod may emit independently but in the far zone the emission is overlapped by the emission of other regions<sup>3)</sup> and the emission maxima have a more uniform distribution.

C. The complexity of the distribution pattern of the emission in the near and far zones is correlated with the number of frequency packets in the spectrum of a spike: if one packet is generated, the distribution pattern consists clearly of alternating maxima and minima but in the presence of a larger number of packets the distribution pattern is spread out. Changes in the position of a packet along the frequency axis from one spike to another are usually accompanied by changes in the transverse distribution; and measurements, carried out on a large number of photographic frames, indicate a definite tendency for a maximum to appear at that point of the rod end (or along that angular direction) at which a minimum has been observed in the preceding spike.

This feature is observed easily at the beginning of the laser emission at low pumping levels, when modes

<sup>2)</sup> The variant with a spherical resonator was also considered in a recently published paper by Ratner et al. [10]

<sup>3)</sup> This feature of the emission at higher pumping powers was pointed out in [2].

with small transverse indices are excited (Fig. 1e). It follows that each transverse mode corresponds to a "packet" of frequencies in the spectrum in which each of the frequencies corresponds, in turn, to different axial indices of this mode. Thus, the jumps in the average emission frequency, which govern the integrated width of the spectrum ( $1 \text{ cm}^{-1}$ )—and have been reported in all the investigations of ruby lasers—are associated with a change in the transverse index of the excited modes. The magnitudes of the frequency jumps from one spike to another are, however, usually much larger than the separation between two modes with the transverse index differing by unity, which, in the  $r \gg L$  case, should be given by the relationship<sup>[2]</sup>

$$\Delta\nu_m = \frac{c}{\pi} \sqrt{\frac{2}{rL}}, \quad (3)$$

where  $r$  is the radius of curvature of the mirror;  $L$  is the resonator length;  $c$  is the velocity of light.

In the case of plane mirrors, some equivalent radius of curvature appears due to the inhomogeneities in the ruby rod. According to Leontovich and Veduta,<sup>[2]</sup> this radius may be of the order of  $2 \times 10^2 - 6 \times 10^2 \text{ cm}$ . Then, Eq. (3) yields  $\Delta\nu_m = 10^8 \text{ cps}$ , i.e., the transverse mode separation is of the same order as the separation between axial modes ( $\Delta\nu_q = c/2L \approx 2 \times 10^8 \text{ cps}$ ).

Thus, the transverse inhomogeneity cannot be the cause of the frequency jumps greater than  $\Delta\nu_m - \Delta\nu_q \approx 10^8 \text{ cps}$  because a change in the transverse index would always give rise to the excitation of modes of frequencies close to the center of the line because that is where the gain is greatest. The longitudinal mode index would also vary in the same manner.

In this case, the influence of the longitudinal inhomogeneity is small because in each subsequent spike, due to the similarity of the transverse distribution of the field across any section of the caustic surface, the field maxima are located at the points of the minima in the preceding spike throughout the length of the active rod. Moreover, not one but between three and seven modes with different axial indices are excited in one spike and this produces a more uniform de-excitation than in the transverse direction. These considerations are supported experimentally by the observation that when one of the resonator mirrors is replaced by an uncoated substrate (in this case, the "traveling" property of the waves increases strongly while the transverse inhomogeneity remains unchanged), the general nature of the dynamics of the spectrum (Fig. 1f) as well as its integrated width do not change appreciably.

2. Figure 2 shows the time-resolved slit interferograms of the emission in the following cases: a) the laser resonator contained two selector plates; the mirror substrates and these plates were not plane-parallel but the angles between their surfaces varied within the limits  $1' - 3'$ ; the plates were adjusted so that the normals to their surfaces should make the smallest possible angle with the ruby rod axis; b) the plates were removed from the resonator; c) the plates were removed from the resonator but the ruby rod was turned through  $1^\circ$  and one of the mirror substrates was turned by  $30'$ .

The dispersion region of the Fabry-Perot etalon was  $0.3 \text{ cm}^{-1}$  (a),  $0.25 \text{ cm}^{-1}$  (b), and  $0.16 \text{ cm}^{-1}$  (c). Comparison of the photographs given in Fig. 2 showed that when

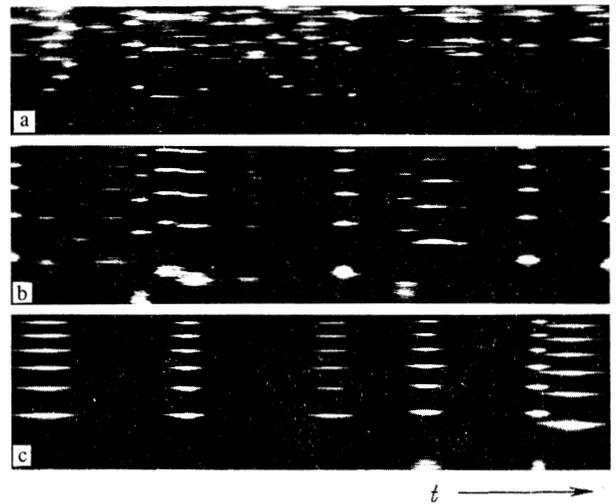


FIG. 2

the number of selector elements was reduced, the frequency jumps from one spike to another decreased strongly.<sup>4)</sup>

Thus, the width of the ruby laser spectrum, integrated over the total emission time, is governed by the simultaneous effect of two factors: "disordered" selection and transverse inhomogeneity. The "disordered" selection is understood to be the following effect. In an ordinary laser with plane mirrors, there are six surfaces, which separate different media (glass, air, ruby) and which have different reflection coefficients. They are not located at equal distances from one another, and, in general, they have different angles of inclination with respect to the ruby rod axis. When the inhomogeneities in the active rod are taken into account, we find that we have a resonance system with complex and, to a considerable degree, random frequency characteristics, which are different for different angular directions. The curve representing the sum of the gain curve of the active medium and the resonance curve of such a system naturally has a series of irregular frequency maxima (somewhere near the center of the line), whose amplitude differences may be smaller than the difference of the gain for different modes due to the transverse inhomogeneity. The first emission spike develops under conditions when there is no inhomogeneity. In this case, a mode is excited corresponding to the strongest of these maxima because during gradual inversion the threshold conditions are reached for this transition first. An investigation of a large number of photographic frames depicting the dynamics of the transverse distribution shows that, in the majority of cases, a mode with  $m \geq 10$  is excited. In the next spike, the energy conditions favor the excitation of the mode whose maxima in the transverse direction coincide as closely as possible with the inversion maxima. Because of a change in the angular characteristics, the effect of the selector elements in a laser will be different for this mode, and this is responsible for the magnitude of the frequency jumps and for the fluctuations of the width of the spectrum from one spike to another.

<sup>4)</sup> A similar effect of the selection in the case of isolation of a single transverse mode by a stop was reported in [11].

The width of the spectral emission line, generated in each spike, may be reduced by the longitudinal and transverse inhomogeneities because they introduce an additional difference in the effective gain of various modes, governed by their distance from the center of the luminescence line and by the value of  $Q$ . Consequently, during the linear development time some modes may overtake others quite considerably and prevent their development during the generation of a spike. Thus, in a laser with a saturable filter,<sup>[12]</sup> in which the linear development time of the emission is of the order of  $10^{-6}$  sec, i.e., approximately of the same order as under free generation conditions, the relative difference in the effective gain of two modes of the order of  $10^{-3}$  is found to produce a threefold difference between their intensities before the beginning of the nonlinear development. This is in agreement with the results presented in<sup>[3]</sup>, where it is reported that in the absence of selection and in the case of a homogeneous initial distribution of inversion, a spike contains modes for which the relative difference in the effective gain is of the same order of magnitude ( $6 \times 10^{-4}$ ). The fact that in the presence of a spatial inhomogeneity, other conditions being equal, the spectrum of a spike becomes narrower (as demonstrated in the case without selection<sup>[3]</sup> and with the disordered selection in our earlier investigation<sup>[13]</sup>), indicates that the spatial inhomogeneity does indeed increase considerably the difference between the effective mode gain and narrows the spectrum of a spike.

In order to check to what extent the relatively large number of modes with different axial indices, excited in a spike in the absence of selection (cf., for example, <sup>[14]</sup>), is the consequence of the spatial inhomogeneity, we carried out the following experiment. In a laser with spherical mirrors having radii of curvature  $r_1 = r_2 = 10^2$  cm reflectivities  $R_1 = R_2 \approx 0.95$ , and a separation of 70 cm, we eliminated the transverse and longitudinal spatial inhomogeneities by the method described in<sup>[13]</sup>. Figure 3 shows oscillograms obtained in ordinary emission (a) and in the case when the spatial inhomogeneities had been smoothed out (b). We can see that, instead of irregular undamped spikes, we obtain regular damped spikes, which—as demonstrated clearly by numerous comparisons of changes in the spectrum and emission kinetics—always indicate the disappearance of frequency jumps from one spike to another. The fact that the in-

tegrated width of the spectrum is then approximately equal to the dispersion region of the etalon ( $0.17 \text{ cm}^{-1}$ ) shows that not less than 20 modes with different axial indices are excited in a spike.

Thus, the following conclusions can be drawn from our discussion.

A. The free emission conditions in a ruby laser are affected mainly not by the longitudinal but by the transverse inversion inhomogeneity.

B. Strictly speaking, the spatial inhomogeneity of the inversion is not the cause of the multimode emission. It is responsible for the jumps from one mode to another during transient emission. Therefore, at a given moment, the emission can be of the single-mode type.

C. The number of modes generated at a given moment is determined by the difference between the effective gain of various modes, by which we understand here the simultaneous effect of the following factors: the  $Q$  of a mode, the distance of a mode from the center of the luminescence line, the selective effect of all the ruby elements, and the spatial distribution of the inversion before the beginning of emission. An increase of this difference reduces the number of emitted modes. Conversely, the presence of the spatial inhomogeneity tends to narrow the emission spectrum of a peak in the absence of selection or in the case of disordered selection.

D. In the cases considered, the smoothing-out of the spatial inhomogeneity cannot produce single-mode emission because, by removing the jumps from one mode to another, we simultaneously produce conditions favorable for the emission of a large number of modes in a single spike; this happens because the leveling of the effective gain of various modes is stronger than the increase of the mode competition due to the merging of the excitation regions of various modes during the uniform de-excitation of each of them.

3. On the basis of the reported results, we may conclude that the single-mode emission can be obtained only when the effective gain for each selected mode exceeds the gain for the neighboring modes by an amount greater than the change in the effective gain introduced by the spatial inhomogeneity of the inversion produced by the field of the standing wave of the selected mode. The results of numerous investigations on the mode selection in a ruby laser indicate that such a situation can

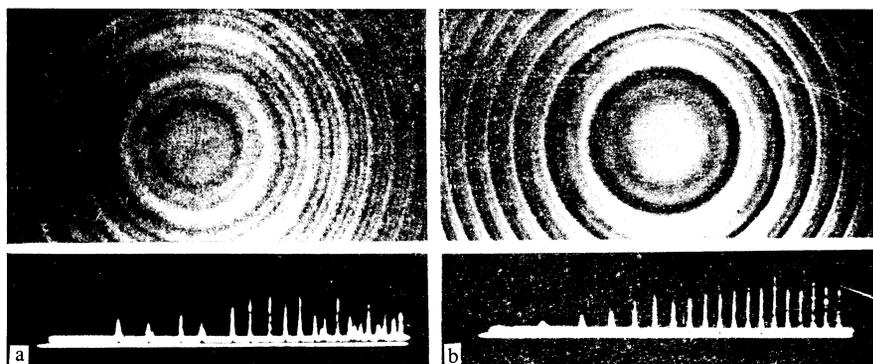


FIG. 3

be obtained only at low pumping levels and, consequently, at low inversion levels. The smoothing-out of the spatial inhomogeneity should result in a considerable increase of the effectiveness of such selection. This point was first made by Galanin et al.<sup>[15]</sup> We carried out an experiment which confirmed this conclusion.

Our laser generator consisted of a single spherical (radius of curvature  $r = 10^2$  cm) mirror, having a reflectivity  $R = 0.95$ , and, at a distance  $L = 70$  cm, a resonance reflector in the form of a glass plate 1 cm thick, on whose surface we deposited reflective coatings with  $R_1 = R_2 = 0.8$ . The transverse and longitudinal inhomogeneities were eliminated by the method described in<sup>[13]</sup>. The ruby rod was turned relative to the surfaces of the resonance reflector through an angle of  $1^\circ$ . Because of this, and because of the weakening of the selectivity due to the sphericity of the wave front within the resonator (in this case, the front would be plane only on the surface of the plane reflector), the stray selection was practically eliminated.

Figure 4a shows an oscillogram and an interferogram of the emitted radiation in the case of an inhomogeneous spatial distribution of the population inversion. We can see that the spectrum consists of two lines  $\sim 0.02$ – $0.03$   $\text{cm}^{-1}$  wide, which evidently correspond to maxima in the reflection of the selector and each of these maxima includes several resonator modes; the spiking kinetics is irregular, which indicates that different modes are excited in different spikes. In the case of uniform de-excitation (Fig. 4b) only one line remains in the spectrum whose width is  $\sim 0.01$   $\text{cm}^{-1}$ , which is of the same order of magnitude as the separation between the modes whose axial indices differ by unity and is also of the same order as the instrumental width of the etalon. In this case, the kinetics is regular with damping to a constant level, which is in qualitative agreement with the conditions predicted by solutions of the rate equations on the assumption that a single mode is produced and that the inversion and field distributions are uniform (cf., for example,<sup>[11]</sup>).

The width of the spectrum is reproduced from one spike to another for pumping levels up to twice as high as the threshold value. (The limit was set by the apparatus.) However, the emission kinetics in some cases, particularly at high pumping levels, is of the type shown in Fig. 5c. We have already investigated such conditions,<sup>[13]</sup> and demonstrated that the features of the observed emission are associated with the effect of the

inhomogeneity of the first type (cf. Introduction), whose influence will be described later. Consequently, in this case we obtain conditions which are close not to the single-mode case but rather to the single-frequency emission. It must be mentioned that at relatively high (3.5 kJ) pumping levels the emitted frequency is the same as in<sup>[13]</sup> and it does not change during the emission (estimates carried out using the data of Veduta, Leontovich, and Smorchkov<sup>[16]</sup> show that under our conditions the luminescence line suffers a temperature shift by an amount  $> 0.15$   $\text{cm}^{-1}$ ).

4. In the preceding sections, we discussed the influence of inhomogeneities of the second and third types on the emission spectrum. It was demonstrated in<sup>[3]</sup> that when a large number of modes is excited under uniform de-excitation conditions, the spiking kinetics is regular. However, the problem of the damping of spikes has not yet been resolved completely. Experimental observations include random undamped spikes (in lasers with resonators under far from critical conditions), and regular undamped as well as damped spikes (in lasers with resonators in which conditions are close to critical). The problem why random spikes are undamped was discussed briefly in<sup>[4]</sup> but the theoretical discussion was limited to a special case. The damping of the regular spikes follows from the solution of the rate equations but there are considerable discrepancies between the theoretical and experimental data and the interpretations are contradictory of the dependence of the rate of damping on the number of excited modes.<sup>[17, 18]</sup> We shall not try to give a rigorous quantitative explanation of the various conditions, which would be pointless in view of the large number of factors which influence emission, but we shall try to determine qualitatively the causes of the described behavior on the basis of the experimental data.

The origin of the random undamped spikes becomes quite obvious when we consider variation of the spectrum (Figs. 1b, 1d, and 1f). It follows from the solution of the rate equations (cf. for example,<sup>[17]</sup>) that at the end of generation of a spike the energy stored in the vibrational modes does not decay to the noise level. This residual field provides the starting point for the development of the next spike after the threshold conditions are reached again. Since, under random conditions, different modes are generated in successive spikes and the excitation regions of these modes are only partly uncoupled from the excitation regions of the

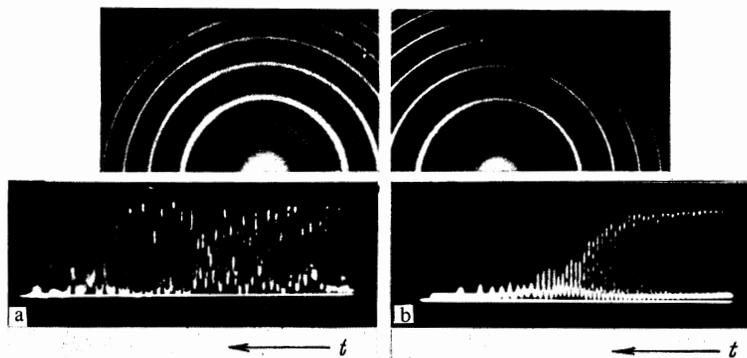


FIG. 4

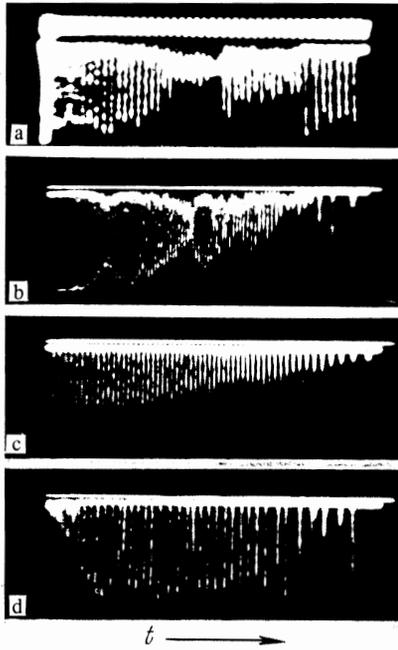


FIG. 5

preceding peaks, the residual field of the modes of the preceding peak may decay completely. Thus, in each peak the emission begins anew, from the noise level, and the undamped spiking is observed. When the radius of curvature of the mirrors is reduced so that the resonator approaches the spherical ( $L = 2r$ ) or confocal ( $L = r$ ) shape, leveling of the  $Q$  values of the modes and weakening of random selection takes place. The conditions are gradually becoming more and more regular.

Figure 5a shows an oscillogram of the radiation emitted by a resonator of nearly confocal shape ( $r_1 = r_2 = 10^2$  cm,  $L = 70$  cm) under conditions such that the threshold pumping level is exceeded by 50%. We can see that at this pumping level almost regular undamped spikes are observed. Figure 5b shows an oscillogram for the same laser under the same pumping conditions but in which the transverse inhomogeneity<sup>5)</sup> has been eliminated by the method described in [13]. We can see that now the spiking is damped. The result obtained is reproduced without exception in a large number of emission spikes. Thus, the main cause of the regular undamped spiking is the alternation, from spike to spike, of modes with different transverse indices but similar values of  $Q$ .

Figure 5c shows a similar oscillogram for a laser with a resonator consisting of a single spherical mirror with  $r = 10^2$  cm and a single plane mirror, separated by a distance  $L = 70$  cm, at a pumping level 100% higher than the threshold value. Figure 5d shows an oscillogram for a resonator consisting of plane mirrors separated by  $L = 70$  cm at the same pumping level. Comparison of Figs. 5b, 5c, and 5d yields the following conclusion. In all cases, changes in the magnitude and sign of the rate of damping of the peaks are observed

<sup>5)</sup> As demonstrated experimentally in Sec. 2, the longitudinal inhomogeneity in such a laser is unimportant because of the large number of the excited axial modes.

during the emission process. As demonstrated in [13], such changes are due to the influence of the inhomogeneity of the first type. Since the rate of damping is proportional to the value of  $Q$ , the modes with higher values of  $Q$  overtake and prevent the generation of modes with lower values of  $Q$  and larger transverse indices. Since the latter modes have, according to Eqs. (1) and (2), larger excitation regions, population inversion begins to take place in part of the active medium and this means that the threshold conditions may be reached again for modes of high transverse order. The replacement of two plane mirrors in the resonator with one, and then with two spherical mirrors, reduces the difference between the values of  $Q$  for modes with different transverse indices and also yields the following expression, from Eqs. (1) and (2), for the difference between the dimensions of the sections of the caustic surface at the resonator center:

$$\Delta D = D_m - D_{m-1} = [\lambda(2rL - L^2)/\pi]^{1/2}, \quad (4)$$

which shows that this difference decreases with decreasing  $r$  when  $L$  is kept constant. Hence, it follows that changes in the damping rate in a resonator with plane mirrors should be stronger, which is indeed observed in the cited oscillograms.

Thus, when the longitudinal and transverse inhomogeneities of the population inversion are smoothed-out, we can obtain regular damped spiking. The rate of damping is affected considerably by the inhomogeneity associated with the difference between the excitation regions of the modes in resonators with spherical or equivalent mirrors, which must be taken into account in comparison of these emission conditions with those obtained from the solutions of the rate equations.

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67