NONSTATIONARY EQUATIONS FOR SUPERCONDUCTORS WITH LOW CONCENTRATION OF PARAMAGNETIC IMPURITIES

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The dynamic properties of the model of a superconductor with a small content of paramagnetic impurities have much in common with the properties of ordinary superconducting alloys. At the same time, in a definite temperature region near T_c, it is possible to obtain for this model equations describing the change of the parameter Δ with time. These equations differ very greatly from those obtained earlier for the case of gapless superconductivity. The obtained equations are used to solve certain problems of importance in the clarification of the dynamic properties of the parameter Δ. In particular, the region of low frequencies of the external field, where Δ varies with the field adiabatically, turns out to be much narrower than expected.

THE level density in the excitation spectrum of pure superconductors and alloys with nonmagnetic impurities have, as is well known, a singularity at the energy $\varepsilon = \Delta$. This is connected to a considerable degree with the fact that there is no simple scheme, such as the Ginzburg–Landau theory in the static case, capable of describing the dynamic behavior of the parameter Δ even near the transition temperature T_c. The introduction of paramagnetic impurities leads to a smoothing of the singularity in the spectrum.[1] Therefore the simplest superconducting system from the point of view of the dynamic properties, is a superconductor with a large concentration of paramagnetic impurities ($\tau_s T_c \ll 1$, where τ_s is the transit time with spin flip). The equations of electrodynamics for such a model, obtained by Gor'kov and the author,[2] are analogous to the Ginzburg–Landau system of equations. In particular, in dimensionless form they contain a single parameter κ, if the time scale is the quantity $(\tau_s \Delta_0^2)^{-1}$, where Δ_0 is the equilibrium value of Δ.

On the other hand, the relative simplicity of this model obscures some important features of the dynamics of superconductors containing no magnetic impurities. Some of these features can be clarified by considering the case of low paramagnetic-impurity concentration, when $\tau_s T_c \gg 1$. The presence of a certain amount of impurities smoothes out partially the singularity in the spectrum, and consequently it becomes impossible, in the temperature region where $\tau_s \Delta \ll 1$, i.e.,

$$(T_c - T) / T_c \ll 1 / (\tau_s \Delta)^2,$$

to simplify greatly the general scheme of the theory.

DEVIATION OF EQUATIONS

From the general equations determining the nonstationary properties of superconductors[2] it follows that the expression for $\Delta(\mathbf{r}, t)$ (and also for the current density $\mathbf{j}(\mathbf{r}, t)$) consists of two parts. The first part is regular and has the property that near the transition temperature it can be expanded in powers of Δ/T_c, and when the field frequency is decreased it goes over into the corresponding static expression. The second part contains integrals of the products of the retarded and advanced Green's functions, and is therefore very sensitive to the details of the spectrum. It depends significantly on the relations between Δ, the frequency, and the spatial gradients.

Taking into account the anomalous part, the equation for Δ near the transition temperature can be written in the form

$$-\frac{\pi}{8T}\left[\frac{\partial}{\partial t} - D_1\left(\nabla - \frac{2ie}{c}A\right)^2\right]\Delta + \left[\frac{T_c - T}{T_c} - \frac{7\zeta(3)}{8(\pi T)^2}|\Delta|^2\right]\Delta + \Delta U = 0, \tag{1}$$

where the last term is the anomalous part. The coefficient D_1 depends on the total concentration of the impurities, including the nonmagnetic ones. Within the framework of the chosen model, we can consider either the case of an alloy, when $\tau_1 T \ll 1$, or the case of a superconductor which is pure with respect to its equilibrium properties: $\tau_1 T \gg 1$. The complete expression for D_1 is of the form

$$D_1 = \frac{v^2\tau_1}{3} f(\tau_1 T), \quad f(\tau_1 T) = \frac{8}{\pi^2}\sum_{n>0}\frac{1}{(2n+1)^2[2\pi(2n+1)\tau_1 T + 1]}. \tag{2}$$

When $\tau_1 \ll 1$ we have $f \approx 1$, and when $\tau_1 T \gg 1$ we get $f(\tau_1 T) \sim 7\zeta(3)/2\pi^3\tau_1 T$. Equation (1) without the anomalous part has already been presented by a number of authors.[3,4] It will be shown below that the role of the anomalous part is very large.

An investigation of the quantity U can be carried out with the aid of the previously described diagram technique.[2] We review here its main rules. In the case of a superconductor with paramagnetic impurities, expanding ΔU in powers of Δ and of the electromagnetic field, we obtain diagrams, some of which are shown in Fig. 1. On these diagrams the electron lines forming the upper part of the diagram correspond to the denominators $(\xi - \varepsilon - i/2\tau_1)^{-1}$ for lines with arrows to the right and $(\xi + \varepsilon + i/2\tau_1)^{-1}$ for lines with arrows to the left; for the lines in the lower part of the diagram we have respectively $(\xi + \varepsilon - i/2\tau_1)^{-1}$ and $(\xi - \varepsilon + i/2\tau_1)^{-1}$. A tri-

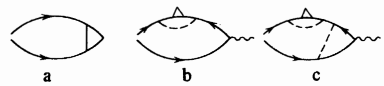

FIG. 1

angle corresponds to the parameter Δ, and a wavy line to the vertex of interaction with the electromagnetic field $(e/mc)\,(\mathbf{p}\cdot\mathbf{A}) - e\varphi$. Dashed lines correspond to impurities. In the case of isotropic scattering, which we assume for simplicity, a dashed line encompassing an even number of vertices Δ corresponds to the factor $(2\pi\tau_1)^{-1}$, and to the factor $(2\pi\tau_2)^{-1}$ in the case of an odd number of vertices Δ. Here τ_1 and τ_2 are the relaxation times introduced by Abrikosov and Gor'kov, and the difference $\tau_1^{-1} - \tau_2^{-1} = 2\tau_S^{-1}$ determines the spin transit time. Finally, the right-hand vertex contains the factor

$$\operatorname{th}\frac{\varepsilon}{2T} - \operatorname{th}\frac{\varepsilon-\omega}{2T} \approx \frac{\omega}{2T}\operatorname{ch}^{-2}\frac{\varepsilon}{2T},$$

where ω is the frequency corresponding to the Δ in this vertex or to the field.

By way of an example we present an expression corresponding to the diagram of Fig. 1, b, from which the remaining details of the technique are clear:

$$\frac{1}{4\pi i}\int_{-\infty}^{\infty} d\varepsilon \int_{-\infty}^{\infty} d\xi \frac{dO_{\mathbf{p}}}{4\pi}\frac{1}{\xi-\varepsilon-i/2\tau_1}[\ldots]\cdot$$
$$\times\frac{1}{\xi-\mathbf{v}(\mathbf{k}-\mathbf{k}_1)+\varepsilon-\omega+\omega_1+i/2\tau_1}\,\frac{1}{\xi-\mathbf{vk}+\varepsilon-\omega-i/2\tau_1}$$
$$\times\left(\operatorname{th}\frac{\varepsilon-\omega+\omega_1}{2T}-\operatorname{th}\frac{\varepsilon-\omega}{2T}\right)\left(\frac{e}{mc}(\mathbf{pA})-e\varphi\right)_{\mathbf{k}_1,\,\omega_1},$$
$$[\ldots]=\frac{1}{2\pi\tau_2}\int d\xi'\int\frac{dO_{\mathbf{p}'}}{4\pi}\cdot$$
$$\times\frac{\Delta(\mathbf{k}-\mathbf{k}_1,\,\omega-\omega_1)}{(\xi'-\varepsilon-i/2\tau_1)[\xi'-\mathbf{v}'(\mathbf{k}-\mathbf{k}_1)+\varepsilon-\omega+\omega_1+i/2\tau_1]}\qquad(3)$$

The method of deriving the equation for U is essentially the same as in the case of large paramagnetic-impurity concentration.[2] The summation of the ladder of dashed lines encompassing the vertex Δ on the upper or lower line makes it necessary to write throughout in lieu of Δ

$$\tilde{\Delta}=\Delta\frac{\varepsilon\pm i/2\tau_1}{\varepsilon\pm i/\tau_s},$$

where the plus and minus signs pertain to the upper and lower lines, respectively. The quantity Δ in the right-hand vertex remains unchanged. The summation of the

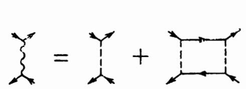

FIG. 2

ladder shown in Fig. 2 leads to the following value of $D_k(\omega)$:

$$D_k(\omega)=\frac{1}{2\pi\tau_1}\frac{1}{(-i\omega+Dk^2)\tau_1},\qquad(4)$$

where $D = v^2\tau_1/3$. It is important to note that (4) contains the usual diffusion coefficient (and not D_1 as in Eq. (1)) even in the case when the ordinary number of impurities is small, for this requires only that the quantities $\omega\tau_1$ and $kv\tau_1$ be small. The small denominator in (4) makes it necessary to sum additionally diagrams containing an arbitrary number of staircases $D_k(\omega)$, separated by parts containing Δ and Δ^*. The assumption that the quantity $\tau_S\Delta$ is small allows us to confine ourselves to separation diagrams of the order of Δ^2. The summation of such diagrams leads to equations for the vertex parts Γ^+ and Γ^- shown in Fig. 3.

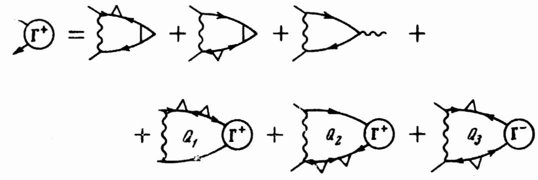

FIG. 3

In calculating the separate terms of these equations, it is necessary to bear in mind formula (3). In addition, the kernels Q should be suitably averaged over the impurities, so that, for example, the kernel Q_1 corre-

FIG. 4

sponds to the diagrams of Fig. 4. As a result of the calculations we obtain for the quantities $\Gamma^+ - \Gamma^-$ and $\Gamma^+ + \Gamma^-$ equations that assume the following form in the coordinate representation:

$$\left(\frac{\partial}{\partial t}-D\nabla^2\right)(\Gamma^+-\Gamma^-)=-\frac{i}{\tau_1}\frac{1}{2T}\operatorname{ch}^{-2}\frac{\varepsilon}{2T}\frac{\varepsilon}{\varepsilon^2+\tau_s^{-2}}\frac{\partial|\Delta|^2}{\partial t},\qquad(5a)$$

$$\left(\frac{\partial}{\partial t}-D\nabla^2\right)(\Gamma^++\Gamma^-)$$
$$=-\frac{1}{\tau_1}\frac{1}{2T}\operatorname{ch}^{-2}\frac{\varepsilon}{2T}\frac{1}{\tau_s(\varepsilon^2+\tau_s^{-2})}\left(\Delta\frac{\partial\Delta^*}{\partial t}-\Delta^*\frac{\partial\Delta}{\partial t}\right)$$
$$-\frac{2i}{\tau_1}\frac{1}{2T}\operatorname{ch}^{-2}\frac{\varepsilon}{2T}\frac{\partial}{\partial t}\left[e\varphi+\frac{e}{c}D\operatorname{div}\mathbf{A}\right]-\frac{2}{\tau_s}\frac{|\Delta|^2(\Gamma^++\Gamma^-)}{\varepsilon^2+\tau_s^{-2}}.\qquad(5b)$$

The quantity ΔU can be expressed in terms of Γ^+ and Γ^-. This connection is shown in Fig. 5. Diagrams

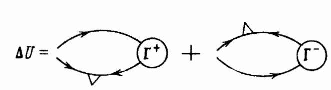

FIG. 5

containing no $\Gamma^{+(-)}$ are small. Simple calculation yields

$$U=\frac{\tau_1}{4i}\left\{\int d\varepsilon\frac{\varepsilon}{\varepsilon^2+\tau_s^{-2}}(\Gamma^+-\Gamma^-)+\frac{i}{\tau_s}\int d\varepsilon\frac{\Gamma^++\Gamma^-}{\varepsilon^2+\tau_s^{-2}}\right\}.\qquad(6)$$

Equations (1), (5), and (6) determine the parameter Δ. A shortcoming of this system is that it contains quantities $\Gamma^{+(-)}$ which depend not only on the coordinates and the time but also on the energy variable ε. This can be partly avoided by changing over to equations for the phase χ and the modulus $|\Delta|$. It then turns out that the equation for $|\Delta|$ contains only quantities integrated with respect to ε:

$$\frac{\pi}{8T}\left\{-\frac{\partial|\Delta|}{\partial t}+D_1\nabla^2|\Delta|-D_1\left(\nabla\chi-\frac{2e}{c}\mathbf{A}\right)^2|\Delta|\right\}$$

$$+\left[\frac{T_c-T}{T_c}-\frac{7\zeta(3)|\Delta|^2}{8(\pi T)^2}\right]|\Delta|+|\Delta|U_1=0,$$

$$\left(\frac{\partial}{\partial t}-D\nabla^2\right)U_1=-\frac{\pi\tau_s}{16T}\frac{\partial|\Delta|^2}{\partial t}, \quad (7)$$

where

$$U_1=\frac{U+U^*}{2}=\frac{\tau_1}{4i}\int d\varepsilon\frac{\varepsilon}{\varepsilon^2+\tau_s^{-2}}(\Gamma^+-\Gamma^-).$$

The equation for the phase

$$\frac{\pi}{8T}\left\{-|\Delta|^2\frac{\partial\chi}{\partial t}+D_1\operatorname{div}\left[|\Delta|^2\left(\nabla\chi-\frac{2e}{c}\mathbf{A}\right)\right]\right\}+|\Delta|^2U_2=0 \quad (8)$$

contains the quantity $U_2 = -i(U - U^*)/2$, for which it is impossible to write down an equation connecting it directly with Δ. It is expressed in terms of $\Gamma^+ + \Gamma^-$:

$$U_2=-\frac{i\tau_1}{4\tau_s}\int d\varepsilon\frac{\Gamma^++\Gamma^-}{\varepsilon^2+\tau_s^{-2}},$$

and $\Gamma^+ + \Gamma^-$ is determined in turn by Eq. (5). There exists, however, a large group of problems (singly-connected superconductors, absence of vortices), in which the change of the modulus is not connected with the oscillations of the phase of the parameter Δ. In these cases Δ can be regarded as a real quantity defined by (7).

The expressions for the current density and the charge density can be obtained in analogy with the derivation of Eq. (6) for U:

$$\mathbf{j}=\sigma\left[-\frac{1}{c}\dot{\mathbf{A}}+\frac{1}{e}\frac{\tau_1}{4i}\nabla\int d\varepsilon(\Gamma^++\Gamma^-)\right]$$
$$-\frac{mp_0e^2}{2\pi c}\frac{|\Delta|^2}{T}D_1\left(\mathbf{A}-\frac{c}{2e}\nabla\chi\right),$$
$$\rho=\frac{mp_0e}{\pi^2}\left[\frac{\tau_1}{4i}\int d\varepsilon(\Gamma^++\Gamma^-)+e\varphi\right].$$

From this and from (5) and (8) follows the continuity equation $\operatorname{div}\mathbf{j}+\dot{\rho}=0$. Taking into account Maxwell's equations $\operatorname{curl}\mathbf{H}=4\pi\mathbf{j}/c$, we find that $\operatorname{div}\mathbf{j}$ and $\dot{\rho}$ vanish individually (electroneutrality condition). It can therefore be assumed that

$$\frac{\tau}{4i}\int d\varepsilon(\Gamma^++\Gamma^-)=-e\varphi,$$

and thus the expression for j assumes the following form:

$$\mathbf{j}=\sigma\mathbf{E}-\frac{mp_0e^2}{2\pi c}\frac{|\Delta|^2}{T}D_1\left(\mathbf{A}-\frac{c}{2e}\nabla\chi\right). \quad (9)$$

The foregoing derivation of the equations pertain to the case of infinite space. To obtain boundary conditions it is necessary to go over to coordinate representation in the calculation of the diagrams. Attention must be called then to those terms containing the averaging of the vector quantities over the directions. For points remote from the surface, such terms vanish, whereas for points on the surface the averaging gives rise to a vector directed normal to the surface. The corresponding terms are larger in order of magnitude than the remaining terms of the equations, and must therefore be set equal to zero. As a result we obtain the following boundary conditions:

$$\left(\mathbf{n}\left(i\nabla+\frac{2e}{c}\mathbf{A}\right)\right)\Delta=0,$$
$$\mathbf{n}\nabla U_1=0,$$
$$\left(\nabla(\Gamma^++\Gamma^-)-\frac{2ie}{c\tau_1}\frac{1}{2T}\operatorname{ch}^{-2}\frac{\varepsilon}{2T}\mathbf{A}\right)\mathbf{n}=0. \quad (10)$$

It is necessary to add to them the ordinary boundary conditions for the current and for the electromagnetic field.

DYNAMIC PROPERTIES OF Δ IN THE CASE OF A HALF-SPACE

In this section we shall find the charge produced in Δ under the influence of a weak high-frequency field parallel to the boundary of a superconducting half-space. This example, as will be shown later, is very useful for a clarification of the dynamic properties of Δ also in the case of a more general formulation of the nonstationary problems. Further calculations pertain to the case of large concentrations of the nonmagnetic impurities ($\tau_1 T_c \ll 1$), when both the diffusion coefficients entering into the equation coincide: $D_1 = D$.

Equations (7) for the parameter Δ, which in this problem can be chosen to be real, and for the field are best reduced to dimensionless form:

$$\left(-\frac{\partial}{\partial t}+\frac{1}{\varkappa^2}\frac{\partial^2}{\partial z^2}\right)\Psi+(1-A^2)\Psi-\Psi^3+\Psi U=0,$$
$$\left(-\frac{\partial}{\partial t}+\frac{1}{\varkappa^2}\frac{\partial^2}{\partial z^2}\right)U=\nu\frac{\partial(\Psi^2)}{\partial t},$$
$$-a\frac{\partial A}{\partial t}+\frac{\partial^2A}{\partial z^2}-\Psi^2A=0. \quad (11)$$

We have chosen here for Δ, A, and the length the scales assumed in the Ginzburg–Landau theory. The frequency is measured in units of

$$\tilde{\omega}=\frac{8}{\pi}(T_c-T)\equiv\frac{7\zeta(3)}{\pi^3}\frac{\Delta_0^2}{T_c}.$$

The numerical factor in the equation for the field is $a = 14\zeta(3)/\pi^4$. An important property of the system (11) is that it contains, besides κ, one more dimensionless parameter ν:

$$\nu=\frac{\pi^3}{14\zeta(3)}\tau_sT\gg1.$$

In a weak field $\Psi = 1 + \Psi_1$, and Ψ_1 are determined from the linearized equation

$$(D^2-2\hat{D}-2i\nu\omega)\Psi_\omega=\hat{D}(A^2)_\omega, \quad (12)$$

where $\hat{D} = i\omega + (\partial^2/\partial z^2)\kappa^{-2}$. Since A^2 contains a constant component and a harmonic with frequency $2\omega_0$, the same pertains also to Ψ_1.

Of greatest interest, from the point of view of formulation of nonlinear problems, is the question of the frequency region in which the transition from the adiabatic variation of Δ to the high-frequency variation takes place. In the case of a superconductor with a large content of paramagnetic impurities, the characteristic frequency is $\tau_S\Delta_0^2$. The same frequency separates the region in which the penetration of the field into the superconductor is determined by the skin effect ($\omega \gg \tau_S\Delta_0^2$) from the region of the Meissner effect ($\omega \ll \tau_S\Delta_0^2$). In the model considered here, as we shall presently show, the frequency characteristic of the variation of Δ turns out to be much smaller, owing to the presence of the large parameter ν, then the frequency at which the skin effect begins to play a role. We therefore confine ourselves to the frequency region $\omega_0 \ll \omega$ or, in dimensionless form, $\omega \ll 1$. Here

$$A^2(z, t) = \frac{H_0^2}{2} e^{-2z}(1 - \cos 2\omega_0 t),$$

The average value of $\Psi_{1,0}$ is determined from the static Ginzburg–Landau equation and is equal to

$$\Psi_{1,0} = \frac{H_0^2}{4} \frac{\varkappa^2}{2 - \varkappa^2}\left(e^{-2z} - \frac{\sqrt{2}}{\varkappa} e^{-\varkappa\sqrt{2}z}\right). \tag{13}$$

The component $\Psi_{2\omega_0}$ is determined from (12) with $\omega = 2\omega_0$. It should be sought in the form of a sum of exponentials

$$\Psi_{2\omega_0} = C_1 e^{k_1 z} + C_2 e^{k_2 z} + C_3 e^{-2z}.$$

As follows from (12)

$$C_3 = -\frac{H_0^2}{8} \frac{x}{x^2 - x - i\nu\omega_0}, \quad x = i\omega_0 + \frac{2}{\varkappa^2}$$

The constants C_1 and C_2 are determined by the boundary conditions (10), which in this case yield

$$k_1^3 C_1 + k_2^3 C_2 = +H_0^2\varkappa^2 + 8C_3,$$
$$k_1 C_1 + k_2 C_2 = +2C_3.$$

The exponents k_1 and k_2 are the roots of the equation

$$u^2 - 2u - 4i\nu\omega_0 = 0, \quad u = 2i\omega_0 + k^2/\varkappa^2$$

with negative real parts. From this equation we see that the low-frequency region for Ψ is the region where $\omega \ll \nu^{-1}$. In this case we get

$$\Psi_{2\omega_0} = \frac{H_0^2\varkappa^2}{2} \frac{2}{4 - 2\varkappa^2 - i\nu\varkappa^4\omega_0}\Big[-e^{-2z} + \frac{2 + i\nu\varkappa^2\omega_0}{\varkappa\sqrt{2}} e^{-\varkappa\sqrt{2}z} - \frac{k_2}{2} e^{k_2 z}\Big],$$
$$k_2 = -(1 - i)\varkappa\sqrt{\nu\omega_0}, \quad \omega_0 \leqslant 1/\varkappa^2.$$

We see that if $\kappa \lesssim 1$, then the condition $\omega_0 \ll \nu^{-1}$ determines the region of the adiabatic behavior of Ψ. If $\kappa \gg 1$, the adiabaticity condition is more stringent: $\kappa^2\nu\omega_0 \ll 1$.

When $\omega_0 \gg \nu^{-1}$ we have

$$k_1 = -\varkappa(4\nu\omega_0)^{1/4} e^{-3\pi i/8}, \quad k_2 = -\varkappa(4\nu\omega_0)^{1/4} e^{\pi i/8},$$

and the corresponding exponentials decrease more rapidly than $\exp(-\kappa\sqrt{2z})$. The expressions for the coefficients C_1, C_2, and C_3 are more cumbersome in this case and will not be written out here. It is important only that they are much smaller than the corresponding coefficients in (13) in the entire region $\omega_0 \gg \nu^{-1}$, although the order of smallness depends on the relations between ω_0, κ, and ν.

Thus, we see that, unlike alloys with large contents of paramagnetic impurities, there exists in our model a region of frequencies low enough to cause the penetration of the field to be determined by the Meissner effect, but at the same time the change of Δ is determined in the main by the square of the field averaged over the time. A similar situation is realized also in alloys without magnetic impurities.[5] This model may therefore be useful to ascertain whether nonlinear problems can be formulated for alloys.

THIN FILM IN A STRONG HIGH-FREQUENCY FIELD

The question of the destruction of superconductivity of a thin film by a high-frequency field was already considered for a model with a large content of paramagnetic impurities.[2] Here we investigate this question on the basis of Eq. (11), for in this case certain new essential aspects come into play. We shall assume that the film is placed in a magnetic field parallel to it, $H = H_0 \sin \omega t$, that the film thickness is so small that Ψ can be regarded as constant over the thickness, and the field can be assumed to coincide with the external field. Introducing the symbol $\rho = \Psi^2$ and omitting the derivatives with respect to the coordinates, we write Eq. (11) in the form

$$-\frac{1}{2}\frac{\partial\rho}{\partial t} + (1 - A^2)\rho - \rho^2 + U\rho = 0,$$
$$-\frac{\partial U}{\partial t} = \nu\frac{\partial\rho}{\partial t}. \tag{14}$$

The second of these equations yields

$$U = -\nu(\rho + C)$$

and thus, in the form given here, Eq. (14) yields for Ψ a solution that depends on the initial conditions. This is caused by the fact that this model does not contain a homogeneous relaxation mechanism. In the case of problems connected with the properties of bulky superconductors, the leading role is played by the diffusion mechanism, since the times of homogeneous relaxation are usually quite large[6] (interaction with the phonons and with the thermostat). On the other hand, it is clear beforehand that at frequencies that are large compared with the reciprocal time of the homogeneous relaxation γ the steady state regime should not depend on γ, whereas the dependence on the initial condition should vanish after a certain transition period, owing to the fact that γ differs from zero. Bearing this in mind, in order to determine the integration constant we write down the equation for U in the form

$$\left(\frac{\partial}{\partial t} + \gamma\right)U = -\nu\frac{\partial\rho}{\partial t}.$$

The solution of this equation is

$$U = -\nu e^{-\gamma t}\Big[e^{\gamma t}\rho(t) - e^{\gamma t_0}\rho(t_0) - \gamma\int_{t_0}^{t} e^{\gamma t'}\rho(t')\,dt'\Big].$$

Assuming that a steady state sets in at times sufficiently remote from t, i.e., that ρ is a periodic function of the time, we break up the integration integral into two regions: the transition region from t_0 to $t = 0$ and the region of periodic regime at $t > 0$. Then

$$e^{-\gamma t}\int_{t_0}^{t} e^{\gamma t'}\rho(t')\,dt' = \mathrm{const}\cdot e^{-\gamma t} + e^{-\gamma t}\int_0^t e^{\gamma t'}\rho(t')\,dt'$$
$$= e^{-\gamma t}\cdot\mathrm{const} + \frac{1 - e^{-\gamma n T}}{1 - e^{-\gamma T}}\int_0^T e^{-\gamma \tau}\rho(t - \tau)\,d\tau,$$

where T is the period of variation of ρ, which in our case equals π/ω, and n is the number of total periods in the interval from zero to t. For time sufficiently large compared with γ^{-1} we obtain, recognizing that $\gamma/\omega \ll 1$,

$$U = -\nu(\rho - \bar{\rho}),$$

where $\bar{\rho}$ is the value of ρ averaged over the period. Substituting this in the equation for ρ, we find that

$$\rho(t) = F(t) \Big/ \left[C_1 + (\nu + 1) \int_0^t F(t') dt' \right],$$

$$F(t) = \exp\left\{ 2 \int_0^t (1 + \nu\bar{\rho} - A^2(t')) dt' \right\}.$$

In order for a steady-state solution to exist, it is necessary, as seen from this, to have

$$1 + \nu\bar{\rho} - \overline{A^2} > 0. \quad (15)$$

In the opposite case, ρ attenuates exponentially after the field is turned on. If condition (15) is satisfied, then at large values of the time the expression for ρ can be reduced to the form

$$\rho(t) = \frac{\omega}{\nu + 1} \left[1 - \exp\left\{ -\frac{2\pi}{\omega} (1 + \nu\bar{\rho} - \overline{A^2}) \right\} \right]$$
$$\times \left[\int_0^{2\pi} dx' \exp\left\{ -\frac{1 + \nu\bar{\rho} - \overline{A^2}}{\omega} x' - \frac{A^2}{\omega} [\sin x - \sin(x - x')] \right\} \right]^{-1}, \quad (16)$$

where $x = 2\omega t$, $\overline{A^2} = H_0^2 d^2/3$, and 2d is the thickness of the film. At large frequencies $\omega \gg \overline{A^2}$ we find that ρ coincides, in the main, with its mean value and its high-frequency component is small:

$$\rho(t) = (1 - \overline{A^2}) \left\{ 1 + \frac{\omega \overline{A^2}}{\omega^2 + (\nu + 1)^2 (1 - \overline{A^2})^2} \left[\sin 2\omega t + \frac{(\nu + 1)(1 - \overline{A^2})}{\omega} \cos 2\omega t \right] \right\}.$$

If $\omega \ll \overline{A^2}$, but at the same time H_0 is so close to its critical value $H_c^2 d^2/3 = 1$ that $\nu(1 - \overline{A^2}) \ll \omega$, then

$$\rho(t) = (1 - \overline{A^2}) 2 \sqrt{\frac{\pi}{\omega}} \exp\left\{ -\frac{1 - \sin 2\omega t}{\omega} \right\}$$

i.e., ρ, being a very small quantity, experiences strong oscillations.

In conclusion I wish to thank L. P. Gor'kov, in constant contact with whom this work was performed, and A. I. Larkin for useful discussions.

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