"PLASMON" MECHANISM OF SUPERCONDUCTIVITY IN DEGENERATE SEMICONDUCTORS AND SEMIMETALS. I

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It is shown that in degenerate semiconductors or semimetals with appreciably different conduction and hole electron masses ($m_n \ll m_p$) and relatively high mobilities, a specific superconductivity mechanism, due to the interaction of the electrons with the low-frequency weakly-damped branch of the collective longitudinal oscillations of the electron-hole plasma (the so called "plasma sound") may exist. The critical temperature of the transition of the semiconductor (semimetal) to the superconducting state is determined by the Langmuir (plasma) frequency or the Fermi energy of the "heavy" holes and, in principle, can be much higher than in metals ($T_c \sim 10^2$ deg K).

1. INTRODUCTION

SUPERCONDUCTIVITY is a rather universal phenomenon, wherein any arbitrarily weak attraction between the electrons near the Fermi surface leads, as is well known to the formation of bound electron pairs (the Cooper phenomenon[1]).

For most pure metals, the main electron-pairing mechanism is electron-phonon interaction[2-5]. In many cases, however there are apparently also other causes of superconductivity, not connected with the crystal-lattice vibrations, as indicated, in particular, by the absence of the isotropic effect in some of the superconductors[6].

In this connection, and principally with an aim at discovering superconducting materials with sufficiently high critical temperatures, various nonphonon conductivity mechanisms have been discussed for ferro- and antiferromagnets[7,8], in transition metals and alloys[9,10], in polymers[11], and also in thin metallic films with dielectric or semiconductor coatings ("sandwiches")[12].

It is shown in this paper that in degenerate semiconductors or semimetals with greatly differing effective masses of the conduction electrons and the holes and with relatively large free-carrier mobilities there can exist a unique superconductivity mechanism due to the interaction of the electrons with the low-frequency weakly-damped branch of the collective longitudinal oscillations of the electron-hole plasma (the so-called "plasma sound"). The width of the energy gap in the conduction-electron spectrum, which characterizes the binding energy of the Cooper pairs, is determined by the Langmuir (plasma) frequency or by the Fermi energy of the "heavy" holes, so that the critical temperature of transition to the superconducting state may, in principle, be much higher in such semiconductors (semimetals) than in metals.

2. "PLASMA SOUND" IN A DEGENERATE ELECTRON-HOLE PLASMA

Let us consider the collective oscillations of a degenerate electron-hole plasma of semiconductors[1)] or semimetals whose band structure is such that the hole effective mass m_p is much larger than the effective mass m_n of the conduction electrons. As is well known, if the free-carrier mobility is sufficiently high, there can exist in such a plasma, besides the electron Langmuir (plasma) oscillations at the frequency $\Omega_n = (4\pi e^2 N_n/m_n)^{1/2}$, where N_n is the concentration of the conduction electrons, also a low-frequency weakly-damped branch of collective longitudinal oscillations with a phase velocity in the interval $v_{Fp} < \omega/q < v_{Fn}$, where v_{Fn} and v_{Fp} are the Fermi velocities of the degenerate electrons and holes (it is assumed that the crystal temperature is close to absolute zero, $T \approx 0$).

In the simplest case of an isotropic semiconductor (semimetal), the dispersion equation of these oscillations is

$$\varepsilon(\mathbf{q}, \omega) - \frac{4\pi e^2}{q^2}\{\Pi_n(\mathbf{q}, \omega) + \Pi_p(\mathbf{q}, \omega)\} = 0. \tag{2.1}$$

Here $\epsilon(\mathbf{q}, \omega)$ is the longitudinal dielectric constant of the crystal, and $\Pi_n(\mathbf{q}, \omega)$ and $\Pi_p(\mathbf{q}, \omega)$ are the polarization operators of the conduction electrons and of the holes, which are respectively equal to[13,14]

$$\Pi_n(\mathbf{q}, \omega) \cong -\frac{3N_n}{2E_{Fn}}\left\{g\left(\frac{q}{2p_{Fn}}\right) + i\frac{\pi}{2}\frac{|\omega|}{qv_{Fn}}\right\} \qquad (\omega \ll qv_{Fn}), \tag{2.2}$$

$$\Pi_p(\mathbf{q}, \omega) \cong \frac{q^2 N_p}{m_p\omega(\omega + i/\tau_p)} \qquad (\omega \gg qv_{Fp}), \tag{2.3}$$

where

$$g(x) = \frac{1}{2}\left\{1 + \frac{1-x^2}{2x}\ln\left|\frac{1+x}{1-x}\right|\right\} \approx 1 - \frac{x^2}{2} \qquad (0 < x < 1); \tag{2.4}$$

$E_{Fn} = p_{Fn}^2/2m_n$ is the electron Fermi energy, reckoned from the bottom of the conduction band, $p_{Fn} = (3\pi^2 N_n)^{1/3}$ is their Fermi momentum ($\hbar = 1$), N_p is the hole density (we assume henceforth for simplicity that $N_p = N_n = N$), and τ_p is the time of hole momentum relaxation due to scattering by impurities (defects) and phonons.

[1)] In intrinsic semiconductors, a degenerate electron-hole plasma can be produced by external carrier injection (for example by "pumping" the crystal with a laser).

Assuming that the frequency of the oscillations in question lies far from the natural frequencies of the crystal (and, in particular, does not coincide with any of the optical or acoustic branches of the lattice vibrations), so that we can approximately assume that $\epsilon \approx$ const, we obtain if $\omega\tau_p \gg 1$, i.e., at a sufficiently high hole mobility, the following expression for the oscillation frequency from (2.1), with allowance for (2.2) and (2.3),

$$\mathrm{Re}\,\omega \equiv \Omega_q = \Omega_p\left\{\frac{q^2}{q^2+\varkappa_n^2(q)}\right\}^{1/2}, \tag{2.5}$$

where $\Omega_p = (4\pi e^2 N/\epsilon m_p)^{1/2}$ is the Langmuir (plasma) frequency of the "heavy" holes ($m_p \gg m_n$), and

$$\varkappa_n^{-1}(q) = \left[\frac{6\pi e^2 N}{\varepsilon E_{Fn}}\, g\left(\frac{q}{2p_F}\right)\right]^{-1/2}$$

is the electron Debye screening radius.

We see that as $q \to 0$ the oscillations have an acoustic character ($\Omega_q \approx qu$, where $u = v_{Fn}(m_n/3m_p)^{1/2}$), whereas in the region $q \gg \kappa_n$ they constitute plasma oscillations of the holes ($\Omega_q \approx \Omega_p$). This is why this branch of oscillations is frequently called "plasma sound"[2)]. The weak damping of plasma sound in the region $\omega > qv_{Fp}$ is due to single-particle electron excitations[3)] and to the finite hole mobility. The damping decrement, according to (2.1)–(2.3), equals ($\Omega_p\tau_p \gg 1$)

$$\mathrm{Im}\,\omega \equiv -\delta_q = -\left\{\frac{\pi}{4}\frac{\varkappa_n^2}{q^2+\varkappa_n^2}\frac{\Omega_q^2}{qv_{Fn}} + \frac{1}{2\tau_p}\right\} \qquad (\delta_q \ll \Omega_q). \tag{2.6}$$

In the region of $q \gtrsim \Omega_p/v_{Fp}$ the resonant damping of oscillations on the holes becomes appreciable ($\mathrm{Im}\,\omega \gtrsim \mathrm{Re}\,\omega$).

We note that at a sufficiently high carrier density the maximum frequency of plasma sound Ω_p may greatly exceed the limiting Debye frequency of the phonons ω_0. But on the other hand $\Omega_p \ll E_{Fn}$ if the condition $p_F a_n \gg m_n/m_p$ is satisfied, where $a_n = \epsilon/m_n e^2$ is the effective Bohr radius of the conduction electron.

3. SUPERCONDUCTIVITY MECHANISM

We shall show in this section that interaction of the conduction electron with the aforementioned low-frequency collective oscillations of a degenerate electron-hole plasma, of the "plasma sound" type, can lead to the formation of bound electron pairs near the Fermi surface, and consequently to the occurrence of superconductivity in certain semiconductors (semimetals).

As is well known, the Coulomb interaction between particles in a degenerate plasma is described by the following vertex part[14]:

$$\Gamma_{ij}(\mathbf{q},\omega) = 4\pi e_i e_j\left(q^2 - \sum_i 4\pi e_i^2 \Pi_i(\mathbf{q},\omega)\right)^{-1} \equiv \frac{4\pi e_i e_j}{q^2\varepsilon_l(\mathbf{q},\omega)}, \tag{3.1}$$

where $\Pi_i(q, \omega)$ is the polarization operator (loop) of the particles of species i, and $\epsilon_l(q, \omega)$ is the longitudinal (self-consistent) dielectric constant of the plasma[4)]. In the general case the vertex (3.1) takes into account effects of the delay of the interaction, and has poles near the natural frequencies of the collective plasma oscillations, when $\mathrm{Re}\,\epsilon_l(q, \omega) = 0$ and $\mathrm{Im}\,\epsilon_l(q, \omega) \to 0$. In particular, the longitudinal dielectric constant of the degenerate electron-hole plasma of an isotropic semiconductor (semimetal) in the frequency (energy) region $qv_{Fp} < \omega < qv_{Fn}$ equals (see (2.1), (2.5), and (2.6))

$$\varepsilon_l(\mathbf{q},\omega) \cong \varepsilon\left(1+\frac{\varkappa_n^2}{q^2}\right)\left(1-\frac{\tilde{\Omega}_q^2}{\omega^2}\right), \qquad \tilde{\Omega}_q = \Omega_q - i\delta_q, \tag{3.2}$$

so that the vertex of the electron-electron interaction in this region is of the form (compare with the "jelly" model[18,19])

$$\Gamma_{nn}(\mathbf{q},\omega) = \frac{4\pi e^2}{\varepsilon(q^2+\varkappa_n^2)}\frac{\omega^2}{\omega^2-\tilde{\Omega}_q^2}. \tag{3.3}$$

It follows therefore that when $\omega < \Omega_q$ (but $\omega > qv_{Fp}$ we have $\mathrm{Re}\,\Gamma_{nn}(q, \omega) < 0$ (but $|\mathrm{Re}\,\Gamma_{nn}|\,\mathrm{Im}\,\Gamma_{nn}|$), i.e., the interaction between conduction electrons has the nature of attraction that results from exchange of virtual "plasmons," which are quanta of collective plasma-sound oscillations. Indeed, the vertex Γ_{nn} can be represented in the form

$$\Gamma_{nn}(\mathbf{q},\omega) = -D(\mathbf{q},\omega) + \Gamma(\mathbf{q}), \tag{3.4}$$

where

$$D(\mathbf{q},\omega) = \Gamma(\mathbf{q})\frac{\tilde{\Omega}_q}{2}\left\{\frac{1}{\tilde{\Omega}_q-\omega}+\frac{1}{\tilde{\Omega}_q+\omega}\right\}, \tag{3.5}$$

$$\Gamma(\mathbf{q}) = \frac{4\pi e^2}{\varepsilon\,[q^2+\varkappa_n^2(q)]}. \tag{3.6}$$

The function D plays here the role of a "Green's function" of the plasmons[5)], whereas Γ describes the Coulomb repulsion.

In the region $\omega \le \Omega_q$ we have $\mathrm{Re}\,D(q, \omega) \le \Gamma(q)$, i.e., the effective attraction due to the "electron-plasmon" interaction prevails over the repulsion[6)]. In the regions $\omega > \Omega_q$ and $\omega \le qv_{Fp}$, to the contrary, the Coulomb repulsion of the electrons prevails ($\mathrm{Re}\,\Gamma_{nn}(q, \omega) > 0$, with $|\mathrm{Im}\,\Gamma_{nn}| \gtrsim \mathrm{Re}\,\Gamma_{nn}$ in the region $\omega \sim qv_{Fp}$). In particular, when $\omega/q \to 0$ we have

$$\Gamma_{nn}(\mathbf{q},0) \equiv \Gamma_0(\mathbf{q}) = \frac{4\pi e^2}{\varepsilon_0[q^2+\varkappa_n^2(q)+\varkappa_p^2(q)]} \tag{3.7}$$

where ϵ_0 is the static dielectric constant of the

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2) This branch of longitudinal oscillations was investigated earlier for the particular cases of strongly-compressed matter [15] and a dense degenerate electron-hole plasma [14]. In a rarefied non-isothermal plasma ($T_e \gg T_i$), a similar branch of collective oscillations is called "ion" (or "non-isothermal") sound [16,17].

3) Such a damping is the quantum analog of resonant Landau damping [16,17].

4) We note that expression (3.1), which is obtained in the high-density approximation (i.e., under the condition $p_F a_0 \gg 1$, where a_0 is the Bohr radius), is valid with sufficiently good accuracy and for not too dense systems, when $p_F a_0 \lesssim 1$ (see [10]).

5) Compare with the phonon Green's function in the case of electron-phonon interaction [5,13].

6) In other words, the presence of "heavy" holes leads in this case to the effect of "antiscreening" of the Coulomb interaction between the conduction electrons, just as the presence of bound d-electrons in transition metals leads to a net attraction between the free s-electrons [9,10].

crystal[7)] and $\kappa_p^{-1}(q)$ is the effective radius of screening of the charge by the "heavy" holes ($\kappa_p^2 \equiv \kappa_n^2 m_p/m_n$). Nonetheless, as will be shown below, the electron-plasmon interaction leads under definite conditions to a pairing of the electrons near the Fermi surface.

The equation for the energy gap in the conduction electron spectrum at T = 0, as is well known[5,10,20], is of the form

$$\Delta_n(\mathbf{k}, \omega) = \frac{1}{i(2\pi)^4} \int d\mathbf{k}'\, d\omega' F_n(\mathbf{k}', \omega')\Gamma_{nn}(\mathbf{k}-\mathbf{k}', \omega-\omega'), \quad (3.8)$$

where

$$F_n(\mathbf{k}, \omega) = \Delta_n(\mathbf{k}, \omega)/\Omega_n(\mathbf{k}, \omega), \quad (3.9)$$

$$\Omega_n(\mathbf{k}, \omega) = [\omega - \xi_n(\mathbf{k}) - \Sigma_n(\mathbf{k}, \omega)][\omega + \xi_n(\mathbf{k}) + \Sigma_n(\mathbf{k}, -\omega)] - \Delta_n^2(\mathbf{k}, \omega), \quad (3.10)$$

$\xi_n(\mathbf{k})$ is the electron energy reckoned from the Fermi surface, and $\Sigma_n(\mathbf{k}, \omega)$ is in turn determined by the equation[5]

$$\Sigma_n(\mathbf{k}, \omega) = \frac{1}{i(2\pi)^4} \int d\mathbf{k}'\, d\omega' G_n(\mathbf{k}', \omega')\Gamma_{nn}(\mathbf{k}-\mathbf{k}', \omega-\omega'); \quad (3.11)$$

here $G_n(\mathbf{k}, \omega)$ is the Green's function of the conduction electrons (with pairing taken into account).

We assume first for simplicity that expression (3.4) for $\Gamma_{nn}(\mathbf{k}-\mathbf{k}', \omega-\omega')$ is valid for all values of $|\omega-\omega'|$ (see below). Then, assuming that $\Omega_p \ll E_{Fn}$, and consequently that $|\mathbf{k}-\mathbf{k}'| \sim p_F$, we obtain with the aid of the method developed by Éliashberg[5] the following equation for the function $C(\omega)$ characterizing the binding energy of the electron Cooper pairs:

$$C(\omega) = \int_{\Delta}^{\infty} \frac{Q(\omega, \omega')C(\omega')}{\sqrt{\omega'^2 - C^2(\omega')}}\, d\omega', \quad (3.12)$$

where

$$Q(\omega, \omega') = \frac{1}{4\pi^2 v_{Fn}}\left[1 - \frac{f(\omega)}{\omega}\right]^{-1} \int_0^{q_m} \Gamma(q)\, q\, dq \times\left\{\frac{\Omega_q}{2}\left[\frac{1}{\omega'-\omega+\Omega_q} + \frac{1}{\omega'+\omega+\Omega_q}\right] - 1\right\}, \quad (3.13)$$

$$f(\omega) = -\frac{1}{8\pi^2 v_{Fn}} \int_0^{q_m} \Gamma(q)\Omega_q \ln\left|\frac{\Omega_q+\omega}{\Omega_q-\omega}\right| q\, dq, \quad (3.14)$$

$\Delta \equiv C(0)$ is the gap parameter on the Fermi surface, $q = |\mathbf{k}-\mathbf{k}'|$, and $q_m \approx 2p_F$.

It is easy to see that when $\omega \to 0$ the kernel $Q(\omega, \omega')$ of the integral equation (3.12) is negative for all values of $\omega' \neq 0$. On the other hand, when $\omega' = 0$, we have in accord with (3.13)

$$Q(\omega, 0) = \frac{\beta(\omega)}{2}\left[1 - \frac{f(\omega)}{\omega}\right]^{-1} \ln\left|\frac{1-\beta(\omega)}{\beta(\omega)}\right|, \quad (3.15)$$

where

$$\beta(\omega) = \frac{\alpha\omega^2}{\Omega_p^2 - \omega^2(1-\alpha/2)}, \qquad \alpha = \frac{\varkappa_n^2(0)}{4p_F^2} \equiv \frac{1}{\pi p_F a_n}. \quad (3.16)$$

The kernel $Q(\omega, 0)$ is positive when $\beta(\omega) \le 1/2$, i.e., in the vicinity of $\omega \le \tilde{\Omega}_p \equiv \Omega_p(1 + 3/2\,\alpha)^{-1/2}$, and is negative when $\omega > \tilde{\Omega}_p$, diverging logarithmically at the point $\omega = \Omega_m \equiv \Omega_p(1 + \alpha/2)^{-1/2}$ (i.e., at $\beta(\omega) = 1$). This divergence disappears, however, when the finite damping of the collective oscillations is taken into account, and the maximum negative value of $Q(\omega, 0)$ has an absolute value ($\delta_q \approx 1/2\,\tau_p$)

$$|Q(\Omega_m, 0)| \approx 1/2 \ln|\alpha\Omega_p\tau_p|. \quad (3.17)$$

According to (3.14), the function $f(\omega)$ in the region $\omega \ll \Omega_p$ equals

$$f(\omega) \cong -\frac{\alpha\omega}{2-\alpha}\ln\left|\frac{2+\alpha}{2\alpha}\right| \qquad (f(\omega) < 0), \quad (3.18)$$

and when $\omega \gg \Omega_p$ its modulus decreases like $1/\omega$ with increasing ω.

It must be borne in mind, however, that Eq. (3.12) has been obtained under the tacit assumption that $\Delta > q_m v_{Fp} \approx 4E_{Fp}$ (where $E_{Fp} = p_F^2/2m_p$ is the Fermi energy of the holes, so that in the entire interval $\omega, \omega' \ge \Delta$ the vertex Γ_{nn} is determined by the expression (3.4). Actually, this condition can be satisfied only subject to the very strong inequality $\Omega_p \gg E_{Fp}$, i.e., at rather large effective-mass ratios ($\mu \equiv m_p/m_n \gg 1$), but small carrier densities ($\alpha \equiv \kappa_n(0)/2p_F \gg 1$). For real semiconductors (semimetals), however, the parameters μ and α are not very large ($\mu \lesssim 10$, $\alpha \sim 1$), and therefore the case when $\Omega_p \gtrsim E_{Fp}$ and $\Delta \ll E_{Fp}$ is of practical interest. It is necessary here to take into account the fact that expressions (3.4) and (3.13) no longer hold in the vicinity of $|\omega - \omega'| \lesssim E_{Fp}$. In particular, when $\omega = \omega' = 0$ we have, according to (3.7)

$$Q(0,0) \equiv -\frac{1}{4\pi^2 v_{Fn}}\int_0^{q_m} \Gamma_0(q)\, q\, dq = -\frac{\varepsilon}{\varepsilon_0}\frac{\alpha}{2-\alpha(1+\mu)}\ln\left|\frac{2+\alpha(1+\mu)}{2\alpha(1+\mu)}\right|. \quad (3.19)$$

But since Eq. (3.8) with the exact interaction

$$\Gamma_{nn}(q, \omega-\omega') = \frac{4\pi e^2}{q^2\varepsilon_l(q, \omega-\omega')}$$

can be solved only by numerical methods, we confine ourselves here to a consideration of a certain model equation for the gap[8)]:

$$\Delta(\xi) = \int \frac{K(\xi, \xi')\Delta(\xi')}{\sqrt{\xi'^2 + \Delta^2(\xi')}}\, d\xi', \quad (3.20)$$

where (cf.[10,21])

$$K(\xi, \xi') = \begin{cases} -\rho_1, & |\xi|, |\xi'| \le \omega_1 \equiv E_{Fp} \\ \rho_2, & \omega_1 < |\xi|, |\xi'| \le \omega_2 \equiv \tilde{\Omega}_p \\ -\rho_3, & \omega_2 < |\xi|, |\xi'| \le \omega_3 \equiv E_{Fn} \\ 0, & |\xi|, |\xi'| > \omega_3 \end{cases}. \quad (3.21)$$

Here $\rho_1 = -Q(0, 0) > 0$ (see (3.19)), and the constants ρ_2 and ρ_3 are the mean values of the kernel $Q(\omega, 0)$ in the positive and negative regions, respectively:

$$\rho_2 = \frac{1}{\omega_2 - \omega_1}\int_{\omega_1}^{\omega_2} Q(\omega, 0)\, d\omega, \qquad \rho_3 = \frac{1}{\omega_3 - \omega_2}\int_{\omega_2}^{\omega_3} |Q(\omega, 0)|\, d\omega. \quad (3.22)$$

Assuming further for simplicity that $\Delta(\xi) = \Delta_1$ when $|\xi| \le \omega_1$, that $\Delta(\xi) = \Delta_2$ at $\omega_1 < |\xi| < \omega_2$, and that $\Delta(\xi) = \Delta_3$ when $\omega_2 < |\xi| \le \omega_3$, i.e., replacing the exact

7)For certain crystals, ϵ_0 may be much larger than the dynamic dielectric constant in the frequency region $\omega \sim \Omega_p$.

8)Such an equation can be obtained, in particular, with the aid of the Bogolyubov transformation [3] by a suitable choice of the effective electron-electron interaction Hamiltonian. A numerical solution of the exact equation for the gap is now in progress.

values of the gap by their mean values in the corresponding regions, we obtain in lieu of the integral equation (3.20) the following system of equations (cf.[10]):

$$\Delta_1 = -\rho_1\Delta_1 \ln\frac{2\Omega_1}{\Delta_1} + \rho_2\Delta_2 \ln\frac{\omega_2}{\Omega_2} - \rho_3\Delta_3 \ln\frac{\omega_3}{\omega_2},$$

$$\Delta_2 = \rho_2\left[\Delta_1 \ln\frac{2\Omega_1}{\Delta_1} + \Delta_2 \ln\frac{\omega_2}{\Omega_2}\right] - \rho_3\Delta_3 \ln\frac{\omega_3}{\omega_2},$$

$$\Delta_3 = -\tilde{\rho}_3\left[\Delta_1 \ln\frac{2\Omega_1}{\Delta_1} + \Delta_2 \ln\frac{\omega_2}{\Omega_2}\right], \tag{3.23}$$

where

$$\Omega_{1,2} = {}^1\!/_2(\omega_1 + \sqrt{\omega_1^2 + \Delta_{1,2}^2}), \tag{3.24}$$

and $\tilde{\rho}_3$ is the renormalized Coulomb-repulsion constant, equal to[9)]

$$\tilde{\rho}_3 = \frac{\rho_3}{1 + \rho_3 \ln(\omega_3/\omega_2)}. \tag{3.25}$$

It was assumed above that $\Delta_2, |\Delta_3| \ll \omega_2, \omega_3$. If furthermore $\Delta_1, \Delta_2 \ll \omega_1$, then the system (3.23) has, subject to the condition

$$\tilde{\rho} \equiv -(\rho_1 + \rho_2) + \frac{(\rho_2 + \rho_3 - \tilde{\rho}_3)}{1 - (\rho_2 + \rho_3 - \tilde{\rho}_3)\ln(\omega_2/\omega_1)} > 0 \tag{3.26}$$

the following solutions:

$$\Delta_1 = 2\omega_1 e^{-1/\tilde{\rho}}, \quad \Delta_2 = \left(1 + \frac{\rho_1 + \rho_2}{\tilde{\rho}}\right)\Delta_1,$$

$$\Delta_3 = -\frac{\tilde{\rho}_3}{\tilde{\rho}}\left[1 + (\tilde{\rho} + \rho_1 + \rho_2)\ln\frac{\omega_2}{\omega_1}\right]\Delta_1. \tag{3.27}$$

We see that in spite of the Coulomb repulsion (which incidentally is strongly screened by the "heavy" holes), the gap in the spectrum of the conduction electrons near the Fermi surface can be finite and positive in the energy region $\omega \lesssim E_{Fp}$.

We note that in the case of the very strong inequality $\omega_2 \gg \omega_1$ (but $\omega_2 \ll \omega_3$) the system (3.23) admits also of solutions in the form $(\omega_1 \ll \Delta_1, \Delta_2 \ll \omega_2)$

$$\Delta_1 \approx \Delta_2 \approx 2\omega_2 e^{-1/\tilde{\rho}_2}, \quad \Delta_3 \approx -\frac{\tilde{\rho}_3}{\tilde{\rho}_2}\Delta_2, \tag{3.28}$$

where $\tilde{\rho}_2 = \rho_2 + \rho_3 - \tilde{\rho}_3$ (these solutions correspond to the case $\alpha, \mu \to \infty$).

By way of an example, let us consider a semiconductor (semimetal) with conduction-electron effective mass $m_n \approx 0.3 m_0$ and hole mass $m_p = 3m_0$ (m_0 is the electron mass), and with a free-carrier density $N \approx 3 \times 10^{19}$ cm^{-3}. As shown by numerical estimates, in this case (subject to the condition $\epsilon \approx 1$ and $\epsilon_0 \approx 2$), the width of the energy gap near the Fermi surface is, according to (3.27), $\Delta_1 \approx 2 \times 10^{-2}$ eV, corresponding to a critical temperature of transition to the superconducting state $T_c \approx 10^2$ deg K[10)].

Thus, the interaction of the conduction electrons with the collective oscillations of an electron-hole plasma in degenerate semiconductors or semimetals can lead, under certain conditions, to the occurrence of superconductivity with relatively high critical temperature. Such a conductivity mechanism can be called "plasmon" mechanism.

In conclusion we note that the formation of bound electron pairs is possible only when the so-called "coherence length" $\xi_0 \sim v_{Fn}/\Delta$ is small compared with the electron mean free path in the crystal, $l_n \sim v_{Fn}\tau_n$ (where τ_n is the electron momentum relaxation time), and the characteristic electron pairing time $\sim\Delta^{-1}$ is much shorter than the damping time of the collective (plasma-sound) oscillations, $\sim\tau_p$, i.e., under the condition

$$\Delta\tau_{n,p} \ll 1. \tag{3.29}$$

For the example considered above, this corresponds to $\tau_{n,p} \gg 3 \times 10^{-14}$ sec.

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9)Compare with the renormalization of the Coulomb interaction in the phonon model of superconductivity [3].

10)Of course, this estimate should be regarded as roughly tentative. More reliable results can be obtained only by numerically solving the problem.

[1] L. N. Cooper, Phys. Rev. **104**, 1189 (1956); J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162, **108**, 1175 (1957).

[2] H. Frohlich, Phys. Rev. **79**, 845 (1950); Proc. Roy. Soc. **A215**, 291 (1952).

[3] N. N. Bogolyubov, Zh. Eksp. Teor. Fiz. **34**, 58 (1958) [Sov. Phys.-JETP **7**, 41 (1958); N. N. Bogolyubov, V. V. Tolmachev, and D. V. Shirkov, Novyi metod v teorii sverkhprovodimosti, (A New Method in the Theory of Superconductivity), AN SSSR, 1958.

[4] L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **34**, 735 (1958) [Sov. Phys.-JETP **7**, 505 (1958)].

[5] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960) [Sov. Phys.-JETP **11**, 696 (1960)].

[6] T. H. Geballe, B. J. Matthias, G. W. Hull, and E. Corenzwitt, Phys. Rev. Lett. **6**, 275 (1961).

[7] A. I. Akhiezer and I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. **36**, 859 (1959) [Sov. Phys.-JETP **9**, 605 (1959)]; A. I. Akhiezer and I. A. Akhiezer, Zh. Eksp. Teor. Fiz. **43**, 2208 (1962) [Sov. Phys.-JETP **16**, 1560 (1963)].

[8] I. A. Privorotskiĭ, Zh. Eksp. Teor. Fiz. **43**, 2255 (1962) [Sov. Phys.-JETP **16**, 1593 (1963)].

[9] J. W. Garland, Phys. Rev. Lett. **11**, 111, 114 (1963).

[10] B. T. Geĭlikman, Zh. Eksp. Teor. Fiz. **48**, 1194 (1965) [Sov. Phys.-JETP **21**, 796 (1965)]; Usp. Fiz. Nauk **88**, 327 (1966) [Sov. Phys.-Usp. **9**, 142 (1966)].

[11] W. A. Little, Phys. Rev. **A135**, 1416 (1964); Scientific American **212** (2), 21 (1965).

[12] V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **47**, 2318 (1964) [Sov. Phys. JETP **20**, 1549 (1965)]; V. L. Ginzburg and D. A. Kirzhnits, Dokl. Akad. Nauk SSSR **176**, 553 (1967) [Sov. Phys.-Dokl. **12**, 880 (1968)]; Usp. Fiz. Nauk **95**, 91 (1968) [Sov. Phys.-Usp. (in print)].

[13] A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys.-JETP **7**, 996 (1958)].

[14] A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyalosh-

inskiĭ, Metody kvantovoĭ teorii polya v statisticheskoi fizike, Fizmatgiz, 1962 [Quantum Field Theoretical Methods in Statistical Physics; Pergamon, 1965].
[15] A. A. Abrikosov, Zh. Eksp. Teor. Fiz. 39, 1797 (1960) [Sov. Phys.-JETP 12, 1254 (1960)].
[16] V. D. Shafranov, in: Voprosy teorii plazmy) (Problems of Plasma Theory) 3, Gosatomizdat, 1963, p. 3.
[17] A. G. Sitenko, Élektromagnitnye fluktuatsii v plazme (Electromagnetic Fluctuations in Plasma) KhGU, Khar'kov, 1965.
[18] J. R. Schrieffer, Theory of Superconductivity, Benjamin, N. Y. (1964).
[19] D. Pines Elementary Excitations in Solids, Benjamin 1963.
[20] A. B. Migdal and A. I. Larkin, Zh. Eksp. Teor. Fiz. 45, 1036 (1963) [Sov. Phys.-JETP 18, 717 (1964)].
[21] P. Morel and W. Anderson, Phys. Rev. 125, 1263 (1962).

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