ANISOTROPY OF THE EARLY STAGES OF COSMOLOGICAL EXPANSION AND OF RELICT RADIATION

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If the early stages of cosmological expansion differed from the Friedman homogeneous isotropic model, then the most general of the known laws is the law (1) for the variation with time of a volume element of matter[11]. It is shown that there exist homogeneous anisotropic cosmological models with the expansion law (1) for $t \to 0$ which tend to the Friedman representation of expansion for an arbitrary observer as $t \to \infty$. An analysis is given of the conditions of thermodynamic equilibrium during the early stages of cosmological expansion subject to the law (1). An investigation is made of the nature of the anisotropy of the relict cosmological radiation during the late stages of expansion. It is shown that in the present epoch the nature of the anisotropy can be complex. The radiation may have an almost constant temperature over the whole sky with the exception of a spot the dimensions of which depend on the parameters of the model. Within the spot ΔT may either have one definite sign, or it may change sign.

1. Lately the possibility is being widely discussed that the early stages of cosmological expansion differ considerably from the behavior of the homogeneous isotropic model. In order to study this possibility cosmological models are constructed which in the course of expansion approach the Friedmann model and which therefore can lay claim to give a description of the real Universe which is characterized at the present stage by a high degree of isotropy and an average homogeneity on a large scale. In such models physical processes during the early stages of expansion proceed in a manner different from that in the Friedmann model. Moreover, dropping the requirement of homogeneity and isotropy enables one to include in the discussion, for example, an ordered magnetic field, directed streams of particles, etc.

An investigation of such cosmological solutions leads to predictions capable of being tested by observation. Such predictions refer, for example, to the chemical composition of primary matter (which has not yet undergone the stage of nucleosynthesis in the stars),[1,2] to a possible existence of directed neutrino streams[3,4], and, finally, to the magnitude and the angular distribution of the anisotropy of the microwave 3°K cosmological radiation[5-8[. The latter is of particular importance since it can be checked by observation easier than the other conclusions. In[9] data are given on the observations of the anisotropy of microwave radiation at $\lambda = 3.2$ cm which turned out to be of the order $\Delta T/T \approx \pm 0.005$ and which is possibly associated with a difference between the cosmological expansion and the Friedmann expansion.

At the present time specific calculations referring to observational tests of the possibility of a deviation of the early stages of expansion from the Friedmann model have been carried out only for a narrow class of particular solutions. This is associated with the fact that very few solutions are known which for $t \to \infty$ approach the Friedmann solution (according to the observed state of affairs for any observer comoving with the matter) and for which one would, moreover, know their properties at the beginning of expansion when the deviations from the Friedman solution are great. Such solutions are, in particular, some cases of nonstationary spherically symmetric solutions and of homogeneous (in the sense of equivalence of all points of comoving space; for details cf.[10]) anisotropic cosmological models without rotation. Such models possess the following common property. Near the singularity (at the beginning of expansion) the comoving volume varies with time as $V \sim t$ for any equation of state of matter in contrast to $V \sim t^{3/2}$ for the Friedmann solution and the equation of state $P = \epsilon/3$.

It is clear that a different law of dependence of V on t, and, therefore, also of the density $\epsilon(t)$ and of the temperature $T(t)$ leads to a different course of nuclear reactions at the beginning of the expansion, and to different characteristic features of the behavior of weakly interacting particles. The course of development of nuclear reactions in the case of the law $V \sim t$ was calculated by Thorne[2], and the characteristic features of the behavior of weakly interacting particles have been discussed in[3-7].

Lifshitz and Khalatnikov[11] have found a wide class of solutions near the singularity which depends on seven physically arbitrary functions. For this class of solutions the following law holds:

$$V \sim t^{3(1-p_3)(3p_3-1)}, \tag{1}$$

where $p_3 = p_3(x^1, x^2, x^3)$ with $2/3 \le p_3 \le 1$. Here the law $V = V(t)$ differs from the one corresponding to the solutions discussed above. The question arises whether solutions exist which for $t \to 0$ behave in accordance with the Lifshitz-Khalatnikov law, and which for $t \to \infty$ tend to the Friedmann model according to all observational properties. Below we construct an example of such a solution. Thus, for conclusions concerning the chemical composition of primary matter and the properties of the interaction of particles at the beginning of expansion one should, generally speaking, utilize the law (1) which is the most general one of all those known[1)].

There is still another aspect of the problem of the observational testing of the deviations from the Friedmann solution in the past. We are concerned here with the aforementioned anisotropy of the microwave cosmological radiation (for brevity—the background). In order to calculate the anisotropy of the background one ordinarily utilizes homogeneous anisotropic models with flat comoving space (A-models). The anisotropy of the background in A-models has a quadrupole nature. In[8] it is shown that in a more general case the anisotropy of the background is no longer quadrupole. In this reference it is shown that for an anisotropic homogeneous cosmological solution with a space of constant negative curvature and with a density lower than the critical density[12] the anisotropy of the background over the celestial sphere is described by the expression

$$\Delta T / T \sim f(\theta) \cos 2\varphi, \tag{2}$$

where T is the temperature of the background, $f(\theta)$ is a function which has a maximum for a certain $\theta = \theta_0$ with a width of the order of θ_0 and close to zero for other values of θ. it is necessary to determine how general is such a difference of the anisotropy of the background from a quadrupole nature. It will be shown below that in the cosmological solution obtained here the nature of the anisotropy of the background differs from (2) and is more general.

Finally, for a solution with the asymptotic law (1) the problem of weakly interacting particles (and of thermodynamic equilibrium generally) can be formulated quite differently, and this problem has until now been discussed only for the case of an anisotropic model with the law $V \sim t$.

At the earliest stages of the cosmological expansion the finiteness of the velocity of propagation of all interactions leads to the fact that an element of matter can be affected only by a small neighborhood, and all processes (for example, those related to nuclear reactions) are determined by the local properties of the solution. An analysis of homogeneous solutions is therefore of interest also from the point of view of studying the properties during the early stages of expansion of an inhomogeneous Universe in which individual parts are regarded over a bounded time interval as approximately homogeneous.

Thus, the object of the present work consists of constructing and investigating a cosmological solution which, firstly, differs considerably during the early stages from the Friedmann solution and obeys the most general law (1) for the variation $V = V(t)$ and, secondly, for $t \to \infty$ tends according to its observational properties to the Friedmann model.

A special investigation is made of the nature of the anisotropy of the background for this solution and of the question of the thermodynamic equilibrium at the early stages of expansion.

2. We construct a cosmological solution for which near the singularity the law (1) holds, and for $t \to \infty$ the observational properties tend to the Friedmann model. We write the metric in the form

$$ds^2 = dt^2 - e^{2x^3}[a^2(t)\,dx^{1^2} + b^2(t)\,dx^{2^2}] - c^2(t)\,dx^{3^2}. \tag{3}$$

At any arbitrary instant of time the three-dimensional space corresponding to a given system of reference is a space of constant negative curvature. The system of the Einstein equations $R_i^k = -(T_i^k + \frac{1}{2}\delta_i^k T)$ for the metric (3) and of the energy-momentum tensor $T_i^k = (\epsilon + P)u_i u^k - P\delta_i^k$ can be written in the form (velocity of light $c = 1$, $8\pi G = 1$)

$$(\alpha+\beta+\gamma)^{\cdot} + \alpha^2+\beta^2+\gamma^2 = -(\varepsilon+P)u_0^2 + \tfrac{1}{2}(\varepsilon-P), \tag{4a}$$

$$u_1 = u_2 = 0, \tag{4b}$$

$$\alpha+\beta-2\gamma = -(\varepsilon+P)u_0 u_3, \tag{4c}$$

$$\frac{1}{abc}(\alpha abc)^{\cdot} - \frac{2}{c^2} = \frac{1}{2}(\varepsilon-P), \tag{4d}$$

$$\frac{1}{abc}(\beta abc)^{\cdot} - \frac{2}{c^2} = \frac{1}{2}(\varepsilon-P), \tag{4e}$$

$$\frac{1}{abc}(\gamma abc)^{\cdot} - \frac{2}{c^2} = \frac{u_3^2}{c^2}(\varepsilon+P) + \frac{1}{2}(\varepsilon-P), \tag{4f}$$

where $\alpha = \dot{a}/a$, $\beta = \dot{b}/b$, $\gamma = \dot{c}/c$, and the dot above a letter denotes differentiation with respect to t.

It can be easily seen that ϵ, P, u_0, u_3 do not depend on the spatial coordinates and are functions of only the time.

The hydrodynamic equations $T^k_{i;k} = 0$ have the form

$$(\varepsilon+P)[\dot{u}_0 + 2u^3 + u_0(\alpha+\beta+\gamma)] + \dot{\varepsilon}u_0 = 0, \tag{5}$$

$$(\varepsilon+P)\dot{u}_3 + \dot{P}u_3 = 0. \tag{6}$$

We consider the model with the equation of state for matter $P = 0$. First of all, from equation (6) we have $u_3 = \text{const}$. From the system (4) it can be seen that for $u_3 \equiv 0$ we obtain the anisotropic Heckmann-Schücking model[12] (we shall call it the B-model) and if, moreover, $a = b = c$, then we obtain the open Friedmann model. Integration of equation (5) leads to

$$\varepsilon = \frac{\varepsilon_0}{abcu_0}F, \quad F = \exp\left[-2u_3\int_0^t \frac{dt}{\sqrt{c^2(c^2+u_3^2)}}\right]. \tag{7}$$

Moreover, we succeed in obtaining the following integrals,

$$\beta-\gamma = \frac{1}{abc}\left(k - \frac{1}{2}u_3\varepsilon_0 F\right), \tag{8}$$

$$\alpha-\gamma = \frac{1}{abc}\left(-k - \frac{1}{2}u_3\varepsilon_0 F\right), \tag{9}$$

$$\alpha\beta+\alpha\gamma+\beta\gamma - \frac{3}{c^2} = \frac{\varepsilon_0 u_0}{abc}F. \tag{10}$$

Here, ϵ_0 and k are arbitrary constants.

We investigate the behavior of the solution of the system (4) for $P = 0$ near the singularity, i.e., for $t \to 0$ (for the origin of time we have adopted the instant of reaching the singularity). It can be proven that

1)We do not dwell here on the discussion of the problem of the possibility of the existence of a singularity of a more general type than that occurring in the Lifshitz and Khalatnikov solution.

the general solution of the system (4) must depend on three physically arbitrary constants (one is associated with the free gravitational field and two with the presence of matter—with its density and velocity at a fixed instant of time).

The following solution possesses the required physical arbitrariness at $t \to 0$:

$$a = a_0 t^{p_1}, \quad b = b_0 t^{p_2}, \quad c = c_0 t^{p_3},$$

$$\varepsilon = \begin{cases} \dfrac{\varepsilon_0}{a_0 b_0 |u_3|} t^{-1+p_3}, & p_3 > 0 \\ \dfrac{\varepsilon_0}{a_0 b_0 c_0} t^{-1}, & p_3 < 0 \end{cases} \tag{11}$$

where the constants p_1, p_2, p_3 are related by two equations $p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1$. The constants a_0 and b_0 are associated with the choice of scale along the x^1 and x^2 axes and with a displacement along the x^3 axis. The constant c_0 is determined with the aid of (8) and (9) in terms of the other constants by the following condition:

$$12k^2 + \varepsilon_0^2 u_3^2 = 4a_0^2 b_0^2 c_0^2.$$

It is not difficult to verify that after substitution of (11) into (7)—(10) the one relation

$$3p_3 - 1 = \varepsilon_0 u_3 / a_0 b_0 c_0,$$

is imposed on the four constants ϵ_0, k, u_3, p_3, so that only three of them, for example k, ϵ_0, u_3 remain arbitrary. The dependence of ϵ on t for $t \to 0$ (11) turns out to be essentially different for $p_3 > 0$ and $p_3 < 0$.

Since the component $u^3 \approx -u_3 t^{-2p_3}$, then for $p_3 < 0$, $u^3 \to 0$ for $t \to 0$, and the system of coordinates is asymptotically comoving ($t \approx \tau$; τ is the proper time), while the energy density behaves like $\epsilon \sim \tau^{-1}$. $\epsilon = \epsilon(\tau)$ has a different nature for $0 < p_3 < 1$. In this case $u^3 \to \infty$ for $t \to 0$, while the ordinary three-dimensional velocity tends to the velocity of light in accordance with the law $(1 - v^2)^{1/2} \sim t^{p_3}$. The proper time τ of moving matter is related to the time t by means of $d\tau = dt(1 - v^2)^{1/2}$. Therefore $\tau \sim t^{1-p_3}$, while the energy density in the comoving reference system tends to infinity in accordance with the law $\epsilon \sim \tau^{-(1-p_3)/(1+p_3)}$. We now turn to the behavior of the solution for $t \to \infty$. As can be seen from (7)—(10) the solution asymptotically tends to the empty Milne model with $a \sim b \sim c \sim t$. In this case, the isotropization of the expansion, i.e., the tendency towards the same velocity of expansion in all directions, takes place in accordance with the law (the scale along the x^1 and x^2 axes is so chosen that for $t \to \infty$ $a = b = c$)

$$\frac{\dot c}{\dot a} \approx 1 + \frac{C_1}{t^2}, \quad \frac{\dot c}{\dot b} \approx 1 + \frac{C_2}{t^2}, \tag{12}$$

where C_1 and C_2 are determined in terms of the given physically arbitrary constants. In particular, for $u_3 \equiv 0$ we have $C_1 = -C_2$. It should be noted that for sufficiently small values of u_3 the solution can become isotropic even at a stage when the effect of matter on the deformation is significant. In this case the local properties of the solution tend not to be the empty Milne model, but to the Friedmann model containing matter.

The transition to a comoving system of reference is defined by the formulas

$$\zeta = x^3 - \int \frac{u_3 dt}{c\sqrt{c^2 + u_3^2}}, \quad \tau = \int \frac{\sqrt{c^2 + u_3^2}}{c} dt - u_3 x^3. \tag{13}$$

It is evident that the spatial cross-sections τ = const of the comoving coordinate system do not coincide with the cross-sections t = const, as a result of which all the three-dimensional scalar quantities, and also the density of matter in the comoving system, depend at each instant of proper time on the spatial coordinates, i.e., they are inhomogeneously distributed. From formulas (13) it may be seen that all the surfaces τ = const contain both a true singularity where the matter density becomes infinite, and also regions where the density of matter is negligibly small. Nevertheless, in restricted domains sufficiently far from the singularity the distribution of matter is close to homogeneous. Indeed, for $t \to \infty$, as follows from (13), we have

$$t \approx \tau + u_3\zeta, \quad x^3 = \zeta - \frac{u_3}{c_0^2 t}, \quad \varepsilon \sim t^{-3} \sim (\tau + u_3\zeta)^{-3}.$$

Therefore in the region $u_3\zeta \ll \tau$ the distribution is close to a homogeneous one.

Thus, for an arbitrary observer comoving with the matter there exists such a τ starting with which the distribution and the deformation of matter in any given region of space will be arbitrarily close to the Friedmann solution. The characteristic features of observations of relict radiation of a hot model are investigated in the next section.

Near the singularity the equation of state P = 0 is not applicable and it should be replaced by $P = \epsilon/3$. However, the qualitative nature of the solution obtained above is retained in the course of this. From Eq. (6) it follows now that $u_3\epsilon^{1/4}$ = const. During the late stages of expansion (i.e., for $\epsilon \to 0$) the product $u_3 u^3$ depending on the sign of the velocity u_3 either tends to zero (for $u_3 > 0$) or to infinity (for $u_3 < 0$). For $t \to \infty$ the following formulas hold

$$u_3 > 0\colon \ \varepsilon \sim (\ln t / t)^4, \ \ u_3 \sim t / \ln t, \ \ u^3 \sim (t \ln t)^{-1},$$

$$u_3 u^3 \sim (\ln t)^{-2} \to 0, \ \ u_3 < 0\colon \ \varepsilon \sim t^{-8}, \ \ u_3 \sim t^2,$$

$$u^3 \sim \text{const}, \ \ u_3 u^3 \sim t^2 \to \infty.$$

This means thta for $u_3 < 0$ an isotropization of the model with the equation of state $P = \epsilon/3$ does not occur (in a comoving coordinate system large density gradients, in particular, occur). This circumstance can be important from the point of view of observational effects. Naturally, $u_3 u^3 \to \infty$ as a result of the pressure gradient. Therefore for the case P = 0 this did not occur. For $u_3 > 0$, the reference system asymptotically tends to a comoving one and isotropization of expansion occurs, although more slowly than for P = 0[2)].

2) Naturally, the solution with $P = \epsilon/3$ is applicable only as long as in the course of the expansion firstly, the radiation density is greater than the matter density and, secondly, the optical thickness of matter along the path $c\tilde\theta$ ($\tilde\theta$ is the time from the beginning of expansion) is greater than unity. In the opposite case for free quanta and neutrinos the pressure would become anisotropic.

In the neighborhood of the singular point ($t = 0$) the replacement of the equation of state leads to a different dependence of the density of matter of time. For $P = \epsilon/3$ we obtain (for $1/3 < p_3 < 1$)

$$u_3 \approx u_{30} t^{(1-p_3)/2}, \quad \sqrt{1 - v^2} \sim t^{(3p_3-1)/2},$$

$$\varepsilon \approx \varepsilon_0 t^{-2(1-p_3)} \sim \tau^{-4(1-p_3)/(3p_3+1)}. \tag{14}$$

For $p_3 < 1/3$, just as in the case of a dustlike medium with $p_3 < 0$, the reference system asymptotically (for $t \to 0$) goes over in to a comoving one and the dynamics of the model is analogous to the case $u_3 \equiv 0$, i.e., it is the same as in the Heckmann-Schücking solution for $P = \epsilon/3$.

A significant feature of the solution (14) obtained above is the fact that near the singularity all the local properties of each element of the medium (dependence on τ of the density, of the deformation, etc.) is exactly the same as in the seven-function solution due to Lifshitz and Khalatnikov[11]. In contrast to the Heckmann-Schücking solution in which $\epsilon \sim \tau^{-4/3}$ in the solution obtained above we have the law $\epsilon \sim \tau^{-4(1-p_3)/(3p_3+1)}$, i.e., an arbitrary parameter p_3 has appeared. This additional degree of arbitrariness which locally leads to a behavior of the solution analogous to the seven-function case has appeared as a result of the motion of matter with respect to the system of coordinates (3)[3)].

We consider in somewhat greater detail the process of isotropization of the model under discussion with $P = \epsilon/3$ in the case which is of interest to us $1/3 < p_3 < 1$. In the system of equations (4) as $t \to 0$ the terms on the left-hand side of the equations are of order t^{-2} (cf. (11)), and the terms on the right-hand side are of a higher order of smallness. This is the so-called vacuum stage, in which, in particular, the solution is essentially anisotropic. For the isotropization of the solution, it is necessary that: a) the terms on the right-hand side of (4) should become comparable with the terms on the left-hand side, and b) in (4f) the term $(\epsilon + P)u_3u^3$ should become small compared to $\frac{1}{2}(\epsilon - P)$, i.e., $u_3u^3 = u_3^2c^{-2} \ll 1$[4)].

Two variants are possible depending upon which condition is satisfied first.

1) If the velocity u_3 is sufficiently small, then even before the time when the solution begins to become isotropic the velocity of matter with respect to the synchronous reference system (3) will become small, and the solution will go over into the usual (vacuum) asymptotic behavior of the expansion of a B-model. Then the expansion process will be divided into three stages. (We consider only the solution with isotropic pressure.)

The first stage is the one during which $\epsilon \sim t^{-2(1-p_3)} \sim \tau^{-4(1-p_3)/(3p_3+1)}$. It lasts during the time interval $0 < t < t_1$ where t_1 is determined by the condition $u_3u^3 \approx 1$. From this it follows that $t_1 = (c_0/u_{30})^{2/(1-p_3)}$.

The second stage is the one for which $\epsilon \sim t^{-4/3} \sim \tau^{-4/3}$. This stage lasts during the time $t_1 < t < t_2$.

The third stage takes place for $t > t_2$ when the solution approaches the isotropic one and $\epsilon \sim t^{-2} \sim \tau^{-2}$. The instant t_2 is determined from the condition $A/t_1^{4/3} = \epsilon_0/t_1^{2(1-p_3)}$, $A/t_2^{4/3} = 1/t_2^2$, whence $t_2 = (\epsilon_0)^{-3/2} t_2^{1-3p_3} = (\epsilon_0)^{-3/2}(c_0/u_{30})^2$. The third stage occurs only for $u_3 > 0$. In the case $u_3 < 0$ for $P = \epsilon/3$ the anisotropy again increases right up to the moment of transition to the equation of state $P = 0$.

2) If condition a) is satisfied before condition b), then the intermediate stage of transition from the vacuum solution to the isotropic one will be different than in the B-model. The end of the vacuum stage is determined by the condition $t^{-2} \approx \epsilon_0 t^{-2(1-p_3)}$, and the transition to the isotropic stage is determined by the condition $u_3u^3 \approx 1$.

3. We consider the problem of thermodynamic equilibrium during the early stages of expansion. Complete equilibrium occurs if the characteristic time t^* for any reactions establishing equilibrium is less than the hydrodynamic time τ (time since the beginning of expansion):

$$t^* = 1/\sigma n < \tau, \quad \sigma n \tau > 1; \tag{15}$$

n is the particle density, σ is the cross-section for the interaction.

In the Friedmann model for $P = \epsilon/3$ the quantity $n \approx 1/V \sim \tau^{-3/2} \sim T^3$ (T is the temperature). Therefore $n\tau \sim \tau^{-1/2} \sim T$. Therefore, if σ increases with energy, or, in any case, falls off not faster than T, then the left-hand side of the inequality (15) tends to infinity for $\tau \to 0$, $T \to \infty$, and in the Friedmann model complete thermodynamic equilibrium occurs during the early stages[5)].

In the open Heckmann-Schücking model[12] $n\tau = \text{const}$ for $\tau \to 0$. Therefore, if σ either tends to a constant as the energy increases, or falls, then situations are possible when for $\tau \to 0$ there is no equilibrium between certain kinds of particles or even between all particles[4]. Finally, in the case under consideration at present $\epsilon \sim \tau^{-4(1-p_3)/(3p_3+1)}$; we have $n\tau \sim \tau^{2(3p_3-1)/(3p_3+1)} \to 0$ for $\tau \to 0$. In this case at the earliest stages of expansion there is no thermodynamic equilibrium and the particles are free (if only the cross-section σ does not tend to infinity as $T \to \infty$ in accordance with the law $\sigma \sim T^{(3p_3-1)/2(1-p_3)}$ or faster). This derivation is valid for the whole seven-function class of solutions of Lifshitz and Khalatnikov[11]. If there is no thermodynamic equilibrium, then as a result of the anisotropy of the deformation the pressure is also anisotropic. We emphasize once again that here we do not consider processes with free particles and the role of the viscosity for $t \to 0$. This is a separate problem.

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3)We note that in an anisotropic model with flat space the condition of being comoving with matter follows from the condition of homogeneity, i.e., $u^\alpha \equiv 0$, and it is not possible to construct an analogous model with $u_3 \neq 0$. In a problem from the book [13] (p. 412) such a model is constructed near the singularity $t = 0$ without taking into account the converse effect of matter on the metric. It is evident, that such a construction is only approximate.

4)In principle, isotropization can occur not as a result of terms on the right-hand side describing matter, but as a result of the terms $2/c^2$ determined by the curvature of space. But in practice this case is not interesting, since it contradicts the observed values of Hubble's constant, of the average density of matter and of the upper limit for the anisotropy of radiation.

5)With the exception, perhaps, of gravitions[14].

In the preceding section it was pointed out that for $\frac{1}{3} < p_3 < 1$ and for isotropic pressure there exist, generally speaking, three stages of expansion during each of which its own law of variation of matter density with time holds. The effect of free weakly interacting particles can alter these relations $\epsilon = \epsilon(t)$. It is clear that all this must be taken into account in calculating the nuclear reactions occurring in the course of the expansion of the hot model. Final conclusions regarding the relation between the predicted chemical composition of prestellar matter and the degree of anisotropy of the present cosmological expansion can in the course of this be considerably altered compared to the first estimates due to Thorne[2] and can now depend on more than one parameter.

4. We now turn to the problem of the anisotropy of microwave cosmological radiation during the late stages of expansion. At a certain stage of expansion the matter becomes transparent for the background radiation and the photons are propagated freely. The different velocity of expansion of matter in different directions at the instant when it becomes transparent leads to the present anisotropy of the background. In models with a flat comoving space photons moving along the principal axes of the tensor of the velocities of the deformation will always move along these axes[6)]. Therefore the angular distribution of the intensity of the background over the celestial sphere will correspond to a triaxial ellipsoid, and the maximum difference of the measured temperatures of the background will be observed in the sky in mutually perpendicular directions. In[8] it is shown that in models with a curved comoving space the anisotropy of the background will, generally speaking, no longer be such. The example considered there refers to the B-model due to Heckmann and Schücking, i.e., to the case when in formulas (4) $u_3 \equiv 0$.

On a schematic diagram representing the (x^1, x^3) coordinate plane solid lines show light rays arriving at the point of observation 0, dashed lines show the direction of the coordinate lines x^3, and the dotted line is the line starting with which the rays of the background, which are at the present arriving at the position of the observer, were propagated freely without scattering. From the diagram it may be seen that the ray BO (or B′O) travelled perpendicular to the x^3 axis at the time when the matter became transparent. Since the velocity of expansion differs most strongly from the average value in the directions of the x^1 and x^2 axes (which are orthogonal to x^3), then it is clear that the greatest difference in the measured temperatures of the background will be along the directions AO (and to the temperature which is equal to it along the direction A′O) and BO (or B′O). In other planes passing through AOA′ the picture is analogous and the anisotropy of the temperature differs by a factor $\cos 2\varphi$, where φ is the angle between the planes. On the whole, (C_1 is determined by formula (12), t_0 is the present instant):

$$\frac{\Delta T}{\bar{T}} = \frac{T-\bar{T}}{\bar{T}} = \frac{C_1}{t_0^2} f(\theta)\cos 2\varphi, \tag{16}$$

where $f(\theta)$ has a maximum for $\theta = \theta_0$ (cf., diagram). The formula for θ_0 is as follows:

$$\sin\theta_0 = \frac{\Omega(1+z)}{2z + \Omega(1-z) - 4z(1-\Omega)/(1+\sqrt{1+\Omega z})},$$

where $\Omega = 3H^2/\rho$, H is the average value of the Hubble parameter for $t = t_0$, ρ is the density at $t = t_0$, $z = \Delta\lambda/\lambda$ is that value for which the intergalactic gas becomes transparent.

Thus, the temperature of the background is almost constant over the whole sky with the exception of a spot of dimensions of the order $4\pi\theta_0^2$ with four alternating sectors of excess and deficiency of intensity. For $\Omega \ll 1$ we have $\theta_0 \ll 1$. For the limiting case when matter becomes transparent at a stage which already differs little in deformation from the empty isotropic Milne model the function $f(\theta)$ has the form

$$f(\theta) = (1+\cos\theta)^2\left\{\frac{\cos\theta_0}{1-\cos\theta\cos\theta_0} - \frac{2}{\sin^2\theta}\ln\left[1 + \frac{\cos\theta_0}{1-\cos\theta_0}(1-\cos\theta)\right]\right\}. \tag{17}$$

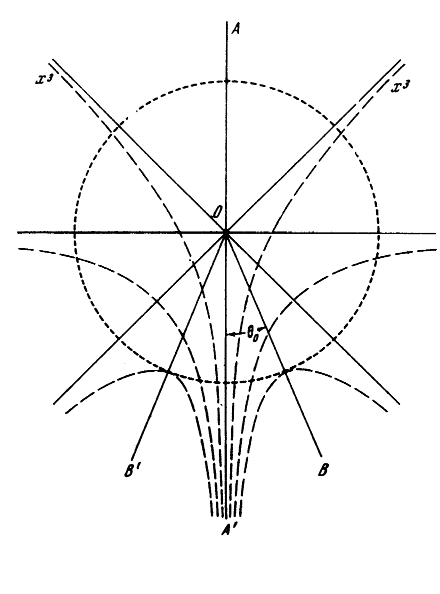
The model constructed in the present paper differs from the one discussed above by the fact that $u_3 \neq 0$ and matter moves along the coordinate lines of x^3. It can be easily seen to what consequences this leads. Firstly, in accordance with (12) the dependence of $\Delta T/T$ on φ will be altered. In place of (16) we obtain

$$\frac{\Delta T}{T} = \frac{\sqrt{C_1^2 + C_2^2}}{t_0^2} f(\theta)\,(\sin\alpha\cos^2\varphi + \cos\alpha\sin^2\varphi), \tag{18}$$

where $f(\theta)$ is, as before, determined by (17)

$$\sin\alpha = C_1/\sqrt{C_1^2 + C_2^2}, \quad \cos\alpha = C_2/\sqrt{C_1^2 + C_2^2}.$$

Secondly, we must take into account the velocity of matter. From the instant $\tilde{\tau}_1$ when matter became transparent until the instant of observation $\tilde{\tau}_0$ the velocity of matter with respect to the synchronous system (3) decreased strongly. Therefore one can neglect the velocity of the observer at the instant $\tilde{\tau}_0$ in comparison with the velocity of matter at the instant $\tilde{\tau}_1$. Thus, on top of the effect of the anisotropy of back-



6)This situation can occur for all models that are homogeneous in the "differential" sense[15].

ground discussed above associated with the anisotropy of deformation there will also be superimposed the Doppler effect associated with the motion of matter along the lines of x^3 [7)]. This effect for $u_3 > 0$ leads to a positive ΔT_D everywhere within the spot indicated above, and to a negative ΔT_D for the remainder of the sky. For $u_3 < 0$ the sign of ΔT_D inside and outside the spot will be opposite. It is clear that for sufficiently great u_3 this effect can become the principal one and can determine the anisotropy of the background. For the effect under consideration at a stage close to the Milne model the following formula holds

$$\frac{\Delta T_D}{T} = \pm \sqrt{u_3 u^3} \frac{\cos\theta - \cos\theta_0}{1 - \cos\theta\cos\theta_0}. \tag{19}$$

The examples discussed above show that the nature of the anisotropy of the background can be complicated. Wilkinson and Partridge[9] have observed a minimum in the background in a relatively small region of the sky with an amplitude which exceeds by a factor of several fold the probable error of measurement. At the present time sufficient data are not yet available in order to draw conclusions regarding the presence of a large scale anisotropy. Experimental investigations are highly desirable.

The model of the Metagalaxy discussed above predicts an anisotropy in the background depending on ten parameters (four parameters θ_0, C_1, C_2, $(u_3u^3)^{1/2}$ determine the anisotropy of the background in accordance with (17)–(19), three parameters determine the unknown motion of the Sun with respect to the radiation field, and three parameters determine the transition to the system of coordinates utilized above from, for example, the galactic system). One should keep in mind that here we have considered a very special model and in actual fact the number of parameters which determine the anisotropy of the background can be greater. As an example, we can indicate the model for which $u_1 \neq 0$ and $u_2 \neq 0$. In such a model the number of parameters will be increased by two more. Still more complicated models are possible. Therefore, a comparison of the predicted anisotropy of the background with observations presents a very complicated problem.

As a possible method of solving such a problem one can propose the following: a portion of the observations can be utilized to determine the parameters of the selected model by means of expanding in terms of a system of orthogonal functions (for example, the Legendre polynomials). A test of whether the selected model is justified will consist in this case of the comparison of the predicted anisotropy (with the parameters already obtained), with the remaining observations.

7)We have assumed that the Doppler effect associated with the motion of the observer is also small in comparison with the anisotropy of the background discussed above for $u_3 \equiv 0$. If this is not the case, then one must take into account also the motion of the observer, and this will lead to an additional dipole effect.

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