GENERATION OF ACOUSTIC WAVES IN PIEZOSEMICONDUCTORS

V. I. PUSTOVOĬT and L. A. CHERNOZATONSKIĬ

All-union Institute of Physico-technical and Radio Engineering Measurements

Submitted January 2, 1968

Zh. Eksp. Teor. Fiz. (U.S.S.R.) 55, 2213–2226 (December, 1968)

The generation of longitudinal and transverse acoustic waves in piezosemiconductors is studied. The kinetic equation is obtained for the noise density in piezosemiconductors for a plasma medium with arbitrary space and time dispersion. By means of solutions of the kinetic equation, it is shown that transverse waves are initially always generated practically independent of the orientation of the crystal in the system. The spectral and angular distributions of the density of generated noise are investigated and it is shown that the indicatrix of the phonon radiation resembles a petal or petals which begin to broaden with increase in the drift velocity of the electrons (holes), and which subsequently begin to narrow. The dependences of the spectral noise density on the frequency, electron concentration, drift veloc-velocity and temperature are studied. It is shown that the characteristic time for noise in the crystal is chiefly determined by the time of flight of the phonons.

In connection with the intense investigation by experiment of the effects of amplification and generation of acoustic waves in piezosemiconductors, it is of interest to obtain formulas that describe this phenomenon. It is known that when the drift velocity of the carriers exceeds the phase velocity of the sound wave, spontaneous oscillations of the lattice, corresponding to sound waves, will grow in the system. To find the intensity of the spontaneously generated sound waves (noise), it is obviously necessary to construct the equation which describes the evolution in space and in time of the mean square of the fluctuating displacement vector, with account of the nonequilibrium properties of the plasma. The state of the electrons and holes in the semiconductor, as also in the plasma,[3] can be completely characterized by specifying the complex dielectric susceptibility of the medium. This in turn determines the acoustic and optical properties of the semiconductors. In a nonequilibrium medium with carrier drift, in addition to the dielectric tensor of the medium, it is also necessary to specify the correlation function of the random currents or of inductions. As has been shown,[2] in certain cases this function can also be expressed in terms of the nonequilibrium complex dielectric susceptibility tensor, with account of drift; thus the problem becomes completely determined if the form of the dielectric susceptibility tensor of the medium is known.

The method of obtaining the equation describing the evolution of the mean square of the fluctuations of the acoustic noise amplitude for a nonequilibrium medium with spatial and temporal dispersion essentially corresponds to the derivation of the law of conservation of energy in the medium.[3]

The aim of the present work is to derive the general kinetic equation for sound fluctuations, and then, with the aid of its solutions, to study the behavior of the generated noises, their spectrum and their directivity pattern, and the dependence of the spectral density of the radiation energy on the temperature, the carrier concentration, drift velocity, and so forth.

1. KINETIC EQUATION FOR PHONONS IN PIEZOSEMICONDUCTORS

The equations describing the fluctuations of sound oscillations and the electric field intensity in piezosemiconductors, with account of the spontaneous "random" oscillations of the electric current

$$j_i^{(s)} = \frac{1}{4\pi}\frac{\partial}{\partial t}D_i^{(s)}$$

and of the elastic stresses $\sigma_{ik}^{(s)}(\mathbf{r}, t)$ have the form[2]

$$\rho\frac{\partial^2 u_i}{\partial t^2} - \lambda_{iklm}\frac{\partial u_{lm}}{\partial r_k} - \mu_{iklm}\frac{\partial^2 u_{lm}}{\partial r_k \partial t} - \beta_{l,ik}\frac{\partial E_l}{\partial r_k} = \frac{\partial \sigma_{ik}^{(s)}}{\partial r_k} \qquad (1.1)$$

$$\varepsilon_0\frac{\partial E_i}{\partial t} - 4\pi\int_{-\infty}^{t} dt' \int d\mathbf{r}'\sigma_{ij}(\mathbf{r}-\mathbf{r}', t-t')E_j(\mathbf{r}', t')$$
$$- 4\pi\beta_{i,kl}\frac{\partial u_{kl}}{\partial t} = -\frac{\partial D_i}{\partial t}, \qquad (1.2)$$

where ρ is the density of the lattice, u_{lm} the deformation tensor, λ_{iklm} the elastic modulus tensor, μ_{iklm} the viscosity tensor, $\beta_{i,kl}$ the piezomodulus tensor "over the deformation," ϵ_0 the static dielectric constant of the lattice, and σ_{ij} the conductivity tensor of the medium. The electric field which accompanies the sound wave is potential; therefore, it is necessary to add the condition curl $\mathbf{E} = 0$ to Eqs. (1.1) and (1.2).

In the representation of wave packets, when the spatial and temporal processes are characterized by fast variables (r, t) and slow variables (μr, μt), the physical quantities of the type $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{u}(\mathbf{r}, t)$ are written in the form[1)]

$$\mathbf{A}(\mathbf{r}, t) \equiv \mathbf{A}(\mu\mathbf{r}, \mu t; \mathbf{r}, t) = \int \frac{d\omega\, d\mathbf{q}}{(2\pi)^4}\mathbf{A}(\mu\mathbf{r}, \mu t; \omega, \mathbf{q})e^{-i\omega t + i\mathbf{q}\mathbf{r}}. \qquad (1.3)$$

Here the time derivative of $\mathbf{A}$ will be

[1)]The method of the derivation of the kinetic equation for phonons is similar to the case of a plasma considered, for example, in the book of Klimontovich. [3]

$$\left(\frac{\partial \mathbf{A}}{\partial t}\right)_{\mu\mathbf{r},\,\mu t;\,\omega,\,\mathbf{q}} = \left\{-i\omega + \frac{\partial}{\partial \mu t}\right\} \mathbf{A}(\mu\mathbf{r}, \mu t; \omega, \mathbf{q}). \quad (1.4)$$

The formula for the spatial derivative is similar. For integral quantities of the type of the electric induction, the transition formulas will have the form (see[3])

$$D_i(\mu\mathbf{r}, \mu t; \omega, \mathbf{q}) = \left\{\varepsilon_{ij}(\omega, \mathbf{q}) + i\frac{\partial}{\partial\omega}\varepsilon_{ij}(\omega, \mathbf{q})\frac{\partial}{\partial\mu t} - i\frac{\partial}{\partial\mathbf{q}}\varepsilon_{ij}(\omega, \mathbf{q})\frac{\partial}{\partial\mu\mathbf{r}}\right\}E_j(\mu\mathbf{r}, \mu t; \omega, \mathbf{q}), \quad (1.5)$$

where

$$D_i(\mathbf{r}, t) = \int_{-\infty}^{t} dt' \int d\mathbf{r}' \varepsilon_{ij}(\mathbf{r} - \mathbf{r}', t - t') E_j(\mathbf{r}', t'). \quad (1.6)$$

The expansions of (1.3)–(1.5), as is known,[3] correspond to the approximation of slowly changing amplitude (the case of geometric optics) and are valid if the conditions

$$\left|\frac{\partial A}{\partial \mu t}\Big/ A\omega_{min}\right| \ll 1, \quad \left|\frac{\partial A}{\partial \mu\mathbf{r}}\Big/ Aq_{min}\right| \ll 1, \quad (1.7)$$

are satisfied, where ω_{min} and q_{min} are the minimal frequency and wave vector of the characteristic variables of the processes in the system.

Eliminating the electric field $\mathbf{E}(\mathbf{r}, t)$ from the system (1.1) and (1.2), we obtain an equation for the amplitude of the sound waves in the piezoelectric medium:

$$\rho\frac{\partial^2 u_i}{\partial t^2} - \frac{\partial}{\partial r_k}\int_{-\infty}^{t} dt' \int d\mathbf{r}' \tilde{\lambda}_{iklm}(\mathbf{r} - \mathbf{r}', t - t') u_{lm}(\mathbf{r}', t') - \mu_{iklm}\frac{\partial^2 u_{lm}}{\partial r_k \partial t}$$

$$= \frac{\partial \sigma_{ik}^{(s)}}{\partial r_k} + \beta_{l,ik}\frac{\partial}{\partial r_k}\int_{-\infty}^{t} dt' \int d\mathbf{r}' \varepsilon_{lj}^{-1}(\mathbf{r} - \mathbf{r}', t - t') D_j^{(s)}(\mathbf{r}', t'), \quad (1.8)$$

where the generalized tensor of the elastic modulus $\tilde{\lambda}_{iklm}$, considered as an operator, will have the form

$$\tilde{\lambda}_{iklm}(\mathbf{r} - \mathbf{r}', t - t') = \lambda_{iklm}\delta(\mathbf{r} - \mathbf{r}')\delta(t - t') - 4\pi\left[\frac{\partial^2}{\partial r_q \partial r_s}\varepsilon_{qs}(\mathbf{r} - \mathbf{r}', t - t')\right]^{-1}\beta_{p,ik}\beta_{j,lm}\frac{\partial^2}{\partial r_p' \partial r_j'}. \quad (1.9)$$

Substituting the expansions (1.3)–(1.5) in the equation of motion (1.8), and using the condition (1.7), we obtain an equation for the damping or growth in space and in time of the amplitude of the wave packet:

$$\left\{L_{ij}(\omega, \mathbf{q}) + i\frac{\partial}{\partial\omega}L_{ij}(\omega, \mathbf{q})\frac{\partial}{\partial\mu t} - i\frac{\partial}{\partial\mathbf{q}}L_{ij}(\omega, \mathbf{q})\frac{\partial}{\partial\mu\mathbf{r}}\right\}u_j = -iq_k\left[\sigma_{ik}^{(s)} - \frac{q_l\beta_{l,ik}\,q_m D_m^{(s)}}{\varepsilon_{rs}(\omega, \mathbf{q})q_r q_s}\right], \quad (1.10)$$

where

$$L_{ij}(\omega, \mathbf{q}) = -\rho\omega^2\delta_{ij} - [-\tilde{\lambda}_{iklj}(\omega, \mathbf{q}) + i\omega\mu_{iklj}]q_k q_l. \quad (1.11)$$

In the absence of random forces, Eq. (1.10) describes the propagation in the piezoelectric medium of acoustic waves with stationary phase and all the results pertaining to the amplification of acoustic waves in piezosemiconductors can be obtained from it. Equation (1.10) is more preferable than, for example, the dispersion equation of the type Det $L_{ij}(\omega, \mathbf{q}) = 0$, inasmuch as it permits us to study a whole class of initial and boundary problems under the amplification conditions; moreover, it can easily be generalized to the case of a weakly inhomogeneous medium.

We are interested in obtaining a kinetic equation for a packet of waves with random phases, i.e., when the phase is repeatedly changed during the interaction and, consequently, the sign of the quantity $u_i(\mu\mathbf{r}, \mu t; \omega, \mathbf{q})$ also changes. Therefore, as in[3,4], we multiply Eq. (1.10) by $i\omega' u_j^*(\mu\mathbf{r}, \mu t; \omega', \mathbf{q}')$ and the complex conjugate of Eq. (1.10), with the replacements $\omega \to \omega'$, $\mathbf{q} \to \mathbf{q}'$, by $i\omega u_i(\mu\mathbf{r}, \mu t; \omega, \mathbf{q})$; combining them, we form the statistical average. For the left side of Eq. (1.10), the statistical average over the ensemble corresponds to averaging over the phase of the wave packets and therefore

$$\langle u_i^*(\mu\mathbf{r}, \mu t; \omega', \mathbf{q}') u_j(\mu\mathbf{r}, \mu t; \omega, \mathbf{q})\rangle = |\mathbf{u}(\mu\mathbf{r}, \mu t; \omega, \mathbf{q})|^2\delta(\omega - \omega')\delta(\mathbf{q} - \mathbf{q}')e_i e_j, \quad (1.12)$$

where e_i is the unit polarization vector of the wave. The relation (1.12) serves essentially as the definition of the quantity $|\mathbf{u}(\mu\mathbf{r}, \mu t, \omega, \mathbf{q})|^2$ which, as it is not difficult to see, characterizes the spectral density of the mean energy of the sound field of the wave packet. In the averaging of (1.12), the phase and the polarization of the waves are regarded as random, i.e., $1/\gamma \ll \delta q \ll q$ and $1/t_0 \ll \delta\omega \ll \omega$, where $t_0 \sim (\mathrm{Im}\,\omega)^{-1}$ and $1/\gamma$ are the characteristic time and length of the considered processes, respectively, $\delta\omega$ and δq are the "widths" of the packet, which determine the change of phase (for more detail, see the book of Tsytovich[4]). Integrating the equation thus obtained for $|\mathbf{u}|^2$ over ω' and $\mathbf{q}'$, and using here the relation (1.12), we obtain the kinetic equation[2)]

$$\frac{\partial}{\partial t}\mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q}) + \frac{\partial}{\partial \mathbf{r}}\mathbf{v}_{\mathrm{rp}}(\omega, \mathbf{q})\mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q}) + 2\gamma(\omega, \mathbf{q})\mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q}) = \int d\omega' d\mathbf{q}' Q^{(s)}(\omega, \mathbf{q}; \omega', \mathbf{q}') \equiv Q^{(s)}(\omega, \mathbf{q}), \quad (1.13)$$

where

$$\mathbf{v}_{\mathrm{lim}}(\omega, \mathbf{q}) = \left(-\frac{\partial}{\partial\omega}\mathrm{Re}\,\{L_{ij}e_i e_j\}\right)^{-1}\frac{\partial}{\partial\mathbf{q}}\mathrm{Re}\,\{L_{ij}e_i e_j\} \quad (1.14)$$

is the group velocity of the waves, and

$$\gamma(\omega, \mathbf{q}) = \left(\frac{\partial}{\partial\omega}\mathrm{Re}\,\{L_{ij}e_i e_j\}\right)^{-1}\mathrm{Im}\,\{L_{ij}e_i e_j\} \quad (1.15)$$

is the damping decrement or the growth increment,

$$\mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q}) = \omega\left(-\frac{\partial}{\partial\omega}\mathrm{Re}\{L_{ij}e_i e_j\}\right)|\mathbf{u}(\mathbf{r}, t; \omega, \mathbf{q})|^2 \quad (1.16)$$

is the energy density of phonons in the dispersive medium with account of the electric field accompanying the sound waves. (The polarization index is omitted in (1.12)–(1.16).)

The right side of Eq. (1.13) determines the spontaneous generation of phonons of a system that is at a finite temperature. To find the explicit form of $Q^{(s)}(\omega, \mathbf{q})$, it is necessary to define the correlation functions of the random inductions and random elastic stresses in a nonequilibrium medium with drift. The correlation function for random elastic stresses can be obtained very simply with the help of the fluctuation-dissipation theorem,[5] inasmuch as there is thermodynamic

2)The index μ will be omitted everywhere below **r** and **t** being already understood as slow variables.

equilibrium of the medium relative to the elastic action of the lattice without electrons (in the linear approximation). Therefore, the correlation function for an infinite, elastically isotropic medium will be[2]

$$\langle q_m{}' \sigma_{mi}^{(s)*}(\omega', \mathbf{q}') q_n \sigma_{nj}^{(s)}(\omega, \mathbf{q}) \rangle = \frac{\hbar\omega}{2\pi} \operatorname{cth} \frac{\hbar\omega}{2T} [\mu_\perp q^2 \delta_{ij} + (\mu_\parallel - \mu_\perp) q_i q_j] \delta(\omega - \omega') \delta(\mathbf{q} - \mathbf{q}'), \quad (1.17)$$

where $\mu_\parallel$ and $\mu_\perp$ are the longitudinal and transverse viscosities, and $\hbar$ is Planck's constant. The correlation function of random inductions (or currents) in a nonequilibrium medium with drift, can be written in the following form, as was shown in[2,6], in the region of low frequencies $ql \ll 1$, which is also the only one considered below:

$$\langle q_m{}' D_m^{(s)*}(\omega', \mathbf{q}') q_n D_n^{(s)}(\omega, \mathbf{q}) \rangle = \frac{T_e}{\pi(\omega - \mathbf{q}\mathbf{v}_d)} \operatorname{Im}\{\varepsilon_{ij}(\omega, \mathbf{q}) q_i q_j\} \delta(\omega - \omega') \delta(\mathbf{q} - \mathbf{q}'). \quad (1.18)$$

It is thus seen that the correlation functions (1.17) and (1.18) are proportional to δ functions of the frequency ω and of the wave vector **q**; therefore the integral on the right side of the kinetic equation (1.13) can be removed. By using the relations (1.17) and (1.18), it is not difficult to find the right side of the kinetic equation (1.13), which determines the spontaneous source of phonons:

$$Q^{(s)}(\omega, \mathbf{q}) = \omega \operatorname{Im} \frac{e_i e_m}{L_{sr} e_s e_r} \left[\xi_{im}(\omega, \mathbf{q}) + \frac{4\pi \beta_{p,mk} q_p q_k \beta_{n,ij} q_n q_j \varphi(\omega, \mathbf{q})}{|\varepsilon_{rs}(\omega, \mathbf{q}) q_r q_s|^2} \right], \quad (1.19)$$

where $\xi_{im}(\omega, \mathbf{q})$ and $\varphi(\omega, \mathbf{q})$ represent the corresponding parts of the correlation functions (1.17) and (1.18), which do not depend on the variables ω' and $\mathbf{q}'$, in other words, without δ functions. In the operator $L_{im}^{-1}(\omega, \mathbf{q})$, which appears in Eq. (1.19), the viscosity and $\operatorname{Im} \epsilon_{ij}$ should approach zero in the given approximation, while the rule for bypassing poles, as was shown in[7], should correspond to the damped solutions.

The presence of the operator $L^{-1}(\omega, \mathbf{q})$ in the source term of the kinetic equation (1.13), which formally goes to infinity at the frequency ω and wave vector **q** for which $\operatorname{Det} L_{ij}(\omega, \mathbf{q}) = 0$, automatically indicates the possibility of propagation and generation of phonons in the system only with those ω and **q** which satisfy the dispersion equation. Physically, this is a well-understood fact. It is essential that in the calculation of such quantities as the total energy density of radiation of phonons, for example,

$$\mathscr{E}(\mathbf{r}, t) = \int \frac{d\omega\, d\mathbf{q}}{(2\pi)^4} \mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q}), \quad (1.20)$$

it is no longer necessary to introduce additional δ functions to satisfy the dispersion law, as was done, in particular, in[8].

Kinetic equations for waves of the type (1.13) are widely used in plasma physics;[3,4,7] for sound waves, such an equation was used in[9] (see also[10]) for specific assumptions on the plasma properties of the medium—weak space and time dispersion.[3)] In contrast with[9] the kinetic equation (1.13) was obtained for piezosemiconductors with arbitrary state of a electron-hole plasma, characterized by the tensor $\epsilon_{ij}(\omega, \mathbf{q})$, and applicable both to the case of weak and to the case of strong space and time dispersion.

If we transform in Eq. (1.13) to the representation of the number of waves $N(\mathbf{r}, t; \omega, \mathbf{q}) = \mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q})/(2\pi)^4 \hbar\omega$, then we get the usual equation for the phonon distribution function $N(\mathbf{r}, t; \omega, \mathbf{q})$.

We now consider the simplest solution of Eq. (1.13). In the absence of electron drift ($v_d = 0$) the medium is in equilibrium and the dependence on the coordinates and time vanishes. Then the spectral density of the radiation energy will be

$$\mathscr{E}_\omega = \int d\mathbf{q} \mathscr{E}(\omega, \mathbf{q}) = \int d\mathbf{q} \frac{Q^{(s)}(\omega, \mathbf{q})}{2\gamma(\omega, \mathbf{q})} = \frac{\omega^2}{2\pi} \left(\frac{1}{v_\parallel{}^3} + \frac{2}{v_\perp{}^3} \right) \left(\frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/T} - 1} \right), \quad (1.21)$$

i.e., it is determined thermodynamically by the equilibrium Planck radiation, independent of the form of the tensor $\epsilon_{ij}(\omega, \mathbf{q})$.

Let us now consider the time solution of the kinetic equation (1.13), which allows us to determine the characteristic time for establishing noise in semiconductors. Using the Laplace transform in time and the initial condition

$$\mathscr{E}(\mathbf{r}, t = 0; \omega, \mathbf{q}) = \mathscr{E}_0(\omega, \mathbf{q}), \quad (1.22)$$

where $\mathscr{E}_0(\omega, \mathbf{q})$ is the initial distribution, which, after integration over the wave vector **q**, gives the Planck distribution (1.21), we find the solution

$$\mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q}) = \begin{cases} \dfrac{Q^{(s)}(\omega, \mathbf{q})}{2\gamma(\omega, \mathbf{q})} (1 - e^{-2\gamma(\omega, \mathbf{q})t}) + \mathscr{E}_0(\omega, \mathbf{q}) e^{-2\gamma(\omega, \mathbf{q})t}, & t < \tau_0 = \dfrac{X}{v_{\lim} \cos\theta)} \\ e^{-\gamma(\omega, \mathbf{q})\tau_0} f(t - \tau_0; \omega, \mathbf{q}) + \dfrac{Q^{(s)}(\omega, \mathbf{q})}{2\gamma(\omega, \mathbf{q})} (1 - e^{-2\gamma(\omega, \mathbf{q})\tau_0}), & t > \tau_0. \end{cases} \quad (1.23)$$

Here $f(t, \omega, \mathbf{q})$ is the density of acoustic oscillations produced on the boundary $X = 0$, θ is the angle between the vector **q** and the X axis. It is assumed here that the drift velocity is directed along X and therefore the growth of the noise also takes place along the X direction. It is seen from Eq. (1.23) that the time for establishing the stationary solution is determined by the time of flight of the phonons from the boundary to a given point with coordinate X. In order to obtain the spectral density of the noise energy, it is necessary to integrate the expression (1.23) over **q**.

We note that for $v_d < v_{ph}$, there is a phonon current carried along by the drifting sound current of electrons;[2] its value can be determined from the equation

$$S_\omega = \int d\mathbf{q}\, \mathbf{v}_{\lim}(\omega, \mathbf{q}) \mathscr{E}(\mathbf{r}, t; \omega, \mathbf{q}).$$

In the stationary case, the solution for the spectral density of the radiation energy, integrated over the wave vector **q**, will be

$$\mathscr{E}_\omega = \frac{\hbar\omega^3}{4\pi v_\parallel{}^3} \int_0^{\pi/2} \frac{d\theta \sin\theta}{2\pi} \left[1 - \exp\left(\frac{-2\gamma_\parallel(\theta) X}{\cos\theta} \right) \right] \left(\frac{\omega}{v_\parallel \gamma_\parallel(\theta)} \right) \left[\frac{\omega \mu_\parallel}{\rho v_\parallel{}^2} \cdot \operatorname{cth} \frac{\hbar\omega}{2T} + \frac{\eta^2 \varepsilon_0 v_\parallel{}^2 \cos^6\theta \varphi(\omega, \omega/v_\parallel, \theta)}{2\hbar\omega^2 |\varepsilon_\parallel(\omega, \omega/v_\parallel, \theta)|^2} \right]$$

$$+ \frac{\hbar\omega^3}{4\pi v_\perp{}^3} \int_0^{\pi/2} \frac{d\theta \sin\theta}{2\pi} \left[1 - \exp\left(\frac{-2\gamma_\perp(\theta) X}{\cos\theta} \right) \right] \left(\frac{\omega}{v_\perp \gamma_\perp(\theta)} \right)$$

[3)]The correlation functions used in [9] for random currents generally do not take into account either space or time dispersion, although, on the other hand, they do play an important role in the theory of amplification and generation of acoustic waves in semiconductors.

$$\times\left[\frac{\omega\mu_\perp}{\rho v_\perp^2}\operatorname{cth}\frac{\hbar\omega}{2T}+\frac{\eta^2\varepsilon_0 v_\parallel^4\cos^4\theta\sin^2\theta\varphi(\omega,\omega/v_\perp,\theta)}{2\hbar\omega^2 v_\perp^2|\varepsilon_\parallel(\omega,\omega/v_\perp,\theta)|^2}\right]+\int dq\, f\exp\left(\frac{-2\gamma_\parallel(\theta)X}{\cos\theta}\right), \quad (1.24)$$

where $v_\parallel$ and $v_\perp$ are the velocities of the longitudinal and transverse sound waves, $\gamma_\parallel(\theta)$ and $\gamma_\perp(\theta)$ are the damping decrements per unit length for longitudinal and transverse waves. In the derivation of (1.24), for the sake of simplicity, it was assumed that only $\beta_{x,xx} \neq 0$, while the remaining $\beta_{i,kl} = 0$. Integration over the wave vector q was accomplished with the use of the method of residues. The poles of $\epsilon_\parallel^{-1}(\omega, q) \equiv \epsilon_{rs}(\omega, q)q_r q_s/q^2$ were not taken into account here, inasmuch as they generally correspond to strongly damped solutions.[4)] The last term in (1.24) corresponds to radiation of waves "from the boundary." For a free boundary, the surface forces on which are equal to zero, while the displacement vector is given, it is easy to show that this term will be equal to

$$2\rho\omega^2|u_x(X=0,\omega)|^2\exp\{-2\gamma_\parallel(\theta=0)X\}, \quad (1.25)$$

which corresponds to amplification of the longitudinal waves "from the boundary."

It is seen from the solution of (1.24) that even in this case, when only $\beta_{x,xx} \neq 0$ and therefore only longitudinal waves can be amplified along the X direction, generation will initially take place of "oblique" transverse waves, the spectral density of the radiation energy of which is described by the second term in (1.24).

In order to make Eq. (1.24) more concrete, it is necessary to substitute the explicit value of the dielectric tensor of the medium and the correlation function. As was shown above, this can be done in only two limiting cases $ql \ll 1$ and $ql \gg 1$, for which the expression for the correlation function is known in terms of the nonequilibrium dielectric tensor of the medium (1.18). As was shown in[12-14], the longitudinal part of this tensor has the form

$$\varepsilon_\parallel(\omega,\mathbf{q})=\varepsilon_0+\frac{4\pi i\sigma_0}{\omega}\left[1-\frac{\mathbf{q}\mathbf{v}_d}{\omega}+\frac{iq^2v_T^2}{\omega\nu}\right]^{-1}, \quad (1.26)$$

$$\varepsilon_\parallel(\omega,\mathbf{q})=\varepsilon_0+\frac{4\pi i\sigma_0}{\omega}\left\{\frac{3\pi(\omega\tau)^2}{2(ql)^3}\left(1-\frac{\mathbf{q}\mathbf{v}_d}{\omega}+i\omega\tau\right)\right\}, \quad (1.27)$$

respectively, for the cases $ql \ll 1$ and $ql \gg 1$. Here σ_0 is the conductivity at constant current, v_T is the thermal velocity of the carriers, $\nu = 1/\tau$ is the effective collision frequency, and l is the free path length. Now, substituting (1.26) or (1.27) in Eqs. (1.24) and (1.23), we obtain the final expression for the spectral density of radiation of phonons in piezosemiconductors, the study of which will be given below. However, we note immediately that we shall consider only the case $ql \ll 1$, which is most frequently satisfied in experiment, in the region of $ql \gg 1$ the generation of phonons is less intense in piezosemiconductors.[5)]

[4)] In crossed electric and magnetic fields, the appearance of "coupled" plasma-acoustical waves is possible[11]; here account of the poles of $\epsilon_\parallel^{-1}(\omega, q)$ is not necessary.

[5)] Apparently this is the reason why in such piezosemiconductors as GaAs, InSb, in spite of the fact that sound amplification is observed, nevertheless, saturation of the current in the volt-ampere characteristic does not take place, because of the relatively small value of the acoustic noise. [15-17].

2. DIRECTIVITY DIAGRAM OF PHONON RADIATION

It is seen from Eq. (1.24) that the directivity diagram of phonon radiation constitutes a rather complicated picture, which is always very difficult to obtain in the general case, even for the simplest situation when $\beta_{x,xx} \neq 0$, unless one resorts to numerical methods. Therefore, we shall consider below only the case in which $|\gamma(\theta)X/\cos\theta| \gg 1$, under the amplification condition and then the behavior of the exponent in Eq. (1.24) will be decisive in the angular dependence of the generation. The opposite case, when one can expand the exponent in a series, is sufficiently simple for analysis, but is less interesting and will be considered but briefly in Sec. 3.

We first consider the case in which only $\beta_{x,xx}$ differs from zero, and the remaining components of the tensor $\beta_{i,kl}$ are equal to zero. For crystals of symmetry C_{6v}, such as CdS and CdSe, this corresponds to the case in which the drift velocity of the electrons is directed along the C_6 axis. Then, using the expression (1.26), we find the growth increments for the longitudinal and transverse sound waves, respectively:

$$\gamma_\parallel(\theta)=\frac{\omega}{2v_\parallel}\left\{\frac{\omega\mu_\parallel}{\rho v_\parallel^2}-\eta^2\cos^6\theta\varepsilon_0\operatorname{Im}\varepsilon_\parallel^{-1}\left(\omega,\frac{\omega}{v_\parallel},\cos\theta\right)\right\}, \quad (2.1)$$

$$\gamma_\perp(\theta)=\frac{\omega}{2v_\perp}\left\{\frac{\omega\mu_\perp}{\rho v_\perp^2}-\eta^2\frac{v_\parallel^2}{v_\perp^2}\sin^2\theta\cos^4\theta\varepsilon_0\operatorname{Im}\varepsilon_\parallel^{-1}\left(\omega,\frac{\omega}{v_\perp}\cos\theta\right)\right\}. \quad (2.2)$$

First we determine the limiting angle θ_0 within which phonons are generated in the system. Substituting the value of the dielectric tensor of the medium $\epsilon_\parallel(\omega, q)$ in (2.1), it is not difficult to obtain an equation that determines $\cos\theta_0 = \xi$ for the longitudinal waves:

$$\xi^6 = C[(\xi\beta-1)^2+A^2]/(\xi\beta-1) \quad (\beta = v_d/v_\parallel), \quad (2.3)$$

where the constants are

$$C=\frac{\omega^2\mu_\parallel\varepsilon_0}{\rho v_\parallel^2\eta^2 4\pi\sigma_0},\quad A=\frac{\omega_0^2}{\omega\nu}\left(1+\frac{\omega^2 r_0^2}{v_\parallel^2}\right),\quad \omega_0^2=\frac{4\pi e^2 n_0}{\varepsilon_0 m},\quad r_0=\frac{v_T}{\omega_0}.$$

It is difficult to find the solution of Eq. (2.3) in explicit form; therefore, we limit ourselves only to qualitative investigation of the behavior of the roots of the equation on the parameters C and A which enter into it. It is immediately seen from (2.3) that, in the absence of viscous or any other non-electron mechanism of sound absorption (C = 0), the limiting angle is determined by the Cerenkov condition $\cos\theta_0 = v_\parallel/v_d$. When $C \neq 0$, it is seen that solutions having physical meaning, for which $\xi \leq 1$, are possible for

$$2CA=\frac{2\omega\mu_\parallel}{\rho v_\parallel^2}\left|\frac{\eta^2}{1+q^2r_0^2}<1\quad\left(q=\frac{\omega}{v_\parallel}\right).\right. \quad (2.4)$$

The roots of Eq. (2.3) can be determined as the points of intersection of the curves $y_1 = \xi^6$ and $y_2 = C[(\xi\beta-1)^2 + A^2]/(\xi\beta-1)$ (see Fig. 1). The latter is bounded by two asymptotes: $\xi = 1/\beta$, $R(\xi) = C(\beta\xi - 1)$. Thus the value of A determines the point $\xi_{min} = (1 + A)/\beta$ at which y_2 takes on a minimum value, while C determines the slope of the line $R(\xi)$, i.e., the rate of "shutting" of the generation cone with increase in the drift velocity β.

It is seen from Fig. 1 that when $1/\beta \lesssim 1$ the point of intersection of the curves y_1 and y_2 initially shifts to the left with increase in β, and then, when the minimum of the curve y_2 is seen to be "to the left" of curve y_1, the point of intersection of the curves begins to move to the right. The latter means that with increase in the drift velocity, the radiation diagram begins to narrow down and for some value of the drift velocity, generation generally stops; schematically, this is shown in Fig. 1 above. The maximum angle inside which phonon generation takes place is determined by the condition $\cos\theta_0^{\max} = (2CA)^{1/6}$, while the maximum value of the drift velocity, for which generation collapses, is equal to $v_{\parallel}(1 + 1/C)$. With increase in frequency, $\theta_0^{\max}(\omega)$ decreases and, consequently, the directivity diagram of the radiation becomes more narrow.

Let us now consider the directivity diagram of the radiation for transverse waves. The equation for the determination of $\cos\theta_0 = \xi$ will have the form

$$\xi^4(1-\xi^2) = C'[(\xi\beta-1)^2 + A'^2] / (\xi\beta - 1) \quad (\beta = v_d / v_\perp),$$
$$C' = -\frac{\omega^2\mu_\perp\varepsilon_0}{\rho v_\parallel^2\eta^2 4\pi\sigma_0}, \quad A' = \frac{\omega_0^2}{\omega\nu}\left(1 + \frac{\omega^2 r_0^2}{v_\perp^2}\right). \tag{2.5}$$

It is seen from Eq. (2.5) that even in this case there exists a minimum value of the drift velocity of the carriers for which phonon generation begins, while the initial value of the angle θ_0 is always different from zero because of the viscous absorption. With increase in β there is initially a broadening of the radiation "petals" and then a narrowing to some limiting value, which is not equal to the initial one, after which the generation stops. Such a behavior of the cone of generation of transverse waves when $\beta_{x,xx} \neq 0$ is connected with a decrease in the electronic growth increment with increase in the drift velocity. Generation is possible here only if (see Fig. 2)

$$C'A' < 2/27. \tag{2.6}$$

Radiation is possible here in the angular range $\theta_0^{\min} < \theta_0 < \theta_0^{\max}$, where $\theta_0^{\min}$ and $\theta_0^{\max}$ are determined as the roots of the equation

$$\cos^4\theta_0(1-\cos^2\theta_0) = 2C'A'.$$

If $C'A' \ll 1$, i.e., the nonelectronic absorption is small, then

$$\cos\theta_0^{\max} \cong (2C'A')^{1/4}, \quad \cos\theta_0^{\min} \cong 1 - C'A', \tag{2.7}$$

and the directivity diagram of the radiation has the form shown in Fig. 2.

We now consider the case in which $\beta_{x,xz}$ (or $\beta_{x,xy}$) is different from zero. Then, evidently, the axial symmetry of the radiation ceases and an additional azimuthal dependence appears. The growth increments for longitudinal and transverse waves will now be equal to

$$\gamma_\parallel(\theta,\varphi) = \frac{\omega}{2v_\parallel}\left\{\frac{\omega\mu_\parallel}{\rho v_\parallel^2} - 4\eta_\perp^2\frac{v_\perp^2}{v_\parallel^2}\varepsilon_0\cos^4\theta\sin^2\theta\sin^2\varphi \times \operatorname{Im}\varepsilon_\parallel^{-1}\left(\omega, \frac{\omega}{v_\parallel}, \cos\theta\right)\right\}, \tag{2.8}$$

$$\gamma_\perp(\theta,\varphi) = \frac{\omega}{2v_\perp}\left\{\frac{\omega\mu_\perp}{\rho v_\perp^2} - \eta_\perp^2\varepsilon_0\cos^2\theta[\cos^2\theta + \sin^2\varphi\sin^2\theta(1-4\cos^2\theta)] \times \operatorname{Im}\varepsilon_\parallel^{-1}\left(\omega, \frac{\omega}{v_\perp}, \cos\theta\right)\right\}, \tag{2.9}$$

where φ is the azimuthal angle measured from the y axis, $\eta_\perp^2 = 4\pi\beta_{x,xz}^2/\rho v_\perp^2\epsilon_0$ is the electromechanical constant for transverse waves. The equation which determines the limiting value of the polar angle θ will also depend on the azimuthal angle φ and for longitudinal waves,

$$\xi^4(1-\xi^2) = C[(\xi\beta-1)^2 + A^2] / (\xi\beta - 1)4\sin^2\varphi \quad (\beta = v_d / v_\parallel). \tag{2.10}$$

Generation is possible for $CA < (8/27)\sin^2\varphi$, so that for small angles φ there is no generation. For a fixed value of the azimuthal angle φ this case is completely analogous to the case already considered for transverse waves at $\beta_{x,xx} \neq 0$.

For transverse waves with $\beta_{x,xz} \neq 0$, the limiting angles will be determined from the equation

$$\xi^2[\xi^2\cos^2\varphi + \sin^2\varphi(2\xi^2-1)^2] = C'[(\xi\beta-1)^2 + A'^2] / (\xi\beta - 1) \quad (\beta = v_d / v_\perp). \tag{2.11}$$

Generation is possible for $2C'A' < 1$. We limit ourselves to finding the limiting angles in the most interesting case $\varphi = \pi/2$. Equation (2.11) then takes the form

$$(2\xi^2-1)^2\xi^2 = C'[(\xi\beta-1)^2 + A'^2] / (\xi\beta - 1). \tag{2.12}$$

It is seen that upon satisfaction of the inequality $C'A' > 1/27$, the behavior of the cone will be similar to the case $\beta_{x,xx} \neq 0$ for longitudinal waves. When $C'A' < 1/27$, additional petals appear on the directivity diagram for sufficiently large drift (see Fig. 3). The fact that the generation vanishes for definite values of the angles is explained by the fall of the electric field (which accompanies the sound wave) to zero for $\theta = \pi/4$, so that the interaction of the elastic displacements with the plasma carriers is cut off.

Upon decrease of the angle φ (the drift velocity is fixed), the petals gradually disappear.

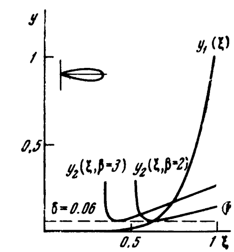

FIG. 1

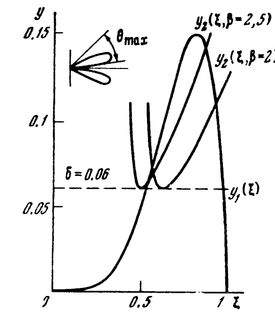

FIG. 2

FIG. 1. Directivity diagram of radiation of longitudinal waves in crystals with $\beta_{x,\,xx} \neq 0$, $1 < \beta_1 < \beta_2 < \beta_3$, $\delta = 2AC$

FIG. 2. Directivity diagram of radiation of transverse waves in crystals with $\beta_{x,\,xx} \neq 0$, $\delta = 2A'C'$

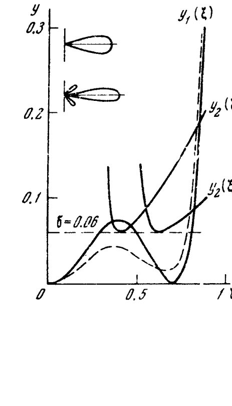

FIG. 3. Directivity diagram of radiation of transverse waves in crystals with $\beta_{x,\,xz} \neq 0$. Solid curves denote the behavior of $y_1(\xi)$ for $\varphi = \pi/2$, the dashed curves, the behavior of $y_1(\xi)$ for $\varphi < \pi/2$; $\delta = 2AC$.

An experimental study of the directivity diagram of phonon radiation in ZnO crystals has been carried out by Zemon, Zucker and Wasko[18] (see also[19]) who, by means of the methods of Rayleigh light scattering, discovered a decrease proportional to θ^2 in the intensity of phonon radiation. As is seen from Eqs. (2.9) and (2.11), the radiation intensity is always proportional to θ^{-2} for small θ.

Consideration was given above of the system when only $\beta_{x,xx} \neq 0$ or $\beta_{x,xz} \neq 0$. If the other components $\beta_{i,kl}$ of the piezomodulus are different from zero, then the directivity diagram can be changed significantly; consideration in this case is best carried out for a specific crystal with known symmetry.

3. INVESTIGATION OF THE SPECTRAL ENERGY DENSITY OF PHONON RADIATION

The spectral energy density of phonon radiation $\mathscr{E}_\omega$ is a rather complicated function of such parameters as the concentration of carriers n_0, the drift velocity v_d and the temperature, so that it can be investigated, most probably, with the help of numerical methods. It is convenient to study this for some specific case or experiment, when a number of the parameters entering into $\mathscr{E}_\omega$ are known. Therefore, we limit ourselves here to some qualitative considerations of the behavior of $\mathscr{E}_\omega$, which can most frequently be realized under the conditions of the experiment.

We consider the behavior of the spectral energy density of the radiation as a function of the drift velocity of the carriers. It is clear that the character of this dependence is determined by two factors: first, by the change in the directivity diagram of the radiation and, second, by the dependence of the increment itself on the drift velocity. Here, inasmuch as the radiation directivity diagrams of the longitudinal and transverse waves are different, the character of the $\mathscr{E}_\omega(\beta)$ dependence should also be different for longitudinal and transverse waves. From the expression (1.24), it is seen that, independently of the orientation of the crystal, the transverse waves with the lower velocity are always the first to be radiated, followed by the longitudinal waves.

If the size of the crystal is not very large, so that $|\gamma(\theta)X/\cos\theta| \ll 1$, then we can expand the exponent in (1.24) in a series, and then, neglecting viscous absorption for simplicity, we obtain the following for transverse waves in the classical limit:

$$\mathscr{E}_\omega^{(\perp)} = \frac{T\omega^2\sigma_0}{\pi\varepsilon_0 v_\perp^4}\eta^2\frac{v_\parallel^2}{v_\perp^2}\int_0^{\theta_0}\frac{d\theta\sin^3\theta\cos^3\theta}{(1-\beta\cos\theta)^2+(\omega_0^4/\omega^2\nu^2)(1+r_0^2\omega^2/v_\perp^2)^2}, \tag{3.1}$$

where θ_0 is the boundary angle that determines the radiation cone, $\cos\theta_0 = v_\perp/v_d = 1/\beta$. It is seen from (3.1) that $\mathscr{E}_\omega$ initially increases as a function of β, chiefly owing to the extension of the limits of integration; it then falls when the first term in the denominator in (3.1) exceeds the last one. The spectrum of generated frequencies here increases at first (as $\omega \to 0$) in proportion to ω^4, then in proportion to the square of the frequency, in the region $r_0\omega/v_\perp \ll 1$, $\omega_0^4/\omega^2\nu^2 \ll (1-\cos\theta)^2$, and virtually ceases to depend on frequency when $r_0\omega/v_T \gg 1$; at some frequency, determined by viscous absorption, the generation stops.

Here, in contrast with the theory of amplification of single frequency signals,[12] the maximum intensity of radiation is no longer determined by the condition $qr_0 \sim 1$. Depending on the carrier concentration or the conductivity, $\mathscr{E}^\omega$, which is determined by Eq. (3.1), has a maximum when

$$\sigma_0^{(opt)} = \frac{\omega}{4\pi}\left[(1-\beta\cos\theta)^2+\frac{\omega^2}{\nu^2}\frac{v_T^4}{v_\perp^4}\right]^{1/2}. \tag{3.2}$$

When $\sigma_0 < \sigma_0^{(opt)}$, the radiation spectral density increases linearly with the conductivity, but when $\sigma_0 > \sigma_0^{(opt)}$ it falls off in proportion to $1/\sigma_0$. As a function of the temperature, $\mathscr{E}_\omega^{(\perp)}$ increases linearly with the temperature for $T < T_{cr}$, where

$$T_{cr} = \frac{mv v_\perp^2}{\omega}\left[(1-\beta\cos\theta)^2+\frac{\omega_0^4}{\omega^2\nu^2}\right]^{1/2}, \tag{3.3}$$

as is seen from (3.1); for $T > T_{cr}$, $\mathscr{E}_\omega^{(\perp)}$ falls off as $1/T$.

We now consider the case of crystals with large dimensions X, where $|\gamma(\theta)X/\cos\theta| \gg 1$ and it is not possible to expand the exponent in Eq. (1.24) in a series. In the general case, the behavior of $\mathscr{E}_\omega(n_0, T, \omega, \beta)$ is rather complicated. Therefore, we consider only the case $\theta = 0$, which corresponds to an experiment in which the noise is recorded by some transducer which detects the normally incident wave. Evidently just this case existed in the experiments of Morozov, Proklov and Stankovskiĭ.[20]

The drift dependence of $\mathscr{E}_\omega$ is determined generally by the behavior of the electronic growth increment, which is conveniently written in the form[6)]

$$\gamma_\parallel^e(\theta=0) = -\frac{Bz}{(1+q^2r_0^2)(1+z^2)}, \quad B = \frac{\eta^2\omega}{2v_\parallel\varepsilon_0},$$
$$z = \frac{\omega\nu(\beta-1)}{\omega_0^2(1+q^2r_0^2)} \quad \left(q = \frac{\omega}{v_\parallel}\right). \tag{3.4}$$

The boundary values of β for which generation cases are determined from the equation $\gamma_\parallel(\theta = 0) = 0$, whence the values of the drift velocity at which generation is initiated and ceases will be equal to

$$v_d^{i} = v_\parallel\left(1+\frac{\mu_\parallel\omega_0^2\varepsilon_0(1+q^2r_0^2)^2}{\rho v_\parallel^2\eta^2\nu}\right), \quad v_d^{c} = v_\parallel\left(1+\frac{\rho v_\parallel^2\omega_0^2\eta^2}{\omega^2\mu_\parallel\nu\varepsilon_0}\right). \tag{3.5}$$

respectively. The maximum value of the spectral density of the noise $\mathscr{E}_\omega$ is achieved at the carrier drift velocity

$$v_d^{m} = v_\parallel\left\{1+\frac{\omega_0^2}{\omega\nu}(1+q^2r_0^2)\left(1-\frac{2v_\parallel}{\omega X}(1+q^2r_0^2)\right)\right\}. \tag{3.6}$$

We now consider the dependence of $\mathscr{E}_\omega$ on the drift for various values of the carrier concentration n_0. In the case of low concentrations, when $qr_0 \gg 1$, the electronic increment increases proportional to n_0; with increase in the carrier concentration, generation takes place sooner, since $(v_d^i/v_\parallel - 1) \sim 1/n_0$, and ends later, while v_d^m slowly shifts in the direction of smaller values. Qualitatively, such a dependence of $\mathscr{E}_\omega$ on the drift was

---

6)For fixed θ, Eq. (3.5) remains in force upon the substitution $B \to B_\theta = B\Phi(\theta)$ where $\Phi(\theta)$ depends on the choice of the piezomodulus, $z \to z_\theta$; $z_\theta = (\omega\nu/\omega_0^2)(\beta\cos\theta - 1)/(1 + q^2r_0^2)$. Thus the dependences of $\mathscr{E}_\omega(\theta)$ on the parameters will be similar to the behavior of $\mathscr{E}_\omega(\theta = 0)$.

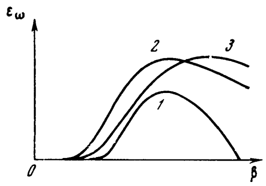

FIG. 4. Behavior of $\mathscr{E}_\omega^{(\parallel)}$ ($\theta = 0, \beta$) for different concentrations (ω, T fixed): 1 – $qr_0 \sim 1$, 2 – $qr_0 \ll 1$, 3 – $qr_0 \gg 1$.

observed in[20] for transverse waves. (The expression $\mathscr{E}^{(\perp)}(\theta = 0)$ for $\beta_{x,xz} \neq 0$ is similar to $\mathscr{E}^{(\parallel)}(\theta = 0)$ for $\beta_{x,xx} \neq 0$, which we shall consider.) For concentration such that $qr_0 \sim 1$, the electronic increment reaches a maximum value in n_0, and with it, $\mathscr{E}_\omega$ also. In this case, the integrated noise density will be proportional to $n_0^{3/2}$, and the spectral one tö n_0. For high concentrations ($qr_0 \ll 1$), the entire picture of the behavior of $\mathscr{E}_\omega(\beta)$ shifts to the region of large values of the drift (see Fig. 4).

In the case of high frequencies ($qr_0 \gg 1$), it is necessary to take into account the viscous absorption, which increases with the frequency, and therefore a decrease in the maximum value of $\mathscr{E}_\omega(\beta^M)$ takes place with increase in frequency, generation begins at high drift: $(v_d^i/v_\parallel - 1) \sim \omega^4$ and disappears sooner: $(v_d^c/v_\parallel - 1) \sim 1/\omega^2$, while v_d^m also increases with frequency.

At low frequencies ($qr_0 \ll 1$), an increase takes place in the electronic increment, proportional to ω^2, the maximum of $\mathscr{E}_\omega(\beta_M)$ is achieved at smaller values of the drift and, at first, generation is practically independent of the frequency.

Let us now consider the frequency dependence of the noise spectral density for $|\gamma_\parallel(\theta = 0)X| \gg 1$. The frequency ω_M for which $\mathscr{E}_\omega$ is a maximum, is determined from the equation $\partial\mathscr{E}_\omega/\partial\omega = 0$, which, with account of viscous absorption will have the form

$$(\zeta^2 - 1)\zeta = 2A(1 - \zeta C)(\zeta + B(1 + \zeta)^2), \tag{3.7}$$

where

$$\zeta = \left(\frac{\omega_M v_T}{\omega_0 v_\parallel}\right)^2, \quad A = \frac{\eta^2 \varepsilon_0 v_\parallel \nu^3 (\beta - 1)^3}{\omega_0^4 X},$$
$$B = \left(\frac{\omega_0 v_T}{\nu(\beta - 1) v_\parallel}\right)^2, \quad C = \frac{\mu_\parallel \omega_0^2 X}{\rho v_T^2 v_\parallel}.$$

If the parameter $\zeta C > 1$, then ω_M shifts in the direction of lower frequencies in comparison with $\omega = \omega_0 v_\parallel / v_T$, i.e., $1/C < \zeta < 1$. If now $\zeta C < 1$, the value of ω_M shifts in the direction of higher frequencies, i.e., $1/C > \zeta > 1$. Thus the generation maximum, even in this very simple analysis, is not determined by the condition $qr_0 \sim 1$ (see, for example,[18]).

The limit frequency for which generation disappears is determined by the equation $\gamma_\parallel(\theta = 0) = 0$ and is equal to

$$\omega_{\lim}^2 = \omega_0^2 \frac{v_\parallel^2}{v_T^2} \left(\sqrt{\alpha^2 + \delta} - \alpha\right), \tag{3.8}$$

where

$$\alpha = \frac{1}{2}\left(\frac{(\beta - 1)\nu v_\parallel}{\omega_0 v_T}\right)^2 + 1, \quad \delta = \frac{(\beta - 1)\rho v_\parallel^2 \eta^2 \nu}{\mu_\parallel \omega_0^2} - 1.$$

We note that the analysis of the generation was carried out only for the case of the absence of trapping and capture centers; if the latter exist, then their account leads to the result that, in place of $\epsilon_{ij}(\omega, q)$, one gets the value of the dielectric tensor with account of the impurity centers. In the presence of an external magnetic field, consideration of noise generation is similar, i.e., it suffices to substitute in the formulas $\epsilon_{ij}(\omega, q)$ and the correlation function with account of the magnetic field.

In conclusion, we express our deep gratitude to L. V. Keldysh and B. N. Levin for discussion and useful comments.

1 V. L. Ginzburg, Teoriya rasprostraneniya élektromagnitnykh voln v plazme (Theory of Electromagnetic Wave Propagation in Plasma) (Gostekhizdat, 1960).

2 V. I. Pustovoĭt, Zh. Eksp. Teor. Fiz. 55, 1784 (1968) [Soviet Phys.-JETP 28, 941 (1969)].

3 Yu. L. Klimontovich, Statisticheskaya teoriya neravnovesnykh protsessov v plazme (Statistical Theory of Nonequilibrium Processes in Plasma) (Moscow State Univ. Press, 1963.

4 V. N. Tsytovich, Nelineĭnnye effekty v plazme (Nonlinear Effects in Plasma), Nauka, 1967.

5 L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka, 1964.

6 V. V. Angeleĭko and I. A. Akhiezer, Zh. Eksp. Teor. Fiz. 53, 689 (1967) [Soviet Phys.-JETP 26, 433 (1968)].

7 B. B. Kadomtsev, Voprosy teorii plazmy (Problems of Plasma Theory). Collection edited by M. A. Leontovich, Atomizdat, 1964, p. 188.

8 V. L. Gurevich and V. D. Kagan, Fiz. Tverd. Tela 6, 2212 (1964) [Sov. Phys.-Solid State 6, 1752 (1965)].

9 V. L. Gurevich, Zh. Eksp. Teor. Fiz. 46, 354; 47, 1291 (1964) [Soviet Phys.-JETP 19, 242 (1964); 20, 873 (1965)].

10 H. Ozaki and N. Mikoshiba, J. Phys. Soc. Japan 21, 2486 (1966).

11 V. P. Orlov and V. I. Pustovoĭt, Fiz. Tekh. Poluprov. 2, 1305 (1968) [Sov. Phys.-Semicond. 2, 1093 (1969)].

12 V. I. Pustovoĭt, Fiz. Tverd. Tela 5, 2490 (1963) [Sov. Phys.-Solid State 5, 1818 (1963).

13 V. I. Pustovoĭt, Zh. Eksp. Teor. Fiz. 43, 2281 (1963) [Sov. Phys.-JETP 16, 1612 (1963)].

14 H. N. Spector, Phys. Rev. 127, 1084 (1962).

15 M. Kikuchi, H. Hayakawa and Y. Abe, J. Appl. Phys. Japan 5, 735 (1966).

16 K. Weller, C. W. Turner and T. van Duzer, Electr. Lett. 3, 418 (1967).

17 P. O. Sliva and R. Bray, Phys. Rev. Lett. 14, 372 (1965).

18 S. Zemon, J. Zucker and J. H. Wasko, Symp. on Sonics and Ultrasonics, Vancouver, Canada, 1967, rep. L.8.

19 J. Zucker and S. Zemon, Appl. Phys. Lett. 9, 398 (1966); 10, 212 (1967).

20 A. I. Morozov, V. V. Proklov, and B. A. Stankovskiĭ, Fiz. Tekh. Poluprov. 1, 895 (1967) [Sov. Phys.-Semicond. 1, 742 (1967)].

Translated by R. T. Beyer
248