STIMULATED MANDEL'SHTAM-BRILLOUIN SCATTERING IN He II

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The question of excitation of first and second sound in He II by light waves is considered. Expressions are obtained for the growth increments and the phase velocities of the sound waves. It is shown that the sound-wave velocities in He II are lower in the presence of a light wave than in the absence of the wave, i.e., the sound waves are focused by the light waves.

As is well known[1,2], weakly damped sound waves, called first and second sound, can exist in He II. The excitation of these waves with the aid of various vibrating bodies was investigated in detail by E. Lifshitz[3]. In the present paper we consider the phenomenon of interaction of light waves with coherent sound waves excited by light in He II. The latter effect is called stimulated Mandel'shtam-Brillouin scattering[4,5].

The stimulated-scattering process can be considered in the following manner: the light wave with wave vector $\mathbf{k}_L$ and frequency ω_L, after emitting a sound wave with wave vector $\mathbf{q}$ and frequency ω_q, is transformed into a light wave with wave vector $\mathbf{k}_S$ and frequency ω_S. The energy and momentum conservation laws call for the satisfaction of the relations

$$\omega_S + \omega_q = \omega_L, \qquad \mathbf{k}_S + \mathbf{q} = \mathbf{k}_L. \tag{1}$$

Since $q \ll k_L \approx k_S$, we get from (1)

$$\cos\varphi = \cos(\mathbf{k}_L\mathbf{q}) = \frac{1}{2n_L}\frac{\omega_q}{\omega_L}\frac{c}{u} \ll 1, \tag{2}$$

where n_L is the refractive index of the incident light wave. We note that it follows from (2) that the first and second sounds are emitted almost perpendicular to the incident light wave.

To investigate the excited Mandel-shtam-Brillouin scattering in He II, we write out the linearized hydrodynamic equations of a two-component liquid, with allowance for the terms connected with the viscosity[2]

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\mathbf{j} = 0, \tag{3}$$

$$\frac{\partial j_i}{\partial t} + \frac{\partial P}{\partial r_i} - \eta\frac{\partial}{\partial r_k}\left(\frac{\partial v_{ni}}{\partial r_k} + \frac{\partial v_{nk}}{\partial r_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_{nl}}{\partial r_l}\right) - \frac{\partial}{\partial r_i}\{\xi_1 \operatorname{div}(\mathbf{j} - \rho\mathbf{v}_n) + \xi_2 \operatorname{div}\mathbf{v}_n\} = f_{1i}, \tag{4}$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\mu_0 - \frac{\partial}{\partial \mathbf{r}}\{\xi_3 \operatorname{div}(\mathbf{j} - \rho\mathbf{v}_n) + \xi_4 \operatorname{div}\mathbf{v}_n\} = \mathbf{f}_2, \tag{5}$$

$$\frac{\partial(\rho\sigma)}{\partial t} + \rho\sigma \operatorname{div}\mathbf{v}_n - \frac{\chi}{T}\Delta T = 0. \tag{6}$$

The employed notation is standard. The right sides of (4) and (5) contain the external forces produced by the light wave and given by[6-8]

$$\mathbf{f}_1 = \frac{\rho}{8\pi}\nabla\left[|E|^2\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right] \tag{7}$$

$$\mathbf{f}_2 = \frac{1}{\rho}\nabla\left(\frac{\varepsilon|E|^2}{8\pi}\right). \tag{8}$$

The expression for $\mathbf{f}_2$ is determined from the expression for the change of the chemical potential

$$d\mu = d\mu_0 + \frac{1}{\rho}d\left(\frac{\varepsilon|E|^2}{8\pi}\right) = -\sigma dT + \frac{1}{\rho}dP + \frac{1}{\rho}d\left(\frac{\varepsilon|E|^2}{8\pi}\right).$$

Recognizing that the coefficient of thermal expansion in He II is anomalously small, we shall henceforth study the first and second sounds separately.

1. We first consider excitation of first sound by a light wave. From the system of equations (3)–(6) for first sound we obtain the following equation for the change of pressure P':

$$\left(q^2 - \frac{\omega^2}{u_{01}^2} - \frac{i2\omega\delta_1}{u_{01}}\right)P' = -i\mathbf{q}\mathbf{f}_1, \tag{9}$$

where $u_{01} = \sqrt{(\partial P/\partial\rho)_S}$ is the propagation velocity of the first sound in the absence of a light wave. The damping decrement of the first sound is given by

$$\delta_1 = \operatorname{Im}\frac{\omega}{u_{01}} = \frac{\omega^2}{2\rho u_{01}^3}\left(\frac{4}{3}\eta + \xi_2\right). \tag{10}$$

The expression for the force $\mathbf{f}_1$ in (9) with allowance for the conservation laws is

$$\mathbf{f}_1(\omega = \omega_L - \omega_S) = i\rho\mathbf{q}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\frac{\mathbf{E}_L\mathbf{E}_S^*}{8\pi},$$

where $\mathbf{E}_L$ and $\mathbf{E}_S$ are the amplitudes of the incident and scattered waves, respectively.

The amplitude $\mathbf{E}_S$ of the scattered wave is determined from Maxwell's equation with allowance for the nonlinear polarizability

$$\Delta\mathbf{E}_S - \frac{1}{c^2}\frac{\partial^2 E_S}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}(\mathbf{p}^{l} + \mathbf{p}^{nl}),$$

$$\mathbf{p} = \alpha(P')\mathbf{E} = \alpha(0)\mathbf{E} + \left(\frac{\partial\alpha}{\partial\rho}\right)_T\frac{P'}{u_{01}^2}\mathbf{E} = \mathbf{p}^{l} + \mathbf{p}^{nl}. \tag{11}$$

Applying the Fourier transformation to Eq. (11) and introducing the symbol $\epsilon_S = 1 + 4\pi\alpha(0)$, we obtain for the scattered wave

$$\left(k_S^2 - \frac{\varepsilon_S\omega_S^2}{c^2}\right)\mathbf{E}_S = \frac{\omega_S^2}{c^2}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\frac{1}{u_{01}^2}P'^*\mathbf{E}_L. \tag{12}$$

In (9) and (12) we obtain the following dispersion equation:

$$(k_{Sz}^2 - k_{Sz}^{02})(q_z^2 - q_z^{02}) = \frac{\omega_L^2\omega_q^2}{c^2u_{01}^4}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2\frac{|E_L|^2}{8\pi}, \tag{13}$$

where

$$k_{Sz}^{02} = \frac{\varepsilon_S\omega_S^2}{c^2} - (k_{xS}^2 + k_{yS}^2),$$

$$q_z^{02} = \frac{\omega_q^2}{u_{01}^2} + i\frac{2\omega_q}{u_q}\delta_1 - (q_x^2 + q_y^2),$$

k_{Sz}^0 and q_z^0 are the proper wave numbers in the absence of pump energy.

The dispersion relation (13) makes it possible to determine the real and imaginary parts of the wave number for the first sound. When solving Eq. (13) we assume that there is no difference between the phase velocities in the z-axis direction, i.e., $\Delta k = k_{zS}^0 - k_{zL} + |q_z^0| = 0$ and $q_z = q_z^0 + \eta$, where η satisfies the inequality $|\eta| \ll |q_z^0|$. Taking these assumptions into account, we obtain for η a cubic equation in the form

$$\eta^3 + 2q_z^0\eta^2 + Q/2k_{zS} = 0, \tag{14}$$

where Q denotes the right side of (13).

One solution of (14) corresponds to a backward sound wave and Re $\eta = -2|q_z^0|$, while the remaining two roots are

$$\eta_1 = \varepsilon_1 u + \varepsilon_1 v - {}^2/_3 q_z^0, \qquad \eta_2 = \varepsilon_2 u + \varepsilon_1 v - {}^2/_3 q_z^0, \tag{15}$$

where

$$\varepsilon_{1,2} = -{}^1/_2 \pm i\sqrt{3}/2, \qquad u = (-a + \sqrt{a^2 + b^2})^{1/3},$$
$$v = (-a - \sqrt{a^2 + b^2})^{1/3}, \qquad a = {}^8/_{27} q_z^{03} + Q/4k_{zS}^0,$$
$$b = -{}^4/_3 q_z^{02}.$$

Let us consider two cases.

A. Let the following inequality be satisfied

$$\left(\frac{\omega_q}{\omega_L}\right)^2 \frac{\rho c^2}{(\rho\partial\varepsilon/\partial\rho)_T^2} \gg \frac{|E_L|^2}{8\pi}. \tag{16}$$

We determine both the real and imaginary parts Re η and Im η in this limiting case. Calculation of the increment γ_1 = Im η yields for the first sound the following expression:

$$\gamma_1^2 = \frac{1}{4}\frac{\omega_L^2}{u_{01}^2}\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_T^2 \frac{|E_L|^2}{8\pi\rho c^2}. \tag{17}$$

Expression (17) can be used for gases, in which the stimulated scattering process is considered. Following[6], we can rewrite Eq. (17) for gases in the form

$$\gamma_1^2 = \frac{\omega_L^2}{c^2}\frac{(n-1)^2}{\rho u_{01}^2}\frac{|E_L|^2}{8\pi}, \tag{18}$$

where $n = \sqrt{\epsilon}$ is the refractive index.

The solution (15) indicates that the light wave not only excites and amplifies the sound waves, Im $\eta \neq 0$, but also influences their phase velocities. The influence is manifest in the fact that the sound-wave propagation velocity decreases, i.e., the sound wave is focused by the light wave.

The velocity of first sound in He II in the presence of a light wave is given by

$$u_1 = u_{01} \Big/ \left(1 + \frac{3}{4}\frac{\gamma_1^2 u_{01}^2}{\omega_q^2}\right). \tag{19}$$

Expression (19) shows, first, that in the presence of a light wave the sound velocity depends on the sound frequency (frequency dispersion), and, second, that it makes it possible to determine directly the increment γ_1 by measuring the velocity of the sound wave u_1 in He II.

B. Let us consider the second limiting case, in which an inequality inverse to (16) is satisfied. In this case the increment and the velocity of the sound wave are

$$\gamma_1 = \frac{\sqrt{3}}{2}\frac{\omega_q}{u_{01}}\left[\frac{\omega_L^2}{2\omega_q^2}\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_T^2 \frac{|E_L|^2}{8\pi\rho c^2}\right]^{1/3}, \tag{20}$$

$$u_1 = u_{01} \Big/ \left(1 + \frac{\gamma_1 u_{01}}{\sqrt{3}\omega_q}\right). \tag{21}$$

It is interesting to note that in this case u_1 depends on the frequency like $\omega_q^{2/3}$, unlike the first case, where the dependence was of the form ω_q^2. We note also that strong focusing of the sound wave takes place in this case.

2. We now consider excitation of second sound. In this sound, $P' = 0$ and from the system (3)–(6), with allowance for the expression for f_2 ($\omega_q = \omega_L - \omega_S$), we obtain for the change of temperature T' the following equation:

$$\left[q^2 - \frac{\omega^2}{u_{02}^2} - \frac{2i\omega\delta_2}{u_{02}}\right]T' = \varepsilon q^2 \frac{\mathbf{E}_L\mathbf{E}_S^*}{8\pi\rho} \tag{22}$$

where

$$u_{02} = \sqrt{\frac{\rho_s}{\rho_n}\frac{\sigma^2}{(\partial\sigma/\partial T)}}$$

is the velocity of second sound in the absence of the light wave. The damping decrement of the second sound is given by

$$\delta_2 = \text{Im}\frac{\omega}{u_{c2}} = \frac{\omega^2}{2\rho u_2^3}\frac{\rho_s}{\rho_n}\left\{\frac{4}{3}\eta + (\xi_2 + \rho^2\xi_3 - 2\rho\xi_1) + \frac{\rho_n}{\rho_s}\frac{\varkappa}{T}\frac{\partial T}{\partial\sigma}\right\}.$$

From Maxwell's equations for the scattering amplitude $\mathbf{E}_S$, with allowance for the nonlinear polarizability, which in our case is of the form

$$\mathbf{p}^{nl} = \frac{1}{4\pi}\left(\frac{\partial\varepsilon}{\partial T}\right)_p T'^* \mathbf{E}_L,$$

and using Eq. (22) for the change of T', we obtain the following dispersion equation:

$$(k_{zS}^2 - k_{zS}^{02})(q_z^2 - q_z^{02}) = \frac{\omega_L^2\omega_q^2}{c^2 u_{02}^2}\frac{1}{\rho\sigma}\left(\frac{\partial\varepsilon}{\partial T}\right)_p \frac{|E_L|^2}{8\pi}. \tag{23}$$

A. The following inequality is satisfied

$$\frac{u_{02}^2}{c^2}\frac{\omega_L^2}{\omega_q^2}\frac{1}{\rho\sigma}\left(\frac{\partial\varepsilon}{\partial T}\right)\frac{|E_L|^2}{8\pi} \ll 1. \tag{24}$$

When (24) is satisfied, the solution of the dispersion equation yields the following expression for the increment of the second sound:

$$\gamma_2^2 = \frac{1}{4}\frac{\omega_L^2}{c^2}\frac{1}{\rho\sigma}\left(\frac{\partial\varepsilon}{\partial T}\right)_p \frac{|E_L|^2}{8\pi}, \tag{25}$$

and for the propagation velocity of the second sound in the presence of a light wave we get

$$u_2 = u_{02} \Big/ \left(1 + \frac{3}{4}\frac{\gamma_2^2 u_{02}^2}{\omega_q^2}\right). \tag{26}$$

B. If an inequality inverse to (24) holds true, we have

$$\gamma_2 = \frac{\sqrt{3}}{2}\frac{\omega_q}{u_{02}}\left[\frac{u_{02}^2}{c^2}\frac{\omega_L^2}{\omega_q^2}\frac{1}{\rho\sigma}\left(\frac{\partial\varepsilon}{\partial T}\right)_p \frac{|E_L|^2}{8\pi}\right]^{1/3} \tag{27}$$

and the velocity of the second sound in the presence of a light wave is

$$u_2 = u_{02} \Big/ \left(1 + \frac{\gamma_2 u_{02}}{\sqrt{3}\,\omega_q}\right). \tag{28}$$

Expressions (26) and (28) show that frequency dispersion exists, and that the second sound is focused by the light waves.

From the foregoing expressions for the increments it follows that if the increment exceeds the damping decrement ($\beta = \gamma - \delta$), i.e., $\beta > 0$, then the amplitudes of the sound waves increase along the z axis, i.e., the wave is amplified. In this case He II can serve as a parametric sound generator with light as the pump. In the opposite case, when $\beta < 0$, the wave is damped. The condition $\beta = 0$ makes it possible to determine the threshold energy of the light beam.

Let us estimate the threshold light-wave power necessary to excite second sound for the following parameters: $T = 1.8°K$, $\sigma = 0.3$ cal/g-deg, $\rho = 0.15$, $u_{02} = 2.3 \times 10^3$ cm/sec, $\omega_q = 10^5-10^6$ Hz, and $\omega_L \approx 5 \times 10^{15}$. For these parameters, the threshold power is

$$W = c|E_L|^2/8\pi \approx 10^{-1} - 10^{-2}\ \text{W/cm}^2.$$

We note that when a light wave passes through He II, it produces a temperature change defined by the expression[6]

$$\Delta T = \frac{T}{c_p}\left(\frac{\partial \varepsilon}{\partial T}\right)_p \frac{|E_L|^2}{8\pi}$$

At light-wave powers necessary to excite second sound under the indicated parameters, there is practically no change of temperature.

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