REABSORPTION OF SYNCHROTRON RADIATION

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Expressions for the increments of the normal waves propagating at an arbitrary angle to a constant magnetic field are obtained by the method of kinetic equation with self-consistent field. These expressions were investigated for the case when in the system consisting of the "cold plasma plus the nonequilibrium electrons" there arises a synchrotron instability. The difference between the increments of the normal waves, which can lead to noticeable circular polarization of the synchrotron radiation at sufficient dimensions of the radiating region, is obtained for the case of quasi-longitudinal propagation.

RECENTLY, Zheleznyakov and one of the authors[2] presented a kinetic analysis of the synchrotron instability observed by Zheleznyakov[1]; the analysis was devoted to the particular case of propagation of electromagnetic waves transversely to the constant magnetic field **H**. Article[2] was devoted to an investigation of the peculiarities of the transition from the instability at individual harmonics to the synchrotron instability, which is determined by an aggregate of many harmonics of the radiation of relativistic electrons, and to a clarification of the degree of acceptability of the Einstein-coefficient method used in[1] (and also in many other papers devoted to investigations of the reabsorption of synchrotron radiation and stability of waves in a plasma). In the present article, the kinetic analysis of the reabsorption of the synchrotron radiation is generalized to include the case of propagation at an arbitrary angle to the magnetic field. In the case of quasilongitudinal propagation, the expressions for the increments of the synchrotron instability coincide with those obtained in[1].

1. Propagation of plane waves $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$ in a homogeneous medium with a dielectric tensor $\epsilon_{\alpha\beta}(\omega, \mathbf{k})$ is described, in the linear approximation, by the equation[3]

$$[n^2\delta_{\alpha\beta} - n_\alpha n_\beta - \varepsilon_{\alpha\beta}(\omega, \mathbf{k})]E_\beta = 0, \tag{1}$$

where $n \equiv ck/\omega$ (c—velocity of light in vacuum) and $\delta_{\alpha\beta}$ is the Kronecker symbol. The dielectric tensor can be obtained by integrating the kinetic equation with a self-consistent field (see[4]).

We shall henceforth consider a medium consisting of a "cold" magnetoactive plasma and non-equilibrium electrons. In a coordinate frame $x'y'z'$, with z' axis directed along the constant magnetic field **H** and x' axis in the **kH** plane, the dielectric tensor of such a medium is given by

$$\varepsilon_{\alpha\beta}(\omega, \mathbf{k}) = \varepsilon_{\alpha\beta}{}^0(\omega) - i^{\alpha+\beta}L_{\alpha\beta}(\omega, \mathbf{k}). \tag{2}$$

Here $\epsilon^0_{\alpha\beta}$ is the dielectric tensor of the "cold" plasma, $L_{\alpha\beta}$ takes into account the contribution of the nonequilibrium electrons to the dielectric tensor. According to[5], the components $\epsilon^0_{\alpha\beta}$ are equal to

$$\overset{0}{\varepsilon}_{x'x'} = \overset{0}{\varepsilon}_{y'y'} = 1 - \frac{v}{1-u}, \quad \overset{0}{\varepsilon}_{z'z'} = 1 - v,$$

$$\overset{0}{\varepsilon}_{x'y'} = -\overset{0}{\varepsilon}_{y'x'} = \frac{iv\sqrt{u}}{1-u}, \quad \overset{0}{\varepsilon}_{x'z'} = \overset{0}{\varepsilon}_{z'x'} = \overset{0}{\varepsilon}_{y'z'} = \overset{0}{\varepsilon}_{z'y'} = 0,$$

where $v = \omega_L^2/\omega^2$, $u = \omega_H^2/\omega^2$ (here $\omega_L = (4\pi e^2 N_0/m)^{1/2}$, $\omega_H = eH/m_0c$ is the plasma and gyrofrequency of the electrons in the "cold" plasma; e and m_0 are the charge and rest mass of the electron, and N_0 is the concentration of the cold-plasma electrons).

The expressions for $L_{\alpha\beta}(\omega, \mathbf{k})$ were found in[6] by the Shafranov method[4]:

$$L_{\alpha\beta}(\omega, \mathbf{k}) = \sum_{s=-\infty}^{\infty} \int_0^\infty dp_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{D_{\alpha\beta}{}^s(\omega, \mathbf{k}, p_\parallel, p_\perp)}{\omega - \omega_s(p_\parallel, p_\perp, \mathbf{k})}. \tag{3}$$

In (3) we used the following symbols:

$$\omega_s = s\Omega_H + \frac{k_{z'}p_\parallel}{m},$$

$$D^s_{\alpha\beta} = 2\pi\frac{m_0\Omega_L{}^2 p_\perp}{m^2\omega^2}\left\{\begin{matrix} Ap_\perp J_s{}^2 s^2\chi^{-2} & -Ap_\perp J_s J_s{}' s\chi^{-1} & Cp_\perp J_s{}^2 s^2\chi^{-2} - Bp_\perp J_s{}^2 s\chi^{-1} \\ Ap_\perp J_s J_s{}' s\chi^{-1} & -Ap_\perp J_s{}'^2 & Cp_\perp J_s J' s\chi^{-1} - Bp_\perp J_s J_s{}' \\ -Ap_\parallel J_s{}^2 s\chi^{-1} & Ap_\parallel J_s J_s{}' & -Cp_\parallel J_s{}^2 s\chi^{-1} + Bp_\parallel J_s{}^2 \end{matrix}\right\},$$

$$A = (m\omega - k_{z'}p_\parallel)\frac{\partial f}{\partial p_\perp} + k_{z'}p_\perp\frac{\partial f}{\partial p_\parallel},$$

$$B = m\omega\frac{\partial f}{\partial p_\parallel}, \quad C = k_{x'}\left(p_\perp\frac{\partial f}{\partial p_\parallel} - p_\parallel\frac{\partial f}{\partial p_\perp}\right),$$

$m = (m_0^2 + p^2c^{-2})^{1/2}$ is the relativistic mass of the electron, $\Omega_L^2 = 4\pi e^2 N/m_0$, N is the concentration of the nonequilibrium electrons with a distribution function $f(p_\parallel, p_\perp)$ ($p_\parallel$ and $p_\perp$ are the longitudinal and transverse components of the momentum **p** relative to **H**), $\Omega_H = eH/mc$, $J_s(\chi)$ is a Bessel function of order s with argument $\chi = k_{x'}p_\perp/m\Omega_H$. Expression (3) is valid when $\mathrm{Im}\,\omega > 0$. The expression for $\mathrm{Im}\,\omega \le 0$ is obtained by analytic continuation of (3) into the lower half plane of ω.

We shall need subsequently an expression for the imaginary part of $L_{\alpha\beta}$ for real ω and **k** (see[6]):

$$\mathrm{Im}\,L_{\alpha\beta} = -\pi\sum_{s=-\infty}^{\infty}\int_0^\infty dp_\perp\int_{-\infty}^{\infty} dp_\parallel D_{\alpha\beta}{}^s(\omega, \mathbf{k}, p_\parallel, p_\perp)\,\delta(\omega - \omega_s(\mathbf{k}, p_\parallel, p_\perp)), \tag{4}$$

where δ is the Dirac delta function.

It is convenient to investigate Eq. (1) by directing one of the coordinate axis along **k**. We therefore change over to the coordinate system x, y, z (the z axis along **k**, y axis in the (**k**, **H**) plane). In the new coordinate system, the dielectric tensor $\epsilon_{\alpha\beta}$ takes the form (θ = kH):

$$\varepsilon_{\alpha\beta}=\left\{\begin{array}{ccc} \varepsilon_{y'y'} & -\varepsilon_{y'x'}\cos\theta+\varepsilon_{y'z'}\sin\theta & \varepsilon_{y'x'}\sin\theta+\varepsilon_{y'z'}\cos\theta \\ -\varepsilon_{x'y'}\cos\theta+\varepsilon_{z'y'}\sin\theta & \varepsilon_{x'x'}\cos^2\theta+\varepsilon_{z'z'}\sin^2\theta- & \varepsilon_{z'x'}\sin^2\theta-\varepsilon_{x'z'}\cos^2\theta \\ & -(\varepsilon_{x'z'}+\varepsilon_{z'x'})\sin\theta\cos\theta & +(\varepsilon_{z'z'}-\varepsilon_{x'x'})\sin\theta\cos\theta \\ \varepsilon_{x'y'}\sin\theta+\varepsilon_{z'y'}\cos\theta & -\varepsilon_{z'x'}\cos^2\theta+\varepsilon_{x'z'}\sin^2\theta+ & \varepsilon_{x'x'}\sin^2\theta+\varepsilon_{z'z'}\cos^2\theta \\ & +(\varepsilon_{z'z'}-\varepsilon_{x'x'})\sin\theta\cos\theta & +(\varepsilon_{z'x'}+\varepsilon_{x'z'})\sin\theta\cos\theta \end{array}\right\}. \tag{5}$$

We assume that the concentration of the high-energy particles is small compared with the concentration of the electrons of the "cold" plasma, so that for nonzero components $\epsilon^0_{\alpha\beta}$ the following conditions are satisfied

$$|\varepsilon_{\alpha\beta}{}^0| \gg |L_{ik}|. \tag{6}$$

Eliminating E_z from the system (1) with allowance for (5), we get

$$\begin{aligned}(A_{xx}-n^2+M_{xx})E_x+(A_{xy}+M_{xy})E_y&=0,\\ (A_{yx}+M_{yx})E_x+(A_{yy}-n^2+M_{yy})E_y&=0.\end{aligned} \tag{7}$$

In (7) we introduced the notations

$$A_{xx}=\varepsilon_{xx}{}^0+\xi^2\varepsilon_{zz}{}^0,\qquad A_{xy}=-A_{yx}=\varepsilon_{xy}{}^0-\xi\eta\varepsilon_{zz}{}^0,$$
$$A_{yy}=\varepsilon_{yy}{}^0-\eta^2\varepsilon_{zz}{}^0;$$
$$M_{xx}=-L_{y'y'}-i\xi[2L_{x'y'}\sin\theta+(L_{y'z'}-L_{z'y'})\cos\theta]$$
$$-\xi^2[L_{x'x'}\sin^2\theta-(L_{x'z'}+L_{z'x'})\sin\theta\cos\theta+L_{z'z'}\cos^2\theta],$$
$$M_{xy}=(\cos\theta+\eta\sin\theta)[iL_{x'y'}+\xi(L_{x'x'}\sin\theta-L_{z'x'}\cos\theta)]$$
$$+(\eta\cos\theta-\sin\theta)[iL_{y'z'}+\xi(L_{z'z'}\cos\theta-L_{x'z'}\sin\theta)],$$
$$M_{yx}=(\cos\theta+\eta\sin\theta)[-iL_{x'y'}-\xi(L_{x'x'}\sin\theta-L_{x'z'}\cos\theta)]$$
$$+(\eta\cos\theta-\sin\theta)[iL_{z'y'}-\xi(L_{z'z'}\cos\theta-L_{z'x'}\sin\theta)],$$
$$M_{yy}=(\cos\theta+\eta\sin\theta)^2L_{x'x'}-(\cos\theta+\eta\sin\theta)(\eta\cos\theta-\sin\theta)(L_{x'z'}$$
$$+L_{z'x'})+(\eta\cos\theta-\sin\theta)^2L_{z'z'};$$
$$\xi=\varepsilon_{xz}{}^0/\varepsilon_{zz}{}^0;\qquad \eta=\varepsilon_{yz}{}^0/\varepsilon_{zz}{}^0.$$

In the expressions for $M_{\alpha\beta}$, we discarded the terms of second and third order in L_{ik}, which are small by virtue of the condition (6). In addition, we used the relations

$$\varepsilon_{yz}{}^0=\varepsilon_{zy}{}^0,\quad \varepsilon_{xy}{}^0=-\varepsilon_{yx}{}^0,\quad \varepsilon_{xz}{}^0=-\varepsilon_{zx}{}^0,\quad L_{x'y'}=-L_{y'x'}.$$

The dispersion equation (the condition under which the system (7) is not trivial) takes, with allowance for the terms linear in $L_{\alpha\beta}$, the form

$$\begin{aligned}&[n_1^2(\omega)-n^2][n_2^2(\omega)-n^2]+(A_{yy}-n^2)M_{xx}\\ &+(A_{xx}-n^2)M_{yy}+A_{xy}(M_{xy}-M_{yx})=0,\end{aligned} \tag{8}$$

where

$$n_{1,2}^2={}^1\!/_2(A_{xx}+A_{yy})\pm{}^1\!/_2[(A_{xx}-A_{yy})^2-4A_{xy}{}^2]^{1/2}$$
$$=1-\frac{2v(1-v)}{2(1-v)-u\sin^2\theta\mp[u^2\sin^4\theta+4u(1-v)^2\cos^2\theta]^{1/2}}$$

are the squares of the refractive indices of the normal waves in the "cold" plasma; the + sign corresponds to the ordinary wave (index 2). The solutions of (8) can be obtained by perturbation theory. Putting $\omega=\Omega_j+\delta_j$, where Ω_j is the solution of the equation $n^2=n_j^2(\omega)$ and $|\delta_j|\ll\Omega_j$, and assuming that

$$|n_2^2(\Omega_j)-n_1^2(\Omega_j)|\gg|L_{\alpha\beta}|, \tag{9}$$

we obtain from (8)

$$\delta_j=-\frac{\gamma_j^2(\mathbf{kv}_{gr})_j}{2n_j^2}[-L_{y'y'}-2\alpha_jL_{x'y'}-\alpha_j\beta_j(L_{x'z'}+L_{z'x'})$$
$$-\beta_j(L_{y'z'}-L_{z'y'})+\alpha_j^2L_{x'x'}+\beta_j^2L_{z'z'}]. \tag{10}$$

Here

$$(\mathbf{kv}_{gr})_j=n_j\Omega_j[\partial n_j(\omega)/\partial\omega]^{-1}_{\omega=\Omega_j},$$

and K_j and Γ_j are the polarization coefficients of the normal waves in the cold plasma (see also[7], Sec. 23):

$$iK_j=\frac{E_y{}^j}{E_x{}^j}=-\frac{A_{xx}-n_j^2}{A_{xy}}$$
$$=-i\frac{2\sqrt{u}(1-v)\cos\theta}{u\sin^2\theta\mp[u^2\sin^4\theta+4u(1-v)^2\cos^2\theta]^{1/2}},$$
$$i\Gamma_j=-\frac{E_z{}^j}{E_x{}^j}=\frac{\varepsilon_{zx}{}^0}{\varepsilon_{zz}{}^0}+iK_j\frac{\varepsilon_{zy}{}^0}{\varepsilon_{zz}{}^0}=-i\frac{v\sqrt{u}\sin\theta-K_juv\sin\theta\cos\theta}{1-u-v+uv\cos^2\theta}.$$

In (10) and throughout, $\omega=\Omega_j$ and $k=\Omega_jn_j(\Omega_j)/c$.

The increments of the normal waves are determined by the imaginary parts of δ_j. Taking (3) and (4) into account, the increments are equal to

$$\operatorname{Im}\delta_j=\frac{\pi^2\Omega_L{}^2m_0}{n_j{}^2\omega^2}\gamma_j{}^2(\mathbf{kv}_{gr})_j\sum_{s=-\infty}^{\infty}\int_0^{\infty}dp_\perp\int_{-\infty}^{\infty}dp_\parallel\frac{1}{m^2}$$
$$\times\{p_\perp J_s'(\chi)+[\alpha_jp_\perp s\chi^{-1}+\beta_jp_\parallel]J_s(\chi)\}^2\,\delta(\omega-\omega_s(p_\parallel,p_\perp))$$
$$\times\left[sm_0\omega_H\frac{\partial f}{\partial p_\perp}+k_{z'}p_\perp\frac{\partial f}{\partial p_\parallel}\right]. \tag{11}$$

The same expression for the increments was obtained earlier by the Einstein-coefficient method in the classical limit[1)][6].

The conditions under which the expressions (11) for the increments were obtained are the conditions for the applicability of the Einstein-coefficient method. Satisfaction of conditions (6) and (9) denotes that the anisotropy of the medium (the polarization of the normal waves and the Faraday rotation) is determined by the "cold" plasma. In addition, in order for the perturbation-theory method used in the derivation of (10) to be applicable, it is necessary that the distribution function be sufficiently smooth (for more details see[2]).

2. Let us examine the reabsorption of synchrotron radiation of relativistic electrons ($E\gg m_0c^2$) in a plasma. For simplicity we assume that $f(\mathbf{p})$ is isotropic[2)].

If the distribution function is sufficiently broad and the frequency spectrum of the emission of the system of relativistic electrons is continuous, it is possible to replace approximately the sum over s in (11) by an integral with infinite limits. Integrating with respect to s with the aid of the δ function, we obtain

1) The expressions for the increments of the normal waves were obtained in [6] also by a kinetic method, but their identity with (11) has been demonstrated only for frequencies close to harmonics of the gyrofrequency.

2) The analysis remains valid also in the case of weak anisotropy, when the distribution function changes little over the width of the angular spectrum of the radiation of the relativistic electron.

$$\operatorname{Im}\delta_j = \frac{\pi^2(\mathbf{k}\mathbf{v}_{gr})_j}{n_j^2}\frac{\Omega_L^2}{\omega\omega_H}\int_0^\infty dp\,p^3\frac{df}{dp}G_j(p), \quad (12)$$

$$G_j(p) = \gamma_j^2\int_0^\pi \sin^3\psi\left[b_j^2 J_s^2 + b_j\frac{dJ_s^2}{d\chi} + \left(\frac{dJ_s}{d\chi}\right)^2\right]d\psi, \quad (13)$$

where

$$s = (\omega/\Omega_H)(1-\beta_\parallel n_j\cos\theta),$$
$$b_j = [K_j(\cos\theta - n_j\beta_\parallel) + \Gamma_j\sin\theta]/\beta_\perp n_j\sin\theta,$$
$$\beta_\parallel = p_\parallel/mc, \quad \beta_\perp = p_\perp/mc.$$

In (12) we have changed over to the integration variables p and ψ ($p_\parallel = p\cos\psi$, $p_\perp = p\sin\psi$).

Going over from the increments to the reabsorption coefficients of the normal waves[3)] and from the distribution function f(p) to the energy spectrum ($Nf(p)4\pi p^2dp = N(E)dE$, $E \approx pc$), we obtain

$$\mu_j = -\frac{2\pi^2c^2}{\omega^2}\int_0^\infty \frac{d}{dE}\left[\frac{N(E)}{E^2}\right]E^2\tilde{Q}_j(\omega,E)\,dE,$$

$$\tilde{Q}_j(\omega,E) = \frac{e^2\omega}{2\pi\sqrt{3}\,c}\Bigg\{(1-n_j^2\beta^2)\left[\int_{\omega/\omega_{jc}}^\infty K_{5/3}(\eta)\,d\eta + \frac{1-K_j^2}{1+K_j^2}K_{2/3}\left(\frac{\omega}{\omega_{jc}}\right)\right]$$
$$+\frac{8}{3}\frac{K_j\operatorname{ctg}\theta}{1+K_j^2}(1-n_j^2\beta^2)^{3/2}\left[K_{1/3}\left(\frac{\omega}{\omega_{jc}}\right)+\left(\frac{\omega}{\omega_{jc}}\right)^{-1}\right.$$
$$\left.\times\int_{\omega/\omega_{jc}}^\infty K_{1/3}(\eta)\,d\eta\right] + \frac{2}{3}\frac{K_j\Gamma_j}{1+K_j^2}\operatorname{ctg}\theta(1-n^2\beta^2)$$
$$\times\left[5\int_{\omega/\omega_{jc}}^\infty K_{1/3}(\eta)\,d\eta - K_{2/3}\left(\frac{\omega}{\omega_{jc}}\right)\right] + \frac{4\Gamma_j}{1+K_j^2}(1-n^2\beta^2)^{1/2}K_{1/3}\left(\frac{\omega}{\omega_{jc}}\right)$$
$$+2\frac{\Gamma_j^2}{1+K_j^2}\int_{\omega/\omega_{jc}}^\infty K_{1/3}(\eta)\,d\eta\Bigg\}. \quad (14)$$

Here $\omega_{jc} = (3/2)\Omega_{H\perp}\sin\theta\,(1-n_j^2\beta^2)^{-3/2}$, K_ν is the Macdonald function of order ν. In the derivation of (14), the integral (13) was transformed with allowance for the known properties of the Bessel functions and their asymptotic expressions in terms of the Macdonald functions (for details see[12]). In addition, it was assumed that the condition

$$(1-n_j^2\beta^2) \ll 1, \quad (15)$$

is satisfied, corresponding to a high energy of the relativistic electrons in a highly rarefied plasma. The expression for $\tilde{Q}_j(\omega, E)$ coincides, accurate to terms of order $(1-\beta^2 n_j^2)$, with the expression obtained in[8] for the total power of the synchrotron radiation of an electron with energy E per unit frequency interval in the j-th wave.

It should be noted that by virtue of the condition (15) $\Gamma_j \to 0$ (the normal waves are almost transverse); we can therefore use for $\tilde{Q}_j(\omega, E)$ the abbreviated equations

$$\tilde{Q}_j(\omega,E) \approx \frac{e^2\omega}{2\pi c\sqrt{3}}\left\{(1-n_j^2\beta^2)\left[\int_{\omega/\omega_{jc}}^\infty K(\eta)\,d\eta + \frac{1-K_j^2}{1+K_j^2}K_{2/3}\left(\frac{\omega}{\omega_{jc}}\right)\right]\right\}. \quad (16)$$

In (16) we have also omitted small terms of order $(1-n_j^2\beta^2)^{3/2}$. The omitted terms must be taken into account only if the reabsorption coefficient (14) is equal to zero when expression (16) is used.

In[1,9-11], the reabsorption of synchrotron radiation was investigated by the Einstein-coefficient method. In[1], attention was called to the fact that the reabsorption coefficient should be referred to a single normal wave[4)]. The latter circumstance was not taken into account in[9-11]. The expression obtained in[1] differs by a factor ½ from the corresponding expressions used in[10,11], and coincides with expressions (14) in the case of quasilongitudinal propagation ($K_j = \pm 1$, $\Gamma_j = 0$), if we neglect the difference between the refractive indices of the normal waves[5)].

The expressions for the reabsorption coefficients, used in[1,10,11], were obtained assuming a δ-like angular emission spectrum of the relativistic electron. Less accurate results are obtained when an approximate account is taken of the finite width of the angular spectrum, as was done in[9], since an additional factor E appears under the integral sign in the expression of the type (14). The expression (14) given above takes correct account of the finite width of the angular spectrum.

It follows from the results of[1,11] that in the case of quasilongitudinal propagation, at a definite choice of the distribution function of the relativistic electrons, negative reabsorption of the synchrotron radiation is possible in a medium (plasma). It is shown in[2] that the reabsorption can be negative also in the case of transverse propagation.

It is easy to show that μ_j can be negative when the propagation is at an arbitrary angle to the magnetic field. It follows from (14) that

$$\mu_j = \frac{2\pi^2c^2}{\omega^2}\int_0^\infty \frac{N(E)}{E^2}\frac{d}{dE}[E^2\tilde{Q}_j(\omega,E)]\,dE. \quad (17)$$

Negative reabsorption is possible if $d[E^2\tilde{Q}_j(\omega, E)]/dE < 0$ in a certain energy interval. In the region of appreciable influence of the medium, where

$$(1-n_j^2)(E/m_0c^2)^2 \gg 1,$$

the expression for $\tilde{Q}_j(\omega, E)$ is given by the function $\Phi_j(\omega, E)$ (Φ_j is obtained from (14) by putting $1-n_j^2\beta^2 = 1-n_j^2$). The expression $d[E^2\Phi_j(\omega, E)]/dE$ is of alternating sign, inasmuch as

$$\int_0^\infty \frac{d}{dE}[E^2\Phi_j(\omega,E)]\,dE = E^2\Phi_j(\omega,E)\,|_0^\infty = 0,$$

from which it follows that $\mu_j < 0$ for certain distribution functions.

3. The normal waves experience different degrees of absorption (amplification) as a result of reabsorption. In radio astronomical applications, an important role is played by the case of quasilongitudinal propagation, where the normal waves are circularly polarized. In this case

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3) In a rarefied plasma ($|1-n_j^2| \ll 1$) the amplitude increment is connected with the reabsorption coefficient by the relation $2\operatorname{Im}\delta_j = -c\mu_j$.

4) Normal waves for a system of relativistic electrons were not investigated, so that the problem of the reabsorption coefficients of a system of relativistic electrons in vacuum still remains open.

5) The reabsorption coefficients of normal waves in transverse propagation ($k_j = 0, \infty$; $\Gamma_j = 0$) coincide with those obtained in [2].

$$\mu_2-\mu_1=\frac{2\pi e^2c}{\sqrt{3}\omega}\int\limits_0^\infty\frac{d}{dE}\left[\frac{N(E)}{E^2}\right]E^2\left\{(n_2-n_1)\left[\int\limits_{\omega/\omega_c}^\infty K_{5/3}(\eta)\,d\eta\right.\right.$$

$$\left.\left.-\frac{3}{2}\frac{\omega}{\omega_c}K_{5/3}\left(\frac{\omega}{\omega_c}\right)\right]+(1-n^2\beta^2)\frac{\sqrt{u}\sin^2\theta}{2\cos\theta}K_{2/3}\left(\frac{\omega}{\omega_c}\right)\right\}dE$$

$$+\frac{8\pi e^2c}{3\sqrt{3}\omega'}\operatorname{ctg}\theta\int\limits_0^\infty\frac{d}{dE}\left[\frac{N(E)}{E^2}\right]E^2(1-n^2\beta^2)^{3/2}\left[K_{1/3}\left(\frac{\omega}{\omega_c}\right)\right.$$

$$\left.+\left(\frac{\omega}{\omega_c}\right)^{-1}\int\limits_{\omega/\omega_c}^\infty K_{1/3}(\eta)\,d\eta\right]dE,\qquad n_{1,2}^2\approx n^2=1-v,$$

$$\omega_c={}^3/_2\Omega_{H\perp}\sin\theta(1-n^2\beta^2)^{-3/2},\qquad n_2-n_1\approx v\sqrt{u}\cos\theta.\quad(18)$$

The first term in (18) is connected with the small deviation of the polarization of the normal waves from circular polarization, and is of the order of $\mu\sqrt{u}$; the second term is of the order of $\mu\sqrt{1-n^2\beta^2}$ and is connected with the nonzero degree of circular polarization of the radiation of the relativistic electron.

The difference $(\mu_2-\mu_1)$ is small (of the order of μ, ω_H/ω and $\mu\sqrt{1-n^2\beta^2}$). However, in the case of negative reabsorption and sufficient dimensions of the radiative region, even a small difference between the reabsorption coefficients can lead to an appreciable difference between the intensities of the normal waves, if $|\mu_2-\mu_1|L \gtrsim 1$.

In a cosmic plasma, the conditions for quasilongitudinal propagation are usually satisfied in a wide range of angles and frequencies. Therefore circular polarization of the synchrotron radiation can serve as an indication of negative reabsorption in the source.

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