GENERALLY COVARIANT QUANTIZATION OF GRAVITATION AND COSMOLOGY

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A method is proposed for a generally covariant quantization of gravitation similar to the quantization of "ordinary" fields in an external field $g_{ik}(x)$. The metric of the Riemann quantization space is defined as the average of the Heisenberg operator for the gravitation field taken over the ground state of the system of interacting quantized fields and satisfies the Einstein equations. A natural definition of the covariant Green's function for the gravitational field is given. The possibility of spontaneous breaking of the group of general transformations of coordinates is considered in analogy with corresponding models of quantum field theory and it is shown that such an approach is equivalent to the condition that all the particles are compound (i.e., to the bootstrap condition). In this case a zero-mass tensor particle (graviton) must exist which is a bound state of other particles. The homogeneous Bethe-Salpeter equation for the vertex of a scalar meson (of mass m) and a graviton is solved on the assumption of a purely gravitational interaction between mesons in the long wavelength region. The graviton mass ρ (introduced for the removal of infrared divergences) is expressed in terms of m and k (k is the gravitational constant) on the basis of the condition of a correct classical limit for the meson-graviton vertex. For $m = m_p$ (m_p is the proton mass) this yields $1/\rho \approx 10^{27}$ cm, which is approximately equal to the radius of curvature of the universe in a real cosmology.

1. THE GREEN'S FUNCTION FOR THE GRAVITATIONAL FIELD AND FOR THE QUANTIZATION OF GRAVITATION

IN[1,2], a covariant formulation of the Mach principle was proposed based on writing the Einstein equations in integral form (without a free term). In this connection a covariant Green's function $G_{ik}^{\alpha\beta}(x, y)$ was utilized which is a two-point tensor in Riemann space with the metric g_{ik} and is defined by the equation

$$\vec{E}_{ik}{}^{mn}G_{mn}{}^{\alpha\beta}(x,y) = \frac{1}{2}(g_i{}^\alpha g_k{}^\beta + g_k{}^\alpha g_i{}^\beta)\frac{\delta^{(4)}(x-y)}{\sqrt{-g}}, \qquad (1)$$

where the differential operator $\vec{E}_{ik}^{mn}$ must satisfy the condition

$$\vec{E}_{ik}{}^{mn}g_{mn} = R_{ik} - {}^1/_2 g_{ik}R. \qquad (2)$$

In[2], a beautiful but a nonunique method for determining $\vec{E}_{ik}^{mn}$ was proposed based on the procedure of linearization of the Riemann tensor. In[1] a number of restrictions on Eq. (1) was obtained starting from the correspondence of the integral form of Einstein equations to the physical Mach principle, but these restrictions also do not define the differential operator uniquely. In this paper we give a natural definition of the operator $\vec{E}_{ik}^{mn}$ as a second variational derivative with respect to the metric of the action of the gravitational field

$$S = \frac{1}{2\chi^2}\int R\sqrt{-g}d^4x, \qquad (3)$$

where $\chi = \sqrt{8\pi}k$, k is the gravitational constant (the system of units is adopted in which h = c = 1).

We perform a small variation of the metric

$$g_{ik} \to g_{ik} + h_{ik} \qquad (4)$$

with the usual subsidiary conditions

$$C_i(h) \equiv (h_i{}^k - {}^1/_2 g_i{}^k h_n{}^n)_{;k} = 0. \qquad (5)$$

The first variation of S gives, as is well known[3],

$$-\delta^{(1)}[2\chi^2 S] = \int h_i{}^k(R_k{}^i - {}^1/_2 g_k{}^i R)\sqrt{-g}d^4x. \qquad (6)$$

Varying (6) taking (5) into account we define $\vec{E}_{ik}^{mn}$ by the relation

$$\int h_i{}^k\delta[(R_k{}^i - {}^1/_2 g_k{}^i R)\sqrt{-g}]\,d^4x \equiv \int h^{ik}\vec{E}_{ikmn}h^{mn}\sqrt{-g}d^4x, \qquad (7)$$

whence it follows that

$$\vec{E}_{ik}{}^{mn} = -{}^1/_2(g_i{}^m g_k{}^n - {}^1/_2 g_{ik}g^{mn})\vec{\Box} - R_{i.\,k.}^{\,m\,n} + {}^1/_2(g_{ik}R^{mn} + g^{mn}R_{ik}) - {}^1/_4 g_{ik}g^{mn}R. \qquad (8)$$

Here $\vec{\Box}$ is the covariant D'Alembertian operator.

The fact that in (7) one should take the variation of the mixed components of the left side of the Einstein equations is uniquely determined by condition (2). The differential operator (8) is symmetric with respect to an interchange of the pairs of indices (ik), (mn), and this guarantees the fact that equation (1) is self-conjugate.

The use of the retarded Green's function enables us to represent any solution of the Einstein equations in the form

$$g_{ik}(x) = a_{ik}(x) + \chi^2\int G^{(ret)\alpha\beta}_{\;i\,k}(x,y)T_{\alpha\beta}(y)\sqrt{-g(y)}d^4y, \qquad (9)$$

where $a_{ik}(x)$ satisfies the homogeneous equation

$$\vec{E}^{mn}_{i\,k}a_{mn}(x) = 0. \qquad (10)$$

Indeed, according to (1), (2) and (10) the application to (9) of the operator $\vec{E}_{ik}^{mn}$ gives the Einstein equations for the metric g_{ik}. It is evident that for $T_{ik} = 0$ the tensor $a_{ik}(x)$ is a solution of the Einstein equations in vacuo. It is shown in[1] with a rotating cylinder as an example that the class of solutions of the Einstein equations satisfying (9) with $a_{ik} = 0$ corresponds to the physical Mach principle only in the case $\vec{E}_{ik}^{mn}$ contains the Riemann tensor with a definite (with respect to the term involving the D'Alembertian) coefficient. It is essential that the procedure proposed in this paper for obtaining the equations for the Green's function gives a result which is automatically correct from the point of view of the Mach principle. We note that the difference between the Green's function defined according to (1), (8) and the Green's function of[1] does not change the results of[1] for the examples discussed there, although the specific form of the equations in the case of the Friedmann model will be different.

The Green's function introduced above goes over in the limiting case of the Minkowski metric into the ordinary Green's function for a free graviton[4] in the gauge (5) and can serve as a basis for quantizing weak gravitational waves in an arbitrary Riemann space. However, according to (9) it is a propagation function not only for weak perturbations, but for the metric as a whole. This enables us to postulate the following procedure for quantizing gravitation. We introduce formally the Heisenberg operator for the gravitational field $\hat{b}_{ik}(x)$ which satisfies the equation

$$\vec{E}_{i\,k}^{mn}\hat{b}_{mn} = \chi\hat{T}_{ik}, \tag{11}$$

where $\hat{T}_{ik}$ is the Heisenberg operator for the energy-momentum tensor the form of which we do not specify further; the differential operator $\vec{E}_{ik}^{mn}$ (cf., (8)) is defined in a Riemann space with the metric

$$g_{ik}(x) \equiv \langle 0|\hat{b}_{ik}(x)|0\rangle\chi, \tag{12}$$

Here the averaging is taken over the ground state of the system of interacting quantized fields.

The space with the metric (12) is, generally speaking, not flat and not even topologically equviavalent to a flat space. In this sense the approach proposed above differs essentially from the method of quantizing the gravitational field as a field of particles of spin 2 in a flat space[5,6]. Recently a generally covariant quantization of gravitation was proposed in a Riemann base space[7]. It is necessary to note that the differential operator F_{ik}^{mn} introduced in this paper which is an analog of $\vec{E}_{ik}^{mn}$ does not satisfy condition (2).

From (2), (11), (12) it follows that for g_{ik} the Einstein equations hold with the right hand side given by

$$T_{ik}(x) \equiv \langle 0|T_{ik}(x)|0\rangle. \tag{13}$$

If we demand that the covariant divergence of the operator $\hat{T}_{ik}$ should vanish, then from (11) it follows that (the vector C_i is defined in (5))

$$\begin{gathered}-{}^1\!/_2\,\vec{\Box}\, C_i(\hat{b}) + {}^1\!/_2\hat{b}^{kn}(R_{in;k} - {}^1\!/_2 g_{in}R_{;k}) \\ + {}^1\!/_2\hat{b}^{kn}{}_{;i}R_{kn} - {}^1\!/_2\hat{b}_i{}^n{}_{;k}R_n{}^k + {}^1\!/_4\hat{b}_n{}^n{}_{;k}R_i{}^k - {}^1\!/_4\hat{b}_n{}^n{}_{;i}R = 0.\end{gathered} \tag{14}$$

From this it can be seen that the subsidiary condition imposed on the allowable state vectors:

$$\langle\Phi_1|C_i(\hat{b})|\Phi_2\rangle = 0, \tag{15}$$

is compatible with (11) only in the case of an empty base space. But if $\langle 0|\hat{T}_{ik}(x)|0\rangle \neq 0$ (i.e., $R_{ik} \neq 0$), then condition (15) cannot be imposed, and this means that longitudinal gravitons must be observable in the non-empty regions of the base space. The reality of the longitudinal components of the gauge field is characteristic of models of quantum field theory in which the ground state is not invariant with respect to the gauge group[8]. A well known example are the plasma excitations in a superconductor which are formally indistinguishable from longitudinal photons[9]. The analogy indicated above requires further development within the framework of some specific model.

We can introduce the operator for the free gravitational field $\hat{a}_{ik}(x)$ which satisfies the condition:

$$\langle 0|\hat{a}_{ik}|0\rangle = 0 \tag{16}$$

and the linear homogeneous equation

$$\vec{E}_{ik}{}^{mn}\hat{a}_{mn} = 0. \tag{17}$$

The set of linearly independent solutions of this equation specifies the spectrum of free gravitons in an external gravitational field $g_{ik}(x)$.

The matrix elements of the operators can be represented in an explicitly generally covariant form. For example, the matrix element of the operator for the energy-momentum tensor between single graviton states is given by

$$\begin{gathered}\langle\gamma_1|\hat{T}_{ik}(x)|\gamma_2\rangle = \int a_{\alpha\beta}^{(\gamma_1)}(y)\bar{a}_{pq}^{(\gamma_2)}(z) \\ \times \Gamma_{i\,k}^{\alpha\beta pq}(x,y,z)\sqrt{-g(y)}\sqrt{-g(z)}\,d^4y\,d^4z.\end{gathered} \tag{18}$$

Here the index "γ" enumerates the solutions of equation (17) and in flat space denotes the polarization and the momentum of the graviton, $\bar{a}_{pq}^{(\gamma_2)}(z)$, $a_{\alpha\beta}^{(\gamma_1)}(y)$ are suitably normalized positive and negative frequency solutions of (17), i.e., wave functions of the gravitons in the states $|\gamma_2\rangle$, $|\gamma_1\rangle$. $\Gamma_{ik}^{\alpha\beta pq}(x, y, z)$ is a three-graviton vertex which is a three-point tensor. The correspondence between its indices and arguments is clear from (18).

The free causal Green's function for a graviton, as well as the retarded one, is defined by Eqs. (1) and (8). If the form of the perturbation is given, then the use in Feynman diagrams of covariant causal Green's functions for internal graviton lines and for internal lines of other fields gives a generally covariant perturbation theory for the S-matrix. In this case the coefficient functions in the expansion of the S-matrix in terms of the free in-fields are covariant multipoint quantities in a Riemann space with metric $g_{ik}(x)$ which transform with respect to each argument in accordance with the method of transformation of the corresponding field.

One need not specify the form of the interaction, but follow axiomatic quantum field theory in which the current operators (and, in particular, $\hat{T}_{ik}$) must, in principle, be determined on the basis of a number of general postulates[10]. In this case it is necessary that all possible states of free gravitons together with the free states of "ordinary" fields should constitute a complete set in Hilbert space (the completeness postulate). This gives meaning to the Heisenberg operators introduced above formally, since this defines the Hilbert space in which they operate. In a coordinate transformation, which is not a group of motions, the ground state is altered (cf., (12)) on the basis of which the Hilbert space is defined, and also the free equations are altered, i.e., the spectrum of single-particle excitations is altered. In the usual canonical quantization scheme this denotes a transition to a nonequivalent representation of the computation relations[11].

We write (11) in integral form

$$\chi\hat{b}_{ik}(x) = \chi\hat{a}_{ik}(x) + a_{ik}(x) + \chi^2 \int G^{(ret)\alpha\beta}_{ik}(x, y)\hat{T}_{\alpha\beta}(y)\sqrt{-g(y)}\,d^4y. \quad (19)$$

Averaging this equation over the ground state yields (9). C is a numerical tensor $a_{ik}(x)$ and, in particular, can define a flat base space with the Minkowski metric which physically corresponds to a Bose condensation of free long-wave gravitons in the ground state. Mach's principle requires that the whole gravitational field should be produced by matter. In other words, the ground state should not contain free gravitons ($a_{ik} = 0$). In this case the metric g_{ik} satisfies the equation

$$g_{ik}(x) \equiv \chi\langle 0|\hat{b}_{ik}(x)|0\rangle = \chi^2 \int G_{ik}{}^{\alpha\beta}(x, y)\langle 0|\hat{T}_{\alpha\beta}(y)|0\rangle\sqrt{-g}\,d^4y, \quad (20)$$

which is a mathematical formulation of Mach's principle[1,2]. Below we propose an approach one consequence of which is equation (20).

2. SPONTANEOUS BREAKING OF THE GROUP OF GENERAL TRANSFORMATIONS OF COORDINATES

In quantum field theory a spontaneous breaking of a certain group of transformations means that the spectrum of physical particles does not have the symmetry of the basic equations[12]. For example, γ_5—an invariant theory in which the mass of the "bare" fermion is equal to zero, admits the existence of a physical fermion of finite mass[11]. In this case a pseudoscalar meson of zero mass appears which is a bound state of fermions. The solution with the broken γ_5—symmetry can be formulated phenomenologically in a γ_5 covariant form by introducing the local field for this meson[11].

In the general case, according to Goldstone's theorem, if spontaneous breaking of a certain symmetry occurs a zero mass collective excitation[13] necessarily appears. One can always introduce a Heisenberg field[14] corresponding to this particle for which the average over the ground state satisfies a condition of self consistency of type (20). In quantum electrodynamics the possibility of a spontaneous breaking of Lorentz-invariance was considered[15], and also of translational invariance[16].

The spectrum of physical particles is evidently noninvariant with respect to the group of general transformations of coordinates (GTC) and this means, in particular, that inertial forces will appear in the accelerated reference system. In complete analogy with the example given above the GTC group can be regarded as being spontaneously broken if the initial Hamiltonian for "bare" particles is not invariant with respect to it. This means that the latter cannot contain a kinetic energy term involving derivatives of the fields and in the language of field theory this is equivalent to the vanishing of the renormalization constants for the wave functions of the particles. We illustrate this by considering the equation for the renormalized Green's function S for a Fermi-field with an initial mass m_0[10],

$$1/S(p) = Z(p - m_0) + \Sigma(p), \quad (21)$$

where p is the four-momentum of the fermion, Z is the renormalization constant for its wave function, Σ is the exact self-energy part the form of which we do not specify, $(\hat{p} - m_0)^{-1}$ is the Green's function for the "bare" fermion. The theory is a priori γ_5-invariant if, following[11], we set $m_0 = 0$ (the interaction is assumed to be invariant). In exactly the same manner the theory is a priori invariant with respect to the GTC group if the first term on the right-hand side of (21) does not contain the kinetic energy, and for this it is necessary to set Z = 0. The condition for the vanishing of the renormalization constants leads to a theory with complete self-consistency (i.e., to a bootstrap theory[17]) within the framework of which all the particles are compound[18]. A bibliography relevant to this problem may be obtained in[19]. We note that at present there does not exist a consistent bootstrap theory.

From a consideration of the variation of (21) in the case of a general transformation of coordinates it also follows that for Z = 0 there must exist a tensor particle (graviton) which is a bound state of other particles. The possibility of considering a graviton as a Goldstone particle was discussed in[20]. The introduction of a Heisenberg field $\hat{b}_{ik}(x)$ corresponding to this particle leads to the usual generally covariant formulation of the theory. The noninvariance of the ground state of a system of interacting particles with respect to the GTC group means that the average $\chi\langle 0|\hat{b}_{ik}(x)|0\rangle = g_{ik}(x)$ is different from zero. The four-space metric g_{ik} must be completely created by the source $\langle 0|\hat{T}_{ik}|0\rangle$ (cf. (20)), where $\hat{T}_{ik}$ is the Heisenberg operator for the energy-momentum tensor for matter (in just the same way as the magnetic field of a ferromagnetic is created by a nonzero magnetic moment density in the ground state and is not introduced externally). Physically (20) means that the metric g_{ik} is created by the vacuum fluctuations of the energy-momentum tensor due to the absence of translational invariance of the ground state, i.e., of the base space with the same metric $g_{ik}(x)$. The case $a_{ik}(x) \neq 0$ in formulas (9) and (19) corresponds to an a priori breaking of symmetry (introduction of absolute space).

Equation (20) is equivalent to Mach's principle corresponding to a definite choice of the graviton Green's function. According to section 1 this choice is determined by the form of the gravitational action (3). This means that in a complete theory one must show that in the long wavelength region the effective Lagrangian of the gravitational field is proportional to the scalar curvature (cf. in this connection[21]).

The above discussion was basically of a qualitative nature. The next step should be the creation of a corresponding quantitative theory. However, attempts to consider a specific model of spontaneous breaking of the GTC group encounter a difficulty in principle which consists of the fact that the situation when the given symmetry is not broken is physically senseless ($g_{ik} = 0$).

We consider quantitatively on a simple example the condition for the bound nature of the graviton which is mandatory in this approach. Assuming that the graviton is a bound state of scalar mesons we investigate the homogeneous Bethe-Salpeter equation[10] for the vertex $\Gamma_{\mu\nu}(p)$ for a virtual meson and a graviton of zero energy:

$$\Gamma_{\mu\nu}(p) = \frac{i}{(2\pi)^4}\int V(p^2, (p-q)^2, q^2)\frac{\Gamma_{\mu\nu}(q)}{(q^2 - m^2 + i\varepsilon)^2}\,d^4q, \quad (22)$$

where p, q are the four-momenta of the meson; p^2

$\equiv p_0^2 - \mathbf{p}^2$ etc; V is the potential for the interaction between mesons. In the right hand side of (22) we have used the free causal Green's function for a meson of mass m.

In subsequent discussion we shall utilize the one-graviton potential for the interaction of mesons calculated with the aid of the usual diagram technique[22,23];

$$V(p, q) = 16\pi k \frac{[p^2q^2 + 2(pq)m^2 - 2m^4]}{(p-q)^2 + i\varepsilon} \quad (23)$$

where k is the gravitational constant.

The vertex $\Gamma_{\mu\nu}(p)$ describes gravitons of spin 0 and 2:

$$\Gamma_{\mu\nu}(p) = \Gamma^{(0)}(p^2)g_{\mu\nu} + \Gamma^{(2)}(p^2)(2p_\mu p_\nu - {}^1/_2 g_{\mu\nu}p^2), \quad (24)$$

and for $p^2 \to m^2$ must go over into the usual expression

$$\Gamma_{\mu\nu}(p) = 2p_\mu p_\nu - g_{\mu\nu}(p^2 - m^2). \quad (25)$$

Here $g_{\mu\nu}$ is the Minkowski metric.

It is necessary to note that in a field theory with vanishing renormalization constants equation (21) loses its meaning. But if the vertex (24) is known then the complete Green's function for the particle can be obtained from Ward's gravitational identity[23] (in analogy to the manner in which this was done by Salam in electrodynamics[24]).

In order to solve (22) we use the method of[25]. In the equations for $\Gamma^{(0,2)}(p^2)$ obtained as a result of substituting (24) into (22) we deform the path of integration in the complex q_0-plane (on the assumption that this is permissible) with the aid of the general formula

$$\int \Phi(p^2, (p-q)^2, q^2)d^4q = \frac{i\pi}{2}\int_0^\infty \frac{dv}{u}\int_{(\sqrt{u}-\sqrt{v})^2}^{(\sqrt{u}+\sqrt{v})^2} \Phi(-u, -z, -v)\sqrt{R}dz, \quad (26)$$

where

$$u \equiv -p^2, \quad v \equiv -q^2; \quad z = -(p-q)^2, \quad u, v, z > 0;$$
$$R = [(\sqrt{u}+\sqrt{v})^2 - z][z - (\sqrt{u}-\sqrt{v})^2].$$

Taking in (22) the potential (23) and integrating in (26) with respect to the variable z we obtain finally

$$\Gamma^{(0)}(u) = \frac{k}{\pi}\int_0^\infty \Big\{K(u,v)\Big[\frac{v}{u}\theta(u-v) + \theta(v-u)\Big] + m^2v\Big\}\frac{\Gamma^{(0)}(v)dv}{(v+m^2)^2}, \quad (27)$$

$$\Gamma^{(2)}(u) = \frac{k}{3\pi}\int_0^\infty K(u,v)\Big[\frac{v^3}{u^3}\theta(u-v) + \theta(v-u)\Big]\frac{\Gamma^{(2)}(v)}{(v+m^2)^2}dv, \quad (28)$$

where

$$K(u, v) \equiv uv - (u+v)m^2 - 2m^4. \quad (29)$$

These equations can be investigated in the domain of negative values of the arguments, and, in particular, near the mass shell of the meson ($u, v \to -m^2$). The existence in the theory of ultraviolet divergences means that (27) and (28) are not Fredholm equations, and consequently do not give a good boundary value problem (in the non-relativistic case a similar situation denotes a fall towards a center). This difficulty is removed by the introduction of a form-factor which cuts off the potential for the interaction (23) in the region of large transferred momenta, i.e., for $z \equiv -(p-q)^2 > \Lambda^2$. It can also be shown that the use in (22) of a potential with a form-factor leads to an exponential falling off of the Bethe-Salpeter wave function for $u > \Lambda^2$. However, the problem of ultraviolet divergences goes beyond the framework of the present paper, particularly since equations (27), (28) have no physical sense in the high frequency region where we cannot neglect nongravitational interactions in the potential (for example, the strong interaction).

It is essential for us that (27) and (28) are exact in the infrared region, i.e., for u, v close to $(-m^2)$. The real interaction potential in this region is determined by the graviton pole and the diagram with the three-graviton vertex neglected on the right-hand side of (22) is indeed equal to zero as a result of the smallness of the interaction between soft gravitons[26]. In order to investigate (27) and (28) in the infrared region we replace expression (29) by its value on the mass shell:

$$K(u, v) \to m^4. \quad (30)$$

It is essential that the part of (27) and (28) neglected in this replacement is free of infrared divergences. We substitute (30) into (27) and (28) and differentiate the resultant integral equations twice, and this gives for $\Gamma^{(s)}(u)$ (cf.[25])

$$\Big[\frac{d^2}{du^2} + \frac{s+2}{u}\frac{d}{du} + \frac{km^4}{\pi}\frac{1}{u(u+m^2)^2}\Big]\Gamma^{(s)}(u) = 0. \quad (31)$$

Here s = 0, 2. Solutions of these equations free of singularities at u = 0 are given by the hypergeometric functions:

$$\Gamma^{(s)}(u) = \mathrm{const}\cdot F(\alpha, \beta, s+2, u/(u+m^2)), \quad (32)$$

where

$$\alpha = 1 - \beta, \quad \beta = {}^1/_2 - {}^1/_2\sqrt{1 + 4km^2/\pi} \approx -km^2/\pi. \quad (33)$$

From (32) it is easy to determine the behavior of $\Gamma^{(s)}(u)$ as $u \to -m^2$

$$\Gamma^{(s)}(u) = C^{(s)}(u+m^2)^\beta. \quad (34)$$

The boundary conditions for $\Gamma^{(s)}(u)$ follow from the condition that the vertex (24) should have the correct long-wave limit, i.e., so that (24) should go over into (25) for $u = -m^2$. In order to eliminate the infrared divergences (cf., (34)) the whole discussion can be carried out in an external gravitational field with an average curvature ρ^2, and this also means that the graviton will acquire a mass ρ. Qualitatively this can be easily taken into account by requiring that (24) should go over into (25) for $u = -(m+\rho)^2$. Contracting (24) and (25) with the metric tensor we obtain for $\Gamma^{(0)}(u)$ two conditions which must be satisfied for $u = -(m+\rho)^2$:

$$\Gamma^{(0)} = {}^1/_2 m^2, \quad d\Gamma^{(0)}/du = {}^1/_2. \quad (35)$$

The first of these conditions determines the normalization constant in (34) while the second one gives the relation between k, m, ρ (cf., (33), (34)):

$$\rho/m = km^2/2\pi. \quad (36)$$

If we substitute here $m = m_p$ (m_p is the proton mass) we obtain $1/\rho = 0.8 \times 10^{27}$ cm, which approximately coincides with the radius of curvature of the universe. This means that the general procedure for self-consistency in a Riemann space with real cosmology gives the correct value for the mass of an elementary particle (the gravitational constant fixes a quantity of the dimensions of length). The relation (36) between the radius of the universe, the gravitational constant and the proton mass was first discussed in[27]. The use of the graviton Green's function introduced above in principle enables us to discuss the Bethe-Salpeter equation in a generally covariant form and to obtain the mass m as a functional of the cosmological metric. In particular, m must vary with time. Physically this implies renormalization in a variable external gravitational field. Such an approach is essentially different from a theory with a variable mass based on the introduction of a long range scalar field[28].

Relation (36) was recently discussed from another point of view in connection with the introduction into the theory of the cosmological constant Λ ($\rho^2 \approx \Lambda$)[29,30]. In this case relation (36) does not imply variability of the mass m, but is simply a definition of Λ. The coincidence between $1/\sqrt{\Lambda}$ and the radius of the universe is not accidental as a result of long delay in the expansion of the universe at this stage of cosmological evolution[30].

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