THEORY OF TUNNELING IN ANISOTROPIC SUPERCONDUCTORS

A. E. GORBONOSOV and I. O. KULIK

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

Submitted December 19, 1967; resubmitted April 23, 1968

Zh. Eksp. Teor. Fiz. 55, 876–888 (September, 1968)

Some features of the current-voltage curves of the tunnel current in anisotropic superconductors are investigated. Two groups of characteristic features can be discerned: 1) those connected with the extremal values of the gap on the Fermi surface (minima, maxima, and saddle points); 2) those related to the extremal values of Δ (minimum or maximum) on the lines of the Fermi surface $\mathbf{v}\cdot\boldsymbol{\nu} = 0$ where $\boldsymbol{\nu}$ is the direction of the normal to the junction surface. Singularities of the first kind occur in single crystals and polycrystalline substances (with crystallite dimensions $\gg \xi_0$). Singularities of the second kind ("boundary singularities") are manifest only in single crystals; in this case the singularity amplitude may be appreciable for diffuse scattering of the electrons by the tunnel contact surface or if the barriers are very thin. The analysis is valid for Fermi surfaces of arbitrary shape and for arbitrary anisotropy of the gap.

INTRODUCTION

THE study of the detailed singularities of the electronic energy spectrum of metals is one of the central problems of modern solid-state physics. It has been established that the electron spectrum in metals (in a normal state) is very complicated—there exist several branches of the dispersion law $\varepsilon_r(\mathbf{p})$ (energy bands or groups), in each of which the dependence of the energy on the quasimomentum $\mathbf{p}$ is essentially nonquadratic and nonisotropic. On going over to superconductivity, this should lead to the occurrence of a number of branches $\Delta_r(\mathbf{n})$ in the dependence of the energy gap Δ on the direction of the quasimomentum of the electron on the Fermi surface $\mathbf{n} = \mathbf{p}/p$. The existence of appreciable anisotropy of the gap in superconductors was convincingly demonstrated in experiments on the investigation of the absorption of ultrasound,[1–4] surface resistance,[5] the tunnel effect,[6–8] etc. The theory of anisotropic superconductors was constructed in the papers of Pokrovskiĭ.[9,10] Sometimes a distinction is made between the effects of anisotropy (the dependence of Δ on $\mathbf{n}$) and the effects of the presence of many bands (the existence of several branches of the function Δ), but it should be noted that actually these effects can be considered in a unified scheme. In particular, the universal inequalities established by Pokrovskiĭ for certain thermodynamic characteristics of anisotropic superconductors would remain unchanged in the multi-band case. As to the kinetic characteristics, the multi-band effects can lead here to a number of singularities.[11,12]

Although the magnitude of the gap anisotropy in certain superconductors is quite large ($\delta\Delta/\Delta \sim 40\%$), nonetheless the effects determined by the average values of Δ turned out to be little sensitive to changes of the gap. Thus, for example, addition of impurities to the superconductor, leading to "isotropization" of the gap (at $l \ll \xi_0$),[13] causes usually a very small change of the thermodynamic characteristics (in particular, of the critical temperature of the superconducting transition). At the same time, the effects in which selection of the values of the gap on definite lines or points of the Fermi surface is effected (for example the absorption of ultrasound or the tunnel effect), turn out to be much more sensitive to a change of Δ and can therefore be used for an experimental study of the anisotropy of the gap. The tunnel effect[14,15] is from this point of view the most effective, since it makes it possible, in principle, to study simultaneously both the "normal" tunnel current and the Josephson current[16,17] in the same sample, makes it possible to study the change of the anisotropy of the gap on addition of impurities, etc. The first observation of the anisotropy of Δ with the aid of the tunnel effect was made by Zavaritskiĭ.[6] Recently, a number of both direct and indirect methods of studying the anisotropy of the gap by determining the singularities of the current-voltage characteristics of the tunnel current[7,8,18–21] and others, have been proposed and perfected.

The present paper is devoted to a systematic investigation of the influence of gap-anisotropy effects on the single-particle tunnel current in superconductors and to the assessment of the information that this can yield concerning the spectrum of the superconducting state.[1)] Unlike earlier papers on similar topics,[22,23] we start here with the most general premises concerning the electron-dispersion law in the normal state $\varepsilon_r(\mathbf{p})$ (i.e., the form of the Fermi surface) and the character of the anisotropy of the gap in the superconducting state $\Delta_r(\mathbf{n})$. The quantities $\varepsilon_r(\mathbf{p})$ and $\Delta_r(\mathbf{n})$ are assumed to be arbitrary functions satisfying the crystal symmetry elements and the condition that follows from the invariance of the Hamiltonian against time reversal.

Our analysis differs from that of [22,23] also in the following respect. In the calculation of the thermal current, it is customary to start from the so-called thermal Hamiltonian:[24,15]

$$H_T = \sum_{\mathbf{pq}rr'} T_{\mathbf{pq}}^{rr'} a_{\mathbf{p}r}^{+} b_{\mathbf{q}r'} + \text{h.c.}, \tag{1.1}$$

where $T_{\mathbf{pq}}^{rr'}$—matrix elements of the tunneling of the

[1)]The same problem was investigated by us in an earlier paper [17] for the case of the Josephson effect.

electron from the state $\mathbf{p}$ (in band r) to the left of the barrier into the state $\mathbf{q}$ (in band r′) to the right of the barrier. From a microscopic calculation of this quantity (see, for example, [25]) it is clear that the tunneling can occur only when the electron velocity component in the direction normal to the barrier is positive: $u_z = \partial\varepsilon_r/\partial p_z > 0$, $v_z = \partial\varepsilon_{r'}/\partial q_z > 0$ (or $u_z < 0$, $v_z < 0$ for the inverse transition). This means that in the calculation of the tunnel current on the basis of (1.1) it is necessary to restrict the summation over the momenta to those values of $\mathbf{p}$ and $\mathbf{q}$ for which u_z, $v_z < 0$. At the same time, in the tunnel-Hamiltonian method the summation is usually carried out over all values of $\mathbf{p}$ and $\mathbf{q}$. Such a model can apparently describe adequately the tunneling between polycrystalline samples, when all possible orientations of $\mathbf{p}$ and $\mathbf{q}$ relative to the crystallographic axes, for which an electron tunnel transition is possible, exist as a result of the random orientation of the crystallites. Therefore singularities of the tunnel current arise in singular points of the gap—minima, maxima, and saddle points of the function $\Delta(\mathbf{n})$.

We consider in the present paper both this case and the case of tunneling from a single crystal, and in the latter case additional singularities arise on the tunnel-current curve, which we shall call limiting, and which are connected with the extremal values of Δ on the strip $\mathbf{v}\cdot\boldsymbol{\nu} = 0$ of the Fermi surface ($\boldsymbol{\nu}$—direction normal to the junction surface). We note that similar singularities appear also in experiments on the absorption of ultrasound,[26] thus uncovering a possibility of direct comparison of data on the gap anisotropy, obtained with the aid of the tunnel effect, with the data of ultrasound absorption. It should be noted that in the case of specular transmission of the electrons through the barrier, the amplitude of the "limiting" singularities will be quite small, since in this case the tunneling probability decreases exponentially with decreasing v_z (see, for example, [27]). However, in the more realistic case of diffuse passage, such a strong dependence of the probability of tunneling on the electron-velocity direction may also not occur. Indeed, in the latter case we can assume that the surface of the tunnel contact is not ideally plane, but contains sections that are oriented at all possible angles to the averaged (smoothed) junction surface. For any direction of the velocity v of an electron moving towards the barrier, there are thus sections which are oriented practically perpendicular to v, for which the tunneling probability is not small. In addition, a noticeable amplitude of the limiting singularities can be expected also in the specular case for very thin barriers (for example, such as used in the Josephson effect), for in this case the dependence of the tunneling probability on the direction of the electron velocity becomes not so significant.

2. TUNNEL CURRENT OF A NORMAL METAL INTO A SUPERCONDUCTOR

As shown in a number of papers,[24, 28, 29] the use of a tunnel Hamiltonian (1.1) leads in the isotropy case to the following expression for the tunnel current between the two metals:

$$J(V) = C\int_{-\infty}^{\infty} d\varepsilon\, \nu_1(\varepsilon)\nu_2(\varepsilon+V)[F(\varepsilon)-F(\varepsilon+V)], \tag{2.1}$$

where V—bias applied to the barrier (e = 1), $F(\varepsilon) = [1 + \exp((\varepsilon-\mu)/T]^{-1}$—Fermi distribution function; $\nu_1(\varepsilon)$ and $\nu_2(\varepsilon)$—relative densities of the states in the left (1) and right (2) metals normalized in such a way that in the normal state $\nu_1^n = \nu_2^n = 1$; C—conductivity of the tunnel contact at temperatures higher than T_{c1} and T_{c2}, when Ohm's law $J_{NN} = CV$ holds.

As shown in [17], in the anisotropy case Eq. (2.1) is rewritten in the following form (we use simultaneously the explicit expression for the density of states $\nu(\varepsilon)$):

$$J(V) = \frac{C}{2\pi^2}\Big\langle \lambda(\mathbf{n},\mathbf{n}')\int_{-\infty}^{\infty} d\omega\Big(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega-V}{2T}\Big) \times \operatorname{Im} G_1^R(\omega,\mathbf{n}) \operatorname{Im} G_2^R(\omega-V,\mathbf{n}')\Big\rangle_{\mathbf{nn}'}, \tag{2.2}$$

where $G^R_{1,2}$ are the retarded Green's functions of the electron in the left and right metals, integrated over ξ (ξ—electron energy in the normal metal, reckoned from the chemical potential μ; as always, Δ can be assumed independent of ξ), $\lambda(\mathbf{n},\mathbf{n}')$—non-negative function describing the anisotropy of the matrix elements of the tunneling, $\langle\lambda\rangle = 1$. The angle brackets in (2.2) denote averaging over the Fermi surface ($\langle 1\rangle = 1$). Using the explicit expression for $G^R(\omega,\mathbf{n})$ (see [30]), we can rewrite (2.2) in the form

$$J(V) = \frac{C}{2}\Big\langle \lambda(\mathbf{n},\mathbf{n}')\int_{-\infty}^{\infty} d\omega\Big(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega-V}{2T}\Big) \times \frac{|\omega|\,|\omega-V|\,\Theta(|\omega|-\Delta_1(\mathbf{n}))\,\Theta(|\omega-V|-\Delta_2(\mathbf{n}'))}{[(\omega^2-\Delta_1^2(\mathbf{n}))((\omega-V)^2-\Delta_2^2(\mathbf{n}'))]^{1/2}}\Big\rangle_{\mathbf{nn}'}. \tag{2.3}$$

Here $\Theta(x)$ is a function equal to unity when $x > 0$ and to zero when $x < 0$. Formula (2.3) pertains both to the case of a contact of two superconductors and to the case of a contact between a normal metal and a superconductor. In this section we analyze the singularities of the function J(V) for the case of a contact between a superconductor and a normal metal ($J = J_{NS}$). The case of two superconductors will be considered in the next section.

1. We consider first the case of zero temperature: T = 0. Then, putting $\Delta_2(\mathbf{n}') = 0$ in (2.3), we get

$$J_{NS}(V) = C\Big\langle \lambda(\mathbf{n})\int_{\Delta(\mathbf{n})}^{V} d\omega\frac{\omega}{\sqrt{\omega^2-\Delta^2(\mathbf{n})}}\Theta(V-\Delta(\mathbf{n}))\Big\rangle_{\mathbf{n}} = C\langle\lambda(\mathbf{n})\sqrt{V^2-\Delta^2(\mathbf{n})}\rangle_{\mathbf{n}V}, \tag{2.4}$$

where $\lambda(\mathbf{n}) = \langle\lambda(\mathbf{n},\mathbf{n}')\rangle_{\mathbf{n}'}$. In the last expression, the integration is carried out over that part of the Fermi surface, on which (at a given V) $\Delta(\mathbf{n}) < V$. From this it is clear, in particular, that $J_{NS} = 0$ when $V < \Delta_m$, where Δ_m—minimum value of the gap. The nonzero tunnel current appears in this case for the first time only when $V > \Delta_m$. The asymptotic form of the current as $V \to \Delta_m$ is of the form (T = 0):

$$J_{NS}(V) \approx \lambda_m C\sqrt{2\Delta_m}(V-\Delta_m)^{1/2}S(V). \tag{2.5}$$

Here λ_m—amplitude of the tunneling probability for the direction of the quasimomentum of the electron, at which the absolute minimum of the gap takes place, and S(V) is the relative part of the Fermi surface on which $\Delta(\mathbf{n}) < V$:

$$S(V) = [(2\pi)^3 N(0)]^{-1}\int_{(\Delta(\mathbf{n})<V)}\frac{dS}{v_F}$$

(N(0)—density of states on the Fermi surface). At the point $V = \Delta_m$, the function S(V) vanishes, so that the asymptotic form of the tunnel current $J_{NS}(V)$ near the threshold (at T = 0) takes the form (cf. [22]):

$$J_{NS}(V) \approx A(V-\Delta_m)^{3/2}, \quad A = \frac{2\pi\lambda_m C\Delta_m^{1/2}}{\Gamma_m |d^2\Delta/d\theta^2|_m}. \tag{2.6}$$

Here Γ_m is the Gaussian curvature of the Fermi surface at the minimum point (we assume for simplicity that the gap near the minimum is axially symmetrical: $\Delta = \Delta(\theta)$). The asymptotic form (2.6) differs from the corresponding formula of the isotropic model, in which the function $J_{NS}(V)$ near threshold has a vertical tangent: $J_{NS}^{isotr}(V) = C\sqrt{V^2-\Delta^2}$.

2. At nonzero temperatures, the tunnel current differs from zero at all V. It is easy to obtain the asymptotic form at temperatures $T \ll \Delta$ below threshold ($V \ll \Delta_m$). It is of the form

$$J_{NS}(V) \approx \begin{cases} C\gamma(2\pi T\Delta_m)^{1/2} e^{-\Delta_m/T} e^{V/T}, & T \ll V, \\ C\gamma(2\pi\Delta_m/T)^{1/2} V e^{-\Delta_m/T}, & V \ll T, \end{cases} \tag{2.7}$$

where γ is a certain numerical coefficient on the order of unity.[2)] The last formula is valid at not too weak an anisotropy of the gap: $\delta\Delta \gg Z$, where $\delta\Delta$—anisotropic part of the gap. From (2.7) we see that in the case of NS contacts the current-voltage characteristic is ohmic as $V \to 0$, and the conductivity decreases then with decreasing temperature like $\exp(-\Delta_m/T)$, where Δ_m is the absolute minimum of the gap on the Fermi surface.

3. Finally, let us consider the case of large displacements, $V \gg \Delta(T)$, when asymptotic expressions valid under the most general assumptions concerning the character of the gap anisotropy can also be obtained for the current. Expanding the expression for the tunnel current, which follows from (2.3), namely

$$J_{NS}(V) = \frac{C}{2}\left\langle \lambda(\mathbf{n}) \int_{-\infty}^{\infty} d\omega \left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega-V}{2T}\right) \frac{|\omega|\Theta(|\omega|-\Delta(\mathbf{n}))}{\sqrt{\omega^2-\Delta^2(\mathbf{n})}} \right\rangle_{\mathbf{n}} \tag{2.8}$$

in powers of Δ/V, we readily obtain

$$J_{NS} = CV - J_1(V), \tag{2.9}$$

$$J_1(V) = (C/2V)\langle\lambda(\mathbf{n})\Delta^2(\mathbf{n})\rangle_{\mathbf{n}} f(V/2T), \tag{2.10}$$

where f(x) is given by

$$f(x) = -\frac{x}{2}\int_{-\infty}^{\infty} \frac{dy}{y\,\operatorname{ch}^2(x+y)}. \tag{2.11}$$

At small and large values of x, the function f(x) can be represented by the asymptotic formulas:

$$x \gg 1, \quad f(x) \approx 1 + \pi^2/12x^2$$

$$x \ll 1, \quad f(x) \approx Ax^2, \quad A = \int_0^{\infty} dy(\operatorname{th} y/y)^2 = 1.68. \tag{2.12}$$

As seen from (2.10), the addition to the current $J_1(V)$, due to the occurrence of superconductivity in one of the metals of the tunnel pair, is proportional to $\langle\lambda\Delta^2\rangle$, and since the mean value of λ is unity, it yields information concerning the magnitude of the gap (in the isotropic model $\langle\lambda\Delta^2\rangle = \Delta^2$). The obtained formula can be used to determine the gap and, in particular, its temperature dependence with the aid of the tunnel effect in the case when one of the metals of the tunnel junction cannot be made superconducting for some reason. In this case, if the temperature is not too low, the threshold of the tunnel current at $V \to \Delta_m$ is not very pronounced, making it impossible to determine Δ reliably; at the same time, formula (2.10) makes it possible to determine Δ directly from the change of the current upon destruction of the superconductivity (for example, by a magnetic field).

3. TUNNEL CURRENT OF A SUPERCONDUCTOR–SUPERCONDUCTOR CONTACT

A study of the features of the current-voltage curves of the tunnel contact between two superconductors makes it possible to obtain much more complete information on the character of the gap anisotropy than in the case of a contact between a superconductor and a normal metal. An analysis of these singularities is based on Eqs. (2.2) and (2.3) of the preceding section. The derivation is similar to that presented, and will be omitted in most cases; only the final results will be given.

1. As is well known,[15] in the isotropic model the tunnel current experiences a jump at a barrier voltage $V = \Delta_1(T) + \Delta_2(T)$ (we emphasize that, in accordance with the theory, such a jump takes place not only at T = 0, but also at finite temperatures). Allowance for the anisotropy of the gap leads to a smoothing of the jump, but in the interval values of voltage between $[\Delta_{1m}(T) + \Delta_{2m}(T)]$ and $[\Delta_{1M}(T) + \Delta_{2M}(T)]$ there will take place a number of singularities of the tunnel-current curve J(V) (or its derivatives dJ/dV, d^2J/dV^2, etc.), connected with the singular points of the gap—minima (m), maxima (M), or saddle points (s). The first to call attention to the existence of such singularities was Bennett,[22] but he considered only the case of zero temperature and assumed that one of the superconductors making up the tunnel contact is isotropic (in addition, the Fermi surface was assumed in [22] to be spherical). Indeed, such singularities take place also at finite temperatures. In the case when both superconductors are anisotropic, the number of singularities increases.

In Table I we summarize the analytic behavior of the first derivative of the current-voltage characteristic of the tunnel current dJ_{SS}/dV as a function of the voltage V near the singular point. In the first column of Table I are given the asymptotic forms of the corresponding functions to the left of the singular point, and in the second column—to the right of the singular point. Figure 1 shows a plot (not to scale) of the corresponding singularities in the first derivatives of the tunnel current with respect to the bias. We note that our derivation pertains to the case of a nonzero temperature and is valid for an arbitrary (smooth) Fermi surface.

In Table I we introduced the symbol

$$\varepsilon_{ik} = \frac{1}{2}\sqrt{\Delta_i\Delta_k}\left(\operatorname{th}\frac{\Delta_i}{2T} + \operatorname{th}\frac{\Delta_k}{2T}\right). \tag{3.1}$$

[2)] In the model in which the angular dependence of the gap is given by the formula $\Delta = \Delta_0 + \delta\Delta\cos\theta$ [22,17], the coefficient γ equals $\pi/4$. It must also be taken into account, however, that the minimum gap Δ_m which enters in (2.7) is itself a function of the temperature, which leads to a certain renormalization of γ (cf. [17]) without a change of the character of the asymptotic expressions (2.7).

Table I. Singularities of the conductivity dJ_{SS}/dV of a tunnel junction made up of anisotropic superconductors.

	dJ_{SS}/dV	
	$V<V_c$	$V>V_c$
$\Delta_{1m}+\Delta_{2m}$	const	const $+ A\varepsilon_{mm}y_{mm}$
$\Delta_{1s}+\Delta_{2s}$	const $- B\varepsilon_{ss}\sqrt{-y_{ss}}$	const $-\frac{4}{\pi}B\varepsilon_{ss}\sqrt{y_{ss}}$
$\Delta_{1M}+\Delta_{2M}$	const	const $- C\varepsilon_{MM}\sqrt{y_{MM}}$
$\Delta_m+\Delta_M$	const	const $+ D\varepsilon_{mM}y_{mM}$
$\Delta_m+\Delta_s$	const $+ E\varepsilon_{ms}y_{ms}\ln\|y_{ms}\|$	
$\Delta_M+\Delta_s$	const $+ F\varepsilon_{Ms}y_{Ms}\ln\|y_{Ms}\|$	

Note. y_{ik} stands for $V_i(\Delta_i+\Delta_k)-1$.

The indices i and k pertain respectively to the first and second metals and characterize the type of the singular point (m, M, s). The quantity ε determines the "amplitude" of the singularity (for example, the magnitude of the jump of the corresponding function or its derivative). It is seen from (3.1) that when $T = 0$ this amplitude will be maximal. With increasing temperature, the singularities "smooth out" but do not vanish (their analytic character also remains unchanged).

2. Let us see now what modifications in the behavior in the current-voltage characteristic of the tunnel current are brought about by allowance for anisotropy when the bias on the barrier is on the order of $|\Delta_1-\Delta_2|$, at which there was a logarithmic current singularity in the isotropy case (if $T\neq 0$).[15] For simplicity we assume that one of the superconductors is isotropic (the case when both superconductors are anisotropic, in the model $\Delta_{1,2}=\Delta^0_{1,2}+\delta\Delta_{1,2}\cos\theta$, is considered in the Appendix). The character of the singularity has in the isotropic case the form:[15]

$$J_{SS}^{\text{isotr}}(V)=-{}^1/_4C\gamma_{12}\ln|x_{12}|. \tag{3.2}$$

Here

$$\gamma_{12}=\sqrt{\Delta_1\Delta_2}\left(\operatorname{th}\frac{\Delta_1}{2T}-\operatorname{th}\frac{\Delta_2}{2T}\right)\quad x_{12}=\frac{V}{|\Delta_1-\Delta_2|}-1.$$

as can be shown on the basis of (3.2), in the anisotropic case we get in lieu of formula (3.2)

$$J_{SS}(V)=-{}^1/_4C\gamma_{12}\eta(V), \tag{3.3}$$

where the function $\eta(V)$ is given by

$$\eta(V)=[(2\pi)^3N(0)]^{-1}\int\frac{dS}{v_F}\ln\left|\frac{V}{|\Delta_1-\Delta_2(\mathbf{n})|}-1\right| \tag{3.4}$$

(for simplicity we assume that the tunneling probability does not depend on **n**).

The integral over the solid angle in (3.4) can be readily reduced to an integral with respect to Δ_2 by introducing a function $\varphi(\omega)$ analogous to the "gap-anisotropy function" introduced by Bennett:

$$\varphi(\omega)=[(2\pi)^3N(0)]^{-1}\int d\theta\,d\varphi\frac{\sin\theta}{v_F\Gamma(\theta,\varphi)}\delta[\omega-\Delta(\theta,\varphi)] \tag{3.5}$$

($\Gamma(\theta,\varphi)$—Gaussian curvature of the Fermi surface at the point with angle coordinates θ, φ). With the aid of (3.5) we get

$$\eta(V)=\int\limits_{\Delta_{2m}}^{|V-\Delta_1|}d\omega\varphi(\omega)\ln\left|\frac{V}{|\Delta_1-\omega|}-1\right|. \tag{3.6}$$

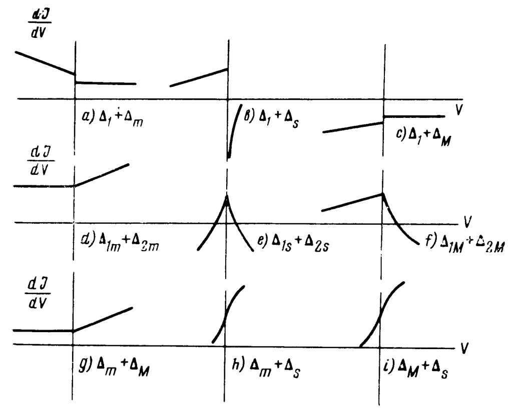

FIG. 1. a–c) Singularities of the conductivity of the tunnel contact dj/dv, in which one of the superconductors is isotropic (see [22]); d–i) singularities of the conductivity of the tunnel contact, in which both superconductors are anisotropic (the curves g–i are symmetrical with respect to both superconductors).

Calculating (3.5) asymptotically near the singular points of the gap Δ_{2c} (Δ_1 = const) and substituting then (3.5) in (3.6), we obtain the form of the current-voltage curves J(V) near the singular points:

$$\begin{aligned} &A+B\gamma_Mx_M\ln|x_M|, && V\to|\Delta_1-\Delta_M|,\\ &D-F\gamma_m\ln|x_m|, && V\to|\Delta_1-\Delta_m|,\\ &C\gamma_s\ln|x_s|, && V\to|\Delta_1-\Delta_s|,\end{aligned}$$

Here

$$\gamma_i=\sqrt{\Delta_1\Delta_i}\left(\operatorname{th}\frac{\Delta_1}{2T}-\operatorname{th}\frac{\Delta_i}{2T}\right),$$
$$x_i=\frac{V}{|\Delta_1-\Delta_i|}-1,$$
$$(i=m,s,M).$$

As seen from Fig. 2, in the anisotropy case the singularity "splits" near $V\sim|\Delta_1-\Delta_2|$, and on the basis of the position of the singular points of the derivative dJ_{SS}/dV it is possible to assess the magnitudes of Δ_m, Δ_s, and Δ_M. We note that these singularities are missing when $T = 0$. Their amplitude decreases with increasing temperature like $\exp(-\Delta_{2m}/T)$ when $T\ll\Delta$ and $V<(\Delta_1+\Delta_{2m})$.

3. We present, in analogy with Item 2 of Sec. 2, the asymptotic forms of the tunnel current at low temperatures far from singularities. It can be readily shown on the basis of an analysis of (2.3) that their form is ($\Delta_1=\Delta_2$)

$$J_{NS}(V)\approx\begin{cases}C\gamma'\Delta_m(\pi T/V)^{1/2}\exp(-\Delta_m/T), & T\ll V\\ C\gamma'\Delta_m(V/T)\ln(4T/V)\exp(-\Delta_m/T), & V\ll T\end{cases} \tag{3.7}$$

(Δ_m—absolute minimum of the gap, γ'—a certain constant on the order of unity). It is seen from formula (3.7) that, unlike the case of NS contacts (formula 3.7),

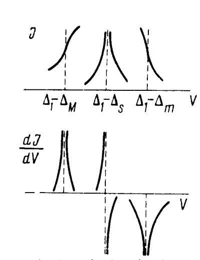

FIG. 2. Form of the J(V) and dj/dv curve in the vicinity of the singular points $V\sim|\Delta_1-\Delta_{2c}|$ for a contact in which Δ_1 = const and $\Delta_2=\Delta_2(\mathbf{n})$ (the central part of the curve on the upper figure is shifted downward along the ordinate axis for convenience).

even in the case of small barrier voltages the current-voltage characteristic of the tunnel current is not ohmic and has at $V \to 0$ the asymptotic form $V \ln V$ (however, since $\ln V$ is a very slowly growing function, the deviation from ohmic behavior can be verified experimentally only by investigating the derivatives of the current with respect to the voltage dJ/dV, d^2J/dV^2, etc.).

4. Finally, as was done for the NS junction, we can obtain a common asymptotic form (valid for an arbitrary form of the Fermi surface and gap anisotropy) at large biases $V \gg \Delta_{1,2}$ and arbitrary temperatures. The asymptotic expression for the addition to the current (see (2.9)) has in this case the form

$$J_1'(V) = (C/2V)\langle\lambda(\Delta_1^2+\Delta_2^2)\rangle f(V/2T), \qquad (3.8)$$

where the function $f(x)$ has the same meaning as in (2.11). This increment is thus additive with respect to the two superconductors. Since the function $f(x)$ can be readily tabulated, formula (3.8) makes it possible to study the temperature dependence of the gap.

4. BOUNDARY SINGULARITIES OF THE TUNNEL CURRENT

In the present section we analyze a class of singularities of the current-voltage curve of the tunnel current $J(V)$, which are connected with the extremal values Δ'_m (minimum) and Δ'_M (maximum) on a Fermi-surface strip $v_z = 0$ (here $\Delta'_m \geq \Delta_m$ and $\Delta'_M \leq \Delta_M$). The general classification of the singularities of the tunnel current $J(V)$ can be established on the basis of an analysis of the gap-anisotropy function $\varphi(\omega)$ (formula (3.5)). Taking into account the fact that v_z is positive, we should write

$$\varphi(\omega) = \frac{2}{(2\pi)^3 N(0)} \int_0^{2\pi} d\varphi \int_0^{\theta(\varphi)} d\theta \frac{\sin\theta}{v_F\Gamma(\theta,\varphi)} \delta[\omega-\Delta(\theta,\varphi)], \qquad (4.1)$$

where $\theta(\varphi)$—curve on a sphere, which is a stereographic projection of the boundary line of integration on the true Fermi surface (see Fig. 3, where the dashed lines show one of such lines for a non-convex Fermi surface and its stereographic projection). The singularities of the function $\varphi(\omega)$ occur at the singular points inside the integration region and on its boundary $\theta = \theta(\varphi)$. The first class of singularities was considered in Secs. 2 and 3. By virtue of the central symmetry of the Fermi surface, the character of these "volume" singularities in the tunnel current does not depend on whether the integration in (4.1) is carried out over the entire Fermi surface or over its half $v_z > 0$, i.e., it does not depend on whether we are dealing with a polycrystal[22, 18] or a single crystal. The second class of singularities is connected with the behavior of $\Delta(\theta, \varphi)$ on the boundary line $\theta = \theta(\varphi)$. In accordance with the foregoing, the total current can be broken up into two parts, $J = J_1 + J_2$, where J_1 was calculated above, and J_2 can be represented with the aid of (2.3) in the form (for simplicity we consider the case when one of the superconductors is isotropic)

$$J_2(V) = \int_0^{2\pi} d\varphi F[\theta(\varphi),\varphi]. \qquad (4.2)$$

Here

$$F[\theta(\varphi),\varphi] = \frac{2}{(2\pi)^3 N(0)} \int_{\theta_0}^{\theta(\varphi)} d\theta \frac{D(\theta)\sin\theta}{v_F\Gamma(\theta,\varphi)} f(\theta,\varphi), \qquad (4.3)$$

with

$$f(\theta,\varphi) = \int_{-\infty}^{\infty} d\omega \left(\operatorname{th}\frac{\omega}{2T} - \operatorname{th}\frac{\omega-V}{2T}\right) \times \frac{|\omega|\,|\omega-V|\,\Theta(|\omega|-\Delta_1)\,\Theta(|\omega-V|-\Delta_2(\theta,\varphi))}{\sqrt{(\omega-V)^2-\Delta_2^2(\theta,\varphi)}\,\sqrt{\omega^2-\Delta_1^2}}, \qquad (4.4)$$

and θ_0 is a certain angle chosen such that the region of integration in (4.3) does not contain "volume" singularities of the function $\Delta_2(\theta, \varphi)$.

As shown above, the amplitude of the boundary singularities is quite sensitive to the character of the passage of electrons through the barrier. In the case of specular passage, we can use for the estimate the angular dependence of Δ, given by the formula[31]

$$D(\theta) \sim \exp\left[-\frac{2d}{\hbar}\sqrt{2m(W-\mu\cos^2\theta)}\right] \qquad (4.5)$$

(W—height of potential barrier, μ—chemical potential). According to this formula, D remains finite at $\theta = \pi/2$, and the order of magnitude of the ratio

$$D(\pi/2)/D(0) \sim \exp\left[-2d\hbar^{-1}\left(\sqrt{2mW}-\sqrt{2m(W-\mu)}\right)\right]$$

is quite small and can be noticeable only at very thin barriers ($d = 10$ Å). In the diffuse case, the quantity D in (4.3) represents the average value of the transparency coefficient over all possible orientations of the elementary areas characterizing the micro-roughnesses of the junction surface. In this case we obtain $D(\theta) \sim D(0)\cos\theta$. The amplitude of the boundary singularities turns out to be not small, but since D vanishes on the boundary line $\mathbf{v}\cdot\boldsymbol{\nu} = 0$, the order of magnitude of the singularities is decreased by unity compared with the case of specular passage. Omitting trivial derivations, we present only the final summary of the singularities of $J_2(Z)$ in the case of NS and SS contacts (Table II). Plots of these singularities are shown schematically in Fig. 4.

Summarizing, it must be stated that a study of the tunnel effect in superconductors, particularly in the case when one of the metals is a single crystal, makes

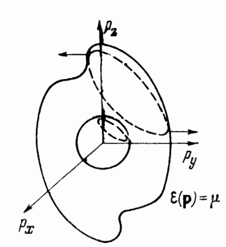

FIG. 3.

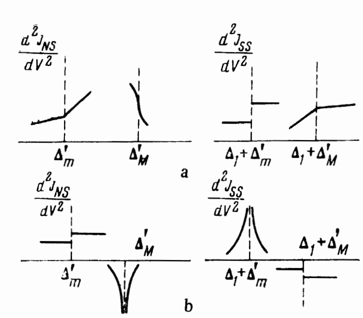

FIG. 4.

FIG. 3. Stereographic projection of the boundary line $v_z = 0$.

FIG. 4. Form of the second derivative of the current d^2J_{SS}/dV^2 near the extremal points Δ'_m and Δ'_M on the strip $v_z = 0$ for NS and SS contacts: a—diffuse case, b—specular case.

Table II. Second derivative d^2J/dV^2 for NS and SS contacts near the extremal points of the gap Δ'_m and Δ'_M on the curve $v_Z = 0$.

	d^2J_{NS}/dV^2		d^2J_{SS}/dV^2	
	$V < V_c$	$V > V_c$	$V < V_c$	$V > V_c$
	Diffuse passage			
Δ'_m	const	const + AZ_m	const	const + $C\delta_{1m}\gamma(V)$
Δ'_M	const + $BZ_M \ln\|Z_M\|$		const + $C_1\tilde{Z}_M$	const + $C_2\tilde{Z}_M$
	Specular passage			
Δ'_m	const	const + A	$A\delta_{1m}\gamma(V)\,\delta(\tilde{Z}_m)$	
Δ'_M	const + $B \ln\|Z_M\|$		const	const − B

Note. $\gamma(V)$—smooth function of the order of unity, $\gamma(\Delta_{1m} + \Delta_{2m}) = 1$. With increasing anisotropy ($\delta\Delta \to 0$) we have

$$(\delta\Delta \to 0), \quad \gamma(V) \to 1; \quad Z_i = V/\Delta'_i - 1,$$

$$\tilde{Z}_i = V/(\Delta'_i + \Delta'_k) - 1, \quad (i, k = m, M), \quad C_1 > C_2.$$

it possible to obtain appreciable information on the local and average values of $\Delta(\mathbf{n})$. Some of the singularities listed above and shown in Figs. 1, 2, and 4 were identified experimentally (see [18]), others are still to be revealed. Experimental study of the "boundary" singularities is of interest not only from the point of view of reconstructing the gap, but also for the investigation of the tunneling mechanism itself, since the character and amplitudes of these singularities should depend strongly on the law governing the interaction of the electrons with the surface of the tunnel junction.

In conclusion the authors thank M. I. Kaganov and I. K. Yanson for a discussion of the results.

APPENDIX

Let us show with a simple model of the angular dependence of the gap (see [15, 22]), namely $\Delta_{1,2} = \Delta^0_{1,2} + \delta\Delta_{1,2}\cos\theta$, that allowance for the anisotropy of both superconductors causes the singularities to appear only in the second derivative d^2J/dV^2, whereas the current $J(V)$ and the conductivity dJ/dV remain continuous. For this case, from the general expression (2.3) for the current $J(V)$ we easily obtain

$$\frac{d^2J}{dV^2} = \frac{C}{4\delta\Delta_1\delta\Delta_2}\gamma_{12}\ln\left|\frac{x_{1,2M}\,x_{1M,2}}{x_{1M,2M}\,x_{1,2}}\right|. \tag{A.1}$$

We note that calculation of the current-voltage curve d^2J/dV^2 in the case of an arbitrary anisotropy of the gap $\Delta(\mathbf{n})$ also leads to the conclusion of the appearance of singularities in the higher derivatives of the current with respect to the bias. Expression (A.1) admits of a transition to the limiting case when one of the superconductors is isotropic; $\Delta_1 = \text{const}$, $\Delta_2(\theta) = \delta\Delta^0_2 + \delta\Delta_2\cos\theta$. Indeed, letting $\delta\Delta_1 \to 0$, we get

$$\frac{dJ}{dV} = \frac{C}{4\delta\Delta_2}\gamma_{12}\ln\left|\frac{x_{1,2M}}{x_{1,2}}\right|, \tag{A.2}$$

$$d^2J/dV^2 = -{}^1/_4\, C\gamma_{12}(x_{1,2}\,x_{1,2M})^{-1}, \tag{A.3}$$

which coincides with the results obtained in Section 3.

[1] P. A. Bezuglyĭ, A. A. Galkin, and A. P. Korolyuk, Zh. Eksp. Teor. Fiz. 36, 1951 (1959) and 39, 7 (1960) [Sov. Phys.-JETP 9, 1388 (1959) and 12, 4 (1961)].

[2] R. W. Morse, T. Olsen, and J. D. Gavenda, Phys. Rev. Lett. 3, 15 (1959).

[3] R. W. Morse, B co. Progress in Cryogenics, 1, Heywood Comp. Ltd. (1959), p. 219.

[4] A. G. Shepelev and G. D. Filimonov, Zh. Eksp. Teor. Fiz. 48, 1033 (1965) [Sov. Phys.-JETP 21, 689 (1965)].

[5] M. A. Biondi, M. P. Garfunkel, and W. A. Thompson, Phys. Rev. 136, A1471 (1964).

[6] N. V. Zavaritskiĭ, Zh. Eksp. Teor. Fiz. 43, 1123 (1962) [Sov. Phys.-JETP 16, 793 (1963)].

[7] N. V. Zavaritskiĭ, ibid. 45, 1839 (1963) and 48, 837 (1965) [18, 1260 (1965) and 21, 557 (1965)].

[8] C. K. Campbell and D. G. Walmsley, Canad. J. Phys. 45, 159 (1967).

[9] V. L. Pokrovskiĭ, Zh. Eksp. Teor. Fiz. 40, 641 (1961) [Sov. Phys.-JETP 13, 447 (1961)].

[10] V. L. Pokrovskiĭ and M. S. Ryvkin, ibid. 43, 92 (1962) [16, 67 (1963)].

[11] B. T. Geĭlikman, R. O. Zaĭtsev, and V. Z. Kresin, Fiz. Tverd. Tela 9, 821 (1967) [Sov. Phys.-Solid State 9, 642 (1967)].

[12] I. I. Fal'ko, ibid. 9, 2124 (1967) [9, 1668 (1968)].

[13] P. W. Anderson, J. Phys. Chem. Solids 11, 26 (1959).

[14] J. Giaver, H. R. Hart, and K. Megerle, Phys. Rev. 126, 941 (1962).

[15] D. Douglas and L. Falicov, in: Sverkhprovodimost' (Superconductivity), Nauka, 1967.

[16] I. O. Kulik, Zh. Eksp. Teor. Fiz. 50, 799 (1966) [Sov. Phys.-JETP 23, 529 (1966)].

[17] A. E. Gorbonosov and I. O. Kulik, Fiz. Metal. Metallov. 23, 803 (1967).

[18] G. I. Rochlin, Phys. Rev. 153, 513 (1966).

[19] I. K. Yanson, Zh. Eksp. Teor. Fiz. 53, 1268 (1967) [Sov. Phys.-JETP 26, 742 (1968)]; ZhETF Pis. Red. 6, 729 (1967) [JETP Lett. 6, 206 (1967)].

[20] A. Bennett, Phys. Rev. 139, A746 (1965).

[21] H. J. Levinstein, V. G. Chirba, and J. E. Kunzler, Phys. Rev. Lett. 24A, 362 (1967).

[22] A. Bennett, Phys. Rev. 140, A1902 (1965).

[23] J. R. Clem, Ann. of Phys. 40, 268 (1966).

[24] M. H. Cohen, L. P. Falicov, and J. S. Phillips, Phys. Rev. Lett. 8, 316 (1962).

[25] A. Schmid, Z. Physik 178, 26 (1964).

[26] V. L. Pokrovskiĭ, Zh. Eksp. Teor. Fiz. 40, 898 (1961) [Sov. Phys.-JETP 13, 628 (1961)].

[27] I. O. Kulik and G. A. Gogadze, ibid. 44, 530 (1963) [17, 361 (1963)].

[28] J. R. Schrieffer, D. J. Scalapino, and J. W. Wilkins, PRL 10, 336 (1963).

[29] I. O. Kulik, Izv. Vuzov Fizika 2, 24 (1967).

Translated by J. G. Adashko
101