SOME REMARKS CONCERNING THE INTERFERENCE OF INDEPENDENT LIGHT BEAMS

V. G. BARYSHEVSKIĬ and M. I. PODGORETSKIĬ

Joint Institute for Nuclear Research

Submitted February 3, 1968

Zh. Eksp. Teor. Fiz. 55, 312—316 (July, 1968)

Some phenomena resulting from the interference of independent photon beams are considered. It is found that depending on the delay time the delayed coincidences undergo damped beats. The nature of the damping depends on the processes occurring in the source. The area under the delayed coincidence curve depends on the frequency difference of the interfering beams and on the damping mechanism.

INTERFERENCE phenomena of light beams produced by independently radiating sources have been discussed and observed in a series of papers.[1-7] The existence of these phenomena is connected with the identical nature of the emitted particles.[3,7] Let us consider two excited atoms located at the points $\mathbf{r}_a$ and $\mathbf{r}_b$, having energies E_a and E_b, and two atoms (two elementary counters) located at points $\mathbf{r}_c$ and $\mathbf{r}_d$.[7] One and the same final state of the system (registration of the photons by the counters at given instants t and τ) can by virtue of the identical nature of the incident particles be produced by two possible paths: (a → c, b → d) and (b → c, a → d). These two paths cannot be differentiated by observation. Therefore the probability $P_{ab}(\mathbf{r}_c, t; \mathbf{r}_d, \tau)$ that at the instant t the radiation will interact with the atom located at the point $\mathbf{r}_c$ and at the instant τ with the atom at the point $\mathbf{r}_d$ contains an interference term and can be written in the form

$$P_{ab}(\mathbf{r}_c, t; \mathbf{r}_d, \tau) = A\left|\frac{\exp\{i(k_a r_{ac} - \omega_a t + \delta_a)\}}{r_{ac}}\,\frac{\exp\{i(k_b r_{bd} - \omega_b \tau + \delta_b)\}}{r_{bd}} + \frac{\exp\{i(k_a r_{ad} - \omega_a \tau + \delta_a)\}}{r_{ad}}\,\frac{\exp\{i(k_b r_{bc} - \omega_b t + \delta_b)\}}{r_{bc}}\right|^2, \quad (1)$$

where A is a constant which is immaterial for this discussion.[7] An expression of the type $r_{ac}^{-1}\exp\{i(k_a r_{ac} - \omega_a t + \delta_a)\}$ is a wave function of a photon with a wave number $k_a = \omega_a/c$ emitted at the point $\mathbf{r}_a$ with an initial phase δ_a and absorbed at the point $\mathbf{r}_c$. For simplicity it is assumed that the radiation emitted by the atoms is monochromatic and has the same polarization. We note also that (1) does not in fact depend on the random phases δ_a and δ_b.

If it is assumed that the distances between the atoms in each pair are much smaller than the distance R between these pairs, then (1) goes over into

$$P_{ab}(\mathbf{r}_c, t; \mathbf{r}_d, \tau) \approx \frac{2A}{R^4}\{1 + \cos[k_a(r_{ac} - r_{ad}) + k_b(r_{bd} - r_{bc}) - (\omega_a - \omega_b)(t - \tau)]\}, \quad (2)$$

where $r_{ac} = |\mathbf{r}_a - \mathbf{r}_c|$, etc. The first term in the curly brackets of expression (2) would describe the probability of the simultaneous registration of two photons if they were not identical particles. The second term corresponds to a change of this probability due to their identical nature. It is seen that allowance for the indistinguishability of the particles leads to the circumstance that the probability $P_{ab}(\mathbf{r}_c, t; \mathbf{r}_d, \tau)$ is an oscillating function of the coordinates and undergoes beats in time at the difference frequency of the emitted photons.

Since real sources and detectors contain many pairs of atoms, (2) must be summed over these pairs. The interference term of interest to us does not vanish as a result of such summing if in changing the positions of the atoms within the volumes of the sources S and detectors D the cosine entering in expression (2) does not undergo oscillations. Consequently, the inequality

$$k_a(r_{ac} - r_{ad}) + k_b(r_{bd} - r_{bc}) \ll 1. \quad (3)$$

should be fulfilled for any pair of atoms within the limits of S and D. Condition (3) under which the interference of independent beams does not vanish can also be written in the form[7]

$$\vartheta_S \vartheta_D \ll \lambda / R \quad (4)$$

or

$$l_S l_D \ll \lambda R, \quad (5)$$

where $\vartheta_{S(D)} = l_{S(D)}/R$ is the angle at which the source (detector) with transverse dimensions $l_S(l_D)$ is seen from the detector (source) located at a distance R from S(D); λ is the wavelength of the radiation (it is assumed that $\Delta\lambda/\lambda \ll 1$). If the inequalities (3)—(5) are fulfilled and if $t = \tau$ (registration of coincidences without delay), then the second term in (2) is equal to the first term. Consequently, the probability of counting identical particles in coincidence differs from the result of the classical calculation by no more than a factor of two. Hence it also follows that the possibility of observing interference is determined by the possibility of registering coincidences in the classical situation. If the beam intensity is such that random coincidences of particles without account of their identity do occur in a given experiment, then interference effects will also be observed.

Since the number of coincidences obtained during some time T fluctuates, the intensity of the sources and the time T must be such that the average number of coincidences $\langle N \rangle$ during the indicated time be larger than the magnitude of the fluctuation of the number of coincidences $\delta N = (\langle N^2 \rangle - \langle N \rangle^2)^{1/2} \sim \langle N \rangle^{1/2} \sim (n_1 n_2 \tau_c T)^{1/2}$ [$n_1(n_2)$ is the number of particles registered by counter 1 (2) per unit time, τ_c is the resolving time of the coincidence circuit][1)]. The inequality $\langle N \rangle/\delta N \approx (n_1 n_2 \tau_c T)^{1/2} > 1$ should consequently be fulfilled. If ρ is the surface

1) For definiteness we consider the case of small loads on the coincidence circuit. In the opposite case one must speak instead of the number of coincidences of the correlation function.[3,4]

intensity of the source, η—the efficiency of the counters, then $n_{1,2} \approx \rho l_S^2(l_D^2/R^2)\eta$. When condition (5) is fulfilled, we have $n_{1,2} \lesssim \rho\lambda^2\eta$. Thus, we finally have

$$\langle N\rangle / \delta N \approx \eta\rho\lambda^2\sqrt{\tau_c T} > 1, \quad (6)$$

which coincides with the expression obtained in[8] for the case when the length of the incident wave train is comparable with the time τ_c.

Expressions (1) and (2) were obtained under the assumption that the atoms of the source are fixed and do not undergo any perturbing interactions. Generally speaking, this is not the case under real conditions. Thermal motion and the collision of atoms in the source will lead to frequency modulation of the emitted photons. In this general case the wave function of the photon can be represented in the following form:

$$\psi_a(t) \sim \exp\{-i[\omega_a t + \Omega_a(t)]\}, \quad (7)$$

where $\Omega_a(t)$ is the change of the photon phase as a result of the thermal motion and the collisions of the radiating atom a.

If the dimensions of the sources and detectors satisfy inequalities (3)—(5), then one can write down with the aid of wave functions of the type (7) for the probability $P_{ab}(\mathbf{r}_c, t; \mathbf{r}_d, \tau)$ averaged over the state of the atoms a and b in the source an expression of the form

$$\begin{aligned} P_{ab}(\mathbf{r}_c, t; \mathbf{r}_d, \tau) &= \text{const}\cdot\langle|\exp\{-i[\omega_a t + \Omega_a(t)]\} \\ &\times \exp\{-i[\omega_b\tau + \Omega_b(\tau)]\} + \exp\{-i[\omega_a\tau + \Omega_a(\tau)]\} \\ &\times \exp\{-i[\omega_b t + \Omega_b(t)]\}|^2\rangle, \end{aligned} \quad (8)$$

where the angle brackets denote the indicated averaging.

Equation (8) includes the quantity

$$G_{ab}(t,\tau) = \langle\exp\{-i(\Omega_a(t) - \Omega_a(\tau) + \Omega_b(\tau) - \Omega_b(t))\}\rangle,$$

characterizing the kinetic processes taking place in the source[2)]. In order to simplify the treatment, we assume below that one can neglect correlations between atoms a and b (for instance, we investigate the radiation of a gaseous source). Then,

$$G_{ab}(t,\tau) = G_a(t,\tau)G_b^*(t,\tau), G_{a,b}(t,\tau) = \langle\exp\{-i[\Omega_{a,b}(t) - \Omega_{a,b}(\tau)]\}\rangle.$$

For homogeneous systems $G_a(t, \tau) = G_b(t, \tau) = G(t, \tau)$. In the majority of practically interesting cases $G(t, \tau)$ can be represented in the following form[9,10]:

$$G(t-\tau) = \exp\{-w^2(t-\tau)/4\}, \quad (9)$$

where $\frac{1}{2}w^2(t-\tau) = \langle[\Omega_a(t) - \Omega_a(\tau)]^2\rangle$.

Utilizing (9), we obtain from (8) the following equality:

$$P_{ab}(t-\tau) = \text{const}\cdot\left\{1 + \exp\left\{-\frac{\text{Re}[w^2(t-\tau)]}{2}\right\} \times \cos[(\omega_a - \omega_b)(t-\tau)]\right\}. \quad (10)$$

Let us sum (10) over all pairs of atoms in the source[3)] and in the detector, assuming that the source emits with equal probability photons of only two frequencies ω_1 and ω_2. As a result we obtain for the probability $P(t-\tau)$ that one photon will be registered at the instant t and another at the instant τ the following expression:

$$P(t-\tau) = \text{const}\cdot\left\{1 + \exp\left\{-\frac{\text{Re}[w^2(t-\tau)]}{2}\right\}\cos^2\left[\frac{\omega_1 - \omega_2}{2}(t-\tau)\right]\right\}. \quad (11)$$

Thus, the delayed coincidence curve experiences modulated beats depending on the delay time $\theta = |t-\tau|$ at a frequency equal to the difference between the frequencies ω_1 and ω_2. In principle, the beat frequency can be regulated with the aid of various external interactions, for example by placing the source in an external magnetic field.

Let us now determine the total number of coincidences N in a given experiment (i.e., let us determine the area under the delayed coincidence curve if the maximum delay time used in the experiment is equal to θ_m). To do this, one must integrate (11) over $\theta = t - \tau$ between the limits $[0, \theta_m]$; this yields an expression of the type

$$N = \text{const}\cdot\left\{\theta_m + \int_0^{\theta_m}\exp\left\{-\frac{\text{Re}[w^2(\theta)]}{2}\right\}\cos^2\left[\frac{\omega_1-\omega_2}{2}\theta\right]d\theta\right\}. \quad (12)$$

The first term proportional to θ_m would correspond to the number of coincidences if the photons were distinguishable. The second term gives a contribution to N which is due to the fact that account has been taken of the identity of the particles. Therefore, allowance for identity leads to the circumstance that the area under the delayed coincidence curve is not proportional to θ_m, as would be the case for distinguishable particles.

Let us consider formula (12) for two limiting cases: in the first the source is an ideal gas at a temperature Q; in the second case the source is such that the main role in modulating the frequency of the radiation is played by collisions whose effect we will take into account in the impact approximation.[11] In the first case $\text{Re}[w^2(\theta)] = k^2\overline{v^2}\theta^2$ ($\overline{v^2}$ is the mean-square thermal velocity of the atoms), in the second case $\text{Re}[w^2(\theta)] = 4\rho(\overline{v^2})^{1/2}\sigma\theta$ (ρ is the density of atoms in the gas and σ is the collision cross section).[11] Substituting the indicated expressions for $w^2(\theta)$ in (12), we find for the first and second case the following expressions:

$$N = \text{const}\cdot\left\{\theta_m + \int_0^{\theta_m}\exp\left(-\frac{k^2\overline{v^2}}{2}\theta^2\right)\cos^2\left[\frac{\omega_1-\omega_2}{2}\theta\right]d\theta\right\}, \quad (13)$$

$$N' = \text{const}\cdot\left\{\theta_m + \frac{1}{2\Gamma} + \frac{\Gamma}{2[(\omega_1-\omega_2)^2+\Gamma^2]} - \frac{e^{-\Gamma\theta_m}}{2\Gamma}\left(1 + \frac{a\Gamma\cos[(\omega_1-\omega_2)\theta_m + \varphi]}{(\omega_1-\omega_2)^2+\Gamma^2}\right)\right\}, \quad (14)$$

where $ae^{i\varphi} = \Gamma + i(\omega_1 - \omega_2)$ and $\Gamma = 2\rho(\overline{v^2})^{1/2}\sigma$ is the impact width of the level[4)].

If $\theta_m(\overline{v^2}k^2)^{1/2} \gg 1$ and $\Gamma\theta_m \gg 1$, then (13) and (14) can be rewritten in the form

$$N \approx \text{const}\cdot\theta_m\left\{1 + \sqrt{\frac{\pi}{\overline{v^2}\theta_m^2k^2}} + \sqrt{\frac{\pi}{\overline{v^2}\theta_m^2k^2}}\exp\left[-\frac{(\omega_1-\omega_2)^2}{\overline{v^2}k^2}\right]\right\}, \quad (15)$$

$$N' \approx \text{const}\cdot\theta_m\left\{1 + \frac{1}{2\Gamma\theta_m} + \frac{1}{2\Gamma\theta_m}\frac{\Gamma^2}{(\omega_1-\omega_2)^2+\Gamma^2}\right\}. \quad (16)$$

2) As is seen from (8), in studying the correlations taking place in the radiation of the source the probability of registering delayed coincidences with two counters depends only on the mutual correlations between the two atoms a and b. If we were to measure triple or multiple correlations, then the corresponding probabilities would depend only on the mutual correlations between three or more atoms. This is somewhat different from the situation occurring in studying the correlation in the radiation scattered by some target when the energy spectrum of the scattered radiation measured in such experiments also depends on the time correlations of the state of a single atom.

3) Such a summation can be carried out if the photon density in the source is such that one can neglect the stimulated emission of the atoms.

4) In cases in which one can neglect the Doppler and impact width of the level, expression (14) in which Γ is the natural width of the level is valid for the area under the delayed coincidence curve.

Thus the area under the delayed coincidence curve depends on the difference frequency $\omega_1 - \omega_2$ and on the modulation mechanism of the frequency of the radiation. If $\omega_1 = \omega_2$, then expressions (15) and (16) differ from the result for classical particles by $2(\pi/\overline{v^2}\theta_m^2 k^2)^{1/2}$ and $1/\Gamma\theta_m$ respectively. If, on the other hand, $(\omega_1 - \omega_2)^2/\overline{v^2}k^2 \gg 1$ or $|\omega_1 - \omega_2| \gg \Gamma$, then the number of coincidences is larger than the classical result by $(\pi/\overline{v^2}\theta_m^2 k^2)^{1/2}$ and $1/2\Gamma\theta_m$ respectively. This means that in fulfilling the indicated inequalities photons of frequency ω_1 can be considered not to be identical with photons of frequency ω_2; at the same time, photons of the same frequency (ω_1 or ω_2) remain of course identical, which manifests itself in the fact that N and N′ differ from the values predicted for classical particles.

We note in conclusion that it would be attractive to carry out such experiments not only for light but, for example, also for Mossbauer γ quanta. However, because of the short wavelength of the γ quanta, it is at present practically impossible to satisfy conditions (5) and (6). Indeed, from inequality (5) we find that whereas for light with $\lambda \approx 10^{-5}$ cm and $l_S \sim l_D \sim 10^{-1}$ cm we must have $R \gtrsim 10^3$ cm, for γ quanta ($\lambda \approx 10^{-8}$ cm) with the same source and detector dimensions we must have $R \gtrsim 10^6$ cm. A more detailed analysis shows that this difficulty could be avoided by means of certain artificial assumptions. However, even more rigorous requirements result from inequality (6). From the latter it follows that with other conditions being equal the time of observation in the region of the x-ray spectrum T_γ should be larger by many orders of magnitude than the corresponding time T_c in the visible region $[T_\gamma \approx (\lambda_c/\lambda_\gamma)^4 T_c$, i.e., $T_\gamma \approx 10^{12} T_c]$. The above also refers to the case when a scattering target[8,12,13] is placed between the sources and the detectors, since the target can be simply considered to be the source of the scattered waves. Analogous serious difficulties also appear for other forms of radiation (electrons, neutrons).

The authors express their deep gratitude to V. L. Lyuboshitz for useful discussions.

[1] G. S. Gorelik, Dokl. Akad. Nauk SSSR **58**, 45 (1947).

[2] G. S. Gorelik, Usp. Fiz. Nauk **34**, 321 (1948).

[3] R. Hanbury-Brown and R. Q. Twiss, Phil. Mag. **45**, 633 (1954).

[4] E. M. Purcell, Nature **178**, 1449 (1956).

[5] A. T. Forrester, R. A. Gundmundsen, and R. Q. Johnson, Phys. Rev. **99**, 1691 (1955).

[6] A. T. Forrester, W. E. Parkins, and E. Gerjuoy, Phys. Rev. **72**, 728 (1947).

[7] U. Fano, Am. J. Phys. **29**, 539 (1961).

[8] M. Goldberger and K. Watson, Phys. Rev. **140**, B500 (1965).

[9] M. I. Podgoretskiĭ and A. V. Stepanov, Zh. Eksp. Teor. Fiz. **40**, 561 (1961) [Soviet Phys.-JETP **13**, 393 (1961)].

[10] K. Singwi and A. Sjölander, Phys. Rev. **120**, 1093 (1960).

[11] I. I. Sobel'man, Vvedenie v teoriyu atomnykh spektrov (introduction to the Theory of Atomic Spectra), Fizmatgiz, 1963.

[12] M. Goldberger, H. Lewis, and K. Watson, Phys. Rev. **132**, 2764 (1963).

[13] M. Goldberger and K. Watson, Phys. Rev. **137**, B1396 (1965).

Translated by Z. Barnea
35