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DYNAMICS OF PROCESSES IN MEDIA WITH INHOMOGENEOUS BROADENING OF THE LINE OF THE WORKING TRANSITION

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We consider the behavior of the field and polarization of a system of two-level molecules having a spread of the transition frequencies. It is shown that when account is taken of the relaxation of the amplitude of the dipole moment and the population difference, the shape of the inhomogeneous broadening line becomes significant. In particular, the presence of a dip in the inhomogeneously-broadened line shape can lead to a pulsating attenuation of the field. We also consider the possibility of high-frequency oscillations connected with the inhomogeneous broadening.

A consistent allowance for inhomogeneous broadening in the solution of problems involving the behavior of an electromagnetic field in a medium made up of active molecules leads in the general case to a system of integro-differential equations for the field and for the dipole moment. Since investigations of such a system are impossible in the general case, processes in media with inhomogeneous broadening are usually considered by assuming a quasistationary dependence of the polarization on the field (see, e.g.[1,2,3]). In this case only the integral characteristics of the inhomogeneously broadened line are significant (total number of particles, average frequency, average scattering, etc.). However, the quasistationary approximation is not applicable in the case of the experimentally realizable systems. In these cases, the character of time variation of the macroscopic dipole moment depends also on the inhomogeneously broadened line shape, leading to different new effects, which cannot be explained within the framework of the indicated approximation. The purpose of the present paper is to call attention to a number of such effects. In particular, it turns out that the presence of a dip in the inhomogeneously broadened line shape can lead to a pulsating attenuation of the field in the medium. We also find that under certain conditions fluctuations of the field amplitude and of the population differences, connected with the inhomogeneous broadening, are produced in the active medium.

1. INITIAL EQUATIONS

We consider a medium made up of two-level molecules, whose transition frequencies have a certain spread. The equations for the density matrix ρ of one such molecule, interacting with the field, are well known:

$$\frac{\partial\rho_{12}}{\partial t} - i\omega_0\rho_{12} + \frac{1}{\tau_2}\rho_{12} = \frac{i}{\hbar}(\mathbf{dE})n,$$

$$\frac{\partial n}{\partial t} + \frac{1}{\tau_1}(n - n_0) = 2\frac{i}{\hbar}\{(\mathbf{dE})\rho_{12} - (\mathbf{d}^*\mathbf{E})\rho_{12}\},$$

$$n = \rho_{22} - \rho_{11}, \quad \rho_{12} = \rho_{21}^*, \quad \mathbf{d} = \mathbf{d}_{12} = \mathbf{d}_{21}^*. \tag{1}$$

The macroscopic polarization $\mathbf{P}$ is determined by the expression

$$\mathbf{P} = \int_{-\infty}^{+\infty} f(\omega_0)\,\mathrm{Sp}(\hat{\mathbf{d}}\hat{\rho})\,d\omega_0,$$

where $f(\omega_0)$ is the distribution function of the particles with respect to the eigenfrequencies of the transitions. After making the substitution $\hat{\rho}_i = f(\omega_0)\rho$, we get

$$\mathbf{P} = \int_{-\infty}^{+\infty} \mathrm{Sp}(\hat{\mathbf{d}}\hat{\rho})\,d\omega_0$$

(we omit the index i). We shall henceforth use this notation, denoting $n_0 f$ by means of the new $f(\omega_0)$. Changing over to new variables $\mathscr{E}_l$, $\mathscr{P}$, and σ,

$$E_l = \mathscr{E}_l e^{i\omega_r t} + \text{c.c.}, \quad \mathbf{P} = \mathbf{d}\mathscr{P}e^{i\omega_r t} + \text{c.c.}, \quad \rho_{12} = \sigma e^{i\omega_r t}$$

(here ω_r – arbitrary frequency lying somewhere in the middle of the line of the medium), we obtain from the system (1) the equations

$$\dot{\sigma} + i\nu\sigma + \frac{1}{\tau_2}\sigma = \mathscr{E}n,$$

$$\dot{n} + \frac{1}{\tau_1}(n - f(\nu)) = -2(\mathscr{E}^*\sigma + \mathscr{E}\sigma^*), \quad \mathscr{P} = \int_{-\infty}^{+\infty}\sigma\,d\nu, \tag{2}$$

where

$$\nu = \omega_r - \omega_0, \quad \mathscr{E} = i(d_l\mathscr{E}_l)/\hbar.$$

These values do not take into account the spatial dispersion, and consequently also the motion of the molecules. However, in those cases when the spatial dispersion of n can be neglected, and it is possible to confine oneself to account of only the Doppler shift $\nu = \mathbf{k}\cdot\mathbf{v}$ for σ, the system (2) describes the behavior of a gas of two-level molecules with inhomogeneous broadening due to thermal motion.

It should be noted that whereas in the quasistationary case, to obtain the average dipole moment it is sufficient to know one distribution function $f(\nu)$, in the present case we need for this purpose three distribution functions: $f(\nu)$, the initial population difference $n_i(\nu)$, and the initial dipole moment $\sigma_i(\nu)$.

An essential feature of the equations in (2) is their linearity with respect to the average functions. Therefore, if for some aggregate $f_1(\nu)$, $n_{i1}(\nu)$, and $\sigma_{i1}(\nu)$ we obtain equations for the macroscopic polarization $\mathcal{P}_1$ and for $f_2(\nu)$, $n_{i2}(\nu)$, and $\sigma_{i2}(\nu)$ we obtain equations for $\mathcal{P}_2$, then the distribution functions $f_1 + f_2$, $n_{i1} + n_{i2}$, and $\sigma_{i1} + \sigma_{i2}$ will correspond to a macroscopic dipole moment $\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2$. In this connection, the arbitrary distribution functions f, n_i, and σ_i are naturally represented as a superposition of functions for which the material equations are obtained in the simplest manner. As will be clear from the problems

The differential equations (2) describe a system with an infinite number of degrees of freedom. Therefore there is no system of differential equations valid for an arbitrary distribution function in the case of the macroscopic polarization. These equations must be obtained individually in each concrete problem. In the present article we consider two such problems: the relaxation of the dipole moment and of the field in a medium with inhomogeneous broadening, and low-frequency population oscillations.

2. PULSATING ATTENUATION OF THE FIELD

We start with the simplest example of the time behavior of the macroscopic polarization of non-interacting particles at specified initial conditions in the absence of a field. In this case the polarization of one molecule has in terms of Laplace transforms the form

$$\overline{\sigma}(p,\nu)=\frac{\sigma_i(\nu)}{p+i\nu+1/\tau_2}; \qquad (3)$$

here $\sigma_i(\nu)$ - initial dipole moment. According to (2), we obtain the transform of the macroscopic polarization:

$$\overline{\mathcal{P}}(p)=\int_{-\infty}^{+\infty}\frac{\sigma_i(\nu)}{p+i\nu+1/\tau_2}d\nu. \qquad (4)$$

If the function $\sigma_i(\nu)$ tends to zero when $\operatorname{Im}\nu \to -\infty$, then the value of $\mathcal{P}(p)$ is determined only by the residues of this function in the singular points of the lower half-plane. One of the simplest functions of this type, having only one simple pole in the lower half-plane, is a distribution function of the Lorentz type:

$$\sigma_i(\nu)=\frac{\mathcal{P}_i\beta}{\pi[(\nu-\alpha)^2+\beta^2]}. \qquad (5)$$

Here α - displacement of the average frequency relative to ω_r, β - width of the distribution function at the 0.5 level, and $\mathcal{P}_i$ determines the initial value of the polarization. For such an elementary function, the average dipole moment is

$$\overline{\mathcal{P}}(p)=\frac{\mathcal{P}_i}{p+i\alpha+\beta+1/\tau_2}, \qquad (6)$$

corresponding to the differential equation

$$\dot{\mathcal{P}}+i\alpha\mathcal{P}+(\beta+1/\tau_2)\mathcal{P}=0 \qquad (7)$$

with initial condition $\mathcal{P}(0) = \mathcal{P}_i$.

Thus, for a distribution function of the Lorentz type, the average polarization is described by a first-order differential equation, the solution $\mathcal{P}(t)$ of which oscillates with the average distribution frequency α and attenuates with a decrement equal to $\beta + 1/\tau_2$. Therefore, the "broader" the distribution, the faster the attenuation of $\mathcal{P}(t)$. It is obvious that a superposition of functions of the type (5),

$$\sigma_i(\nu)=\sum_k\frac{\mathcal{P}_{ik}\beta_k}{\pi[(\alpha_k-\nu)^2+\beta_k^2]}$$

corresponds to the system of equations

$$\mathcal{P}(t)=\sum_k\mathcal{P}_k(t),$$

$$\dot{\mathcal{P}}_k+i\alpha_k\mathcal{P}_k+(\beta_k+1/\tau_2)\mathcal{P}_k=0, \quad \mathcal{P}_k(0)=\mathcal{P}_{ik}.$$

This explains the rather unique behavior of the system of molecules in which the distribution function of the initial conditions is of the form of a sum of two functions of the Lorentz type - with large width β_1 and positive area, and with a small spread β_2 and negative area - i.e., broadening with a narrow dip:

$$\sigma_i=\frac{\mathcal{P}_{i1}\beta_1}{\pi[\nu^2+\beta_1^2]}-\frac{\mathcal{P}_{i2}\beta_2}{\pi[\nu^2+\beta_2^2]},$$

where $\beta_1 \gg \beta_2$ and $\mathcal{P}_{i1} > \mathcal{P}_{i2} > 0$.

The total value of P will be the sum of two exponentially damped quantities:

$$\mathcal{P}=\mathcal{P}_{i1}\exp\{-(\beta_1+1/\tau_2)t\}-\mathcal{P}_{i2}\exp\{-(\beta_2+1/\tau_2)t\}.$$

At the initial instants of time, the polarization is described both by the parameters of the broad distribution and by the contribution of the narrow dip; however, after a time $1/\beta_1$, the first term $\mathcal{P}_1$ can become much smaller than the long-lived polarization $\mathcal{P}_2$ due to the dip, even if $\mathcal{P}_{i2} \ll \mathcal{P}_{i1}$. If furthermore such a dip is produced not at the center of the broadening line, then initially the dipole moment of the system will have two frequencies equal to the frequencies of the center of the broadening line and the center of the dip. Then the component with the central frequency attenuates rapidly with a time on the order of the reciprocal width of the broadening line, and we are left with the component with the average frequency of the dip, the characteristic lifetime of which is equal to the reciprocal width of the dip.

By way of a more complicated example, let us consider the influence of inhomogeneous broadening of Lorentz type

$$f(\nu)=\frac{N_0\beta}{\pi[\nu^2+\beta^2]} \qquad (8)$$

on the change of the amplitude of the field $\mathcal{E}$ in a resonator, assuming that initially $(t \le 0)$ there took place stationary generation, which was then interrupted by a jumplike change in the losses. The distribution functions $\sigma_i(\nu)$ and $n_i(\nu)$ will have dips that depend on the amplitude of the stationary generation $\mathcal{E}_0$:

$$\sigma_i(\nu)=\mathcal{E}_0\frac{(1/\tau_2-i\nu)f(\nu)}{\nu^2+1/\tau_2^2+4|\mathcal{E}_0|^2\tau_1/\tau_2},$$

$$n_i(\nu)=\frac{(\nu^2+1/\tau_2^2)f(\nu)}{\nu^2+1/\tau_2^2+4|\mathcal{E}_0|^2\tau_1/\tau_2}. \qquad (9)$$

For simplicity we confine ourselves to those cases when it is possible to regard the amplitude of the field $\mathcal{E}$ as independent of the coordinates. The change of the amplitude at $t > 0$ is described by the equation

$$\dot{\mathcal{E}}+g\mathcal{E}=\gamma\mathcal{P}, \qquad (10)$$

where g - loss coefficient, equal to $g_0 \ll g$ at $t < 0$, $\gamma = 2\pi d^2\omega_r/\hbar$ is a constant, and $\mathcal{E}(0) = \mathcal{E}_0$. If the jump of the loss coefficient is sufficiently large, then in the initial instants of time the term $\gamma\mathcal{P}$ in (10) can be neglected, and the field will be described with a high degree of accuracy by the expression $\mathcal{E} = \mathcal{E}_0 e^{-gi}$. After a time on the order of $1/g$, the quantities $g\mathcal{E}$ and $\gamma\mathcal{P}$ become comparable, and the term $\gamma\mathcal{P}$ can no longer be described in general. Therefore, to describe the field, a complete system of self-consistent equations for $\mathcal{E}$ and $\mathcal{P}$ is necessary, thus greatly hindering the investigation. However, at that instant of time, if the losses are sufficiently large, the field will be so small that the material equations for $\mathcal{P}$ can be regarded as linear in $\mathcal{E}$. In this approximation, the equations for n in the system (2) will not contain the term $-2(\mathcal{E}^*\sigma + \mathcal{E}\sigma^*)$. Then the Laplace transform of the macroscopic response to the field E is given by

$$\overline{\mathcal{P}}(p) = \int_{-\infty}^{+\infty} \left[\overline{\mathcal{E}} \ast \left\{ \frac{n_i + f/p\tau_1}{p + 1/\tau_1} \right\} + \sigma_i \right] (p + i\nu + 1/\tau_2)^{-1} d\nu, \quad (11)$$

where the symbol $\ast$ denotes convolution with respect to p, and the functions $j(\nu)$, $\sigma_i(\nu)$, and $n_i(\nu)$ are given by expressions (8) and (9). Contribution to the integral will be made by the residues at the singular points: $\nu_1 = -i\beta$, characterizing the distribution function $f(\nu)$, and $\nu_2 = -i\{1/\tau_2^2 + 4|\mathcal{E}_0|^2\tau_1/\tau_2\}^{1/2}$, which determines the width of the dip as a result of stationary generation. Accordingly, the polarization breaks up into two parts: $\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2$, where $\mathcal{P}_2$ is due to the presence of the dip in both distribution functions. After evaluating the integral (11) and taking the inverse transform, we obtain the following system of equations for P(t):

$$\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2,$$

$$\dot{\mathcal{P}}_2 + \left(\beta + \frac{1}{\tau_2}\right)\mathcal{P}_1 = \mathcal{E}N, \quad \dot{N} + \frac{1}{\tau_1}(N - N_0) = 0,$$

$$\dot{\mathcal{P}}_2 + \frac{\alpha + 1}{\tau_2}\mathcal{P}_2 = \mathcal{E}N_2, \quad \dot{N}_2 + \frac{1}{\tau_1}N_2 = 0 \quad \alpha = \sqrt{1 + 4\tau_1\tau_2|\mathcal{E}_0|^2}, \quad (12)$$

with initial conditions

$$\mathcal{P}_1(0) = \frac{-\mathcal{E}_0 N_0}{\tau_2(\beta^2 - \alpha^2/\tau_2^2)}, \quad \mathcal{P}_2(0) = \frac{\mathcal{E}_0 N_0 \beta}{\alpha(\beta^2 - \alpha^2/\tau_2^2)},$$

$$N(0) = N_0 \frac{\beta^2 - 1/\tau_2^2}{\beta^2 - \alpha^2/\tau_2^2}, \quad N_2(0) = -N_0 \frac{\beta(\alpha^2 - 1)}{\alpha\tau_2(\beta^2 - \alpha^2/\tau_2^2)}.$$

The system of equations (10) and (12) determines the behavior of the macroscopic polarization and the field[1)].

For the case of strong inhomogeneous broadening ($\beta \gg 1/\tau_{1,2}, g$) it is possible to assume that $\mathcal{P}_1 \approx \mathcal{E}N/\beta \approx \mathcal{E}N_0/\beta$. The influence of $\mathcal{P}_1$ on the field then reduces in fact to replacement of the coefficient g by the quantity $\gamma N_0/\beta$. In view of the fact that $g \gg \gamma N_0/\beta$, this correction, and consequently also $\mathcal{P}_1$, can be neglected. Taking the foregoing into account, we obtain a solution of the system (10), (12). It is given by the product of an exponential and a cylindrical function of an exponentially damped argument

$$\mathcal{E}(t) = \exp\left\{-\left(g + \frac{\alpha + 1}{\tau_2}\right)t\right\} Z_q\left[2\tau_1\sqrt{\frac{(\alpha^2 - 1)g_0}{\tau_2}} \exp\left(-\frac{t}{2\tau_1}\right)\right].$$

1)We note that the problem of relaxation of the field in a half-space of active medium with inhomogeneous broadening reduces to the same system, if a strong electromagnetic wave of amplitude $\mathcal{E}_0$ is incident normally on its boundary at $t \leq 0$, and if $\mathcal{E}_0 \equiv 0$ at $t > 0$.

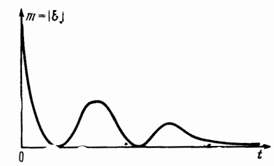

FIG. 1.

Here $Z_q(x) = C_1 J_q$, where J_q and N_q are Bessel functions of the first and second kind, $q = \tau_1(g - (\alpha + 1)/\tau_2)$, and the constants $C_{1,2}$ are determined by the initial conditions.

This function describes a pulsating attenuation of the field, with a finite number of pulsations (see Fig. 1). Such a variation of the field relaxation is the consequence of the fact that the dipole moment is proportional to the area of the dip N_2. In fact, N_2 is a decreasing negative quantity. If N_2 were constant, then at

$$N_2 < -\frac{(g - (\alpha + 1)/\tau_2)^2}{4\gamma}$$

there would be realized a regime of pulsating field attenuation with an infinite number of pulsations. Naturally, such a regime is produced even if N_2 is variable but the dip is sufficiently deep (sufficiently large negative N_2), the only difference being that as a result of the decrease of $|N_2|$ ("clogging" of the dip), the number of oscillations will be finite. Obviously, the smaller the stationary generation, the larger the dip and the larger the number of times that the amplitude of the attenuating field will vanish. In particular, at $\tau_1 = \tau_2 = 1.3 \times 10^{-7}$ sec and $g = 3 \times 10^7$ sec^{-1}, it is necessary to have an excess $\alpha \gtrsim 5$ above threshold to reach at least one zero. The qualitative time dependence of the intensity of the field $m = |E|^2$ is shown in Fig. 1. In the approximation in which "the polarization follows the field," such a pulsating attenuation of the interrupted generation can obviously not be considered.

3. OSCILLATIONS OF ACTIVITY

Another effect connected with inhomogeneous broadening is the possible occurrence of low-frequency oscillations (compared with the optical frequency) of the concentration of the excited particles and of the amplitude of the field $\mathcal{E}$ - oscillations of activity. These oscillations are connected with the delay of the response of the inhomogeneously broadened medium to the field $\mathcal{E}(t)$. Generally speaking, oscillations connected with the finite relaxation time of the response exist also in the case of homogeneous broadening [4]. However, as will be shown below, in media with inhomogeneous broadening there can be low-frequency oscillations that are essentially connected with the broadened line shape, and thus, are due only to the inhomogeneous broadening.

In order not to obscure the effect, let us consider the problem in its simplest formulation. As before, the amplitude of the field is assumed to be independent of the coordinates and determined by Eq. (10). Let also $\tau_1 = \tau_2 = \tau$. We assume that the distribution function $f(\nu)$ has a narrow dip, so that

$$f(\nu) = \frac{N_1\beta_1}{\pi[\nu^2 + \beta_1^2]} - \frac{N_2\beta_2}{\pi[\nu^2 + \beta_2^2]},$$

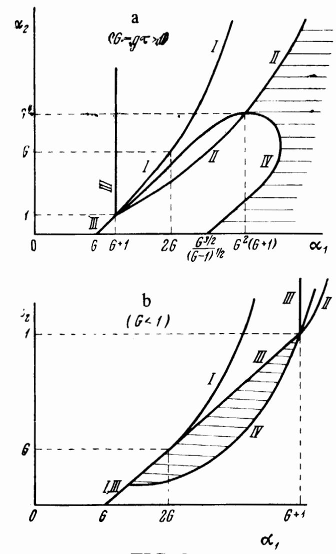

FIG. 2.

where $N_1 > N_2 > 0$ and $\beta_1 \gg \beta_2$. If we find the values the parameters for which all the stationary states in such a system are unstable, then this will be the region where low-frequency oscillations certainly exist.

In the stationary state, the field frequency may not coincide with the central frequency of the broadened line, even if the natural frequency of the resonator does coincide with the center of the line. This frequency shift Δ and the amplitude of the stationary oscillations are determined by the equations

$$(i\Delta + g)\mathscr{E}_{st} = \gamma \int_{-\infty}^{+\infty} \sigma_{st}\, d\nu,$$

$$(i\Delta + i\nu + 1/\tau)\sigma_{st} = \mathscr{E}_{st} n_{st},$$

$$\frac{1}{\tau}(n_{st} - f) = -2(\mathscr{E}_{st}\sigma_{st}^* + \mathscr{E}_{st}^*\sigma_{st}). \tag{13}$$

The calculations are greatly simplified if it is assumed that $\beta_1 \gg g$, $1/\tau$, and $\beta_2 \ll 1/\tau$. Then the conditions for the existence of a stationary regime with $\Delta = 0$ will take the form

$$\gamma N_2\tau + g < \frac{\gamma N_1}{\beta_1} < 2g \quad \text{or} \quad g < \gamma N_2\tau < \left(\frac{\gamma N_1}{\beta_1}\right)^2 \Big/ 4g.$$

The limit of this region is shown by curve I of Fig. 2.

It is easy to verify that for stationary regimes with a detuning we get

$$\Delta = \pm\left\{\gamma N_2 - \left(\frac{\gamma N_1}{\beta_1}\right)^2 \Big/ (g\tau + 1)^2\right\}^{1/2}$$

$$m = |\mathscr{E}_{st}|^2 = \left(\frac{\gamma N_1}{\beta_1}\right)^2 \frac{1}{4(g\tau + 1)^2} - \frac{1}{4\tau^2}.$$

From this we get the condition for the existence of these regimes:

$$\gamma N_1\tau/\beta_1 > g\tau + 1, \qquad \gamma N_2 > (\gamma N_1/\beta_1)^2/(g\tau + 1)^2.$$

These inequalities determine the region between the broken curve III and the parabola II of Fig. 2.

The instability of the zero position of equilibrium ($m = 0$) is satisfied for

$$\gamma N_1/\beta_1 > \max\{g + 1/\tau, \gamma N_2\tau + g\}$$

(self-excitation conditions). In Fig. 2 this region is located to the right of the broken line III. It can be shown that the limit of the instability of the regime with $\Delta = 0$ is determined by a curve that can be expressed parametrically in the form

$$\gamma N_1\tau/\beta_1 = \xi[2g\tau - (g\tau - 1)\xi^2],$$
$$\gamma N_2\tau = \xi^2[g\tau - (g\tau - 1)\xi^2],$$

where ξ is the running parameter. As seen from Fig. 2, this curve differs greatly for $g\tau > 1$ (curve IV of Fig. 2a) and for $g\tau < 1$ (curve IV on Fig. 2b).

Thus, for the parameters $\alpha_1 = \gamma N_1\tau/\beta_1$ and $\alpha_2 = \gamma N_2\tau^2$, which lie in the shaded region on Figs. 2a and b, there exist activity oscillations. It is interesting to note that in order for oscillations to occur when $g < 1/\tau$ it is necessary to have a dip in the distribution function $f(\nu)$ ($\alpha_2 \neq 0$).

4. RESPONSE AT LARGE ν

The simplicity of the equations obtained for the distribution functions of the Lorentz type makes it important in practice to ascertain the possibility of expanding an arbitrary $f(\nu)$ in a sum of a finite number of Lorentz functions. Leaving aside the approximation problems, we consider only the related question of the behavior of the response on the "tails" of the distribution function, which is of independent interest.

For sufficiently large ν, the value of $\sigma(\nu, t)$ can be easily determined for an arbitrary form of $f(\nu)$. Indeed, let $\nu \gg 1/\tau_1$, $1/\tau_2$, $1/\tau_p$, $|\mathscr{E}|$ (τ_p - characteristic time of variation of the field amplitude). Then the solution of the equations for σ can be sought in the form

$$\sigma = C(t)e^{-i\nu t},$$

where $C(t)$ is a function of the time which is slow compared with $e^{-i\nu t}$ and is given by the equation

$$\dot{C} + \frac{1}{\tau_2}C = \mathscr{E}\overline{ne^{i\nu t}}. \tag{14}$$

The superior bar denotes time averaging over the period $2\pi/\nu$. The amplitude of the field $\mathscr{E}$ is taken outside the averaging sign, since $\nu \gg 1/\tau_p$. We determine the value of $n_\nu = ne^{i\nu t}$ from the second equation of the system (2)

$$-i\nu n_\nu + \frac{1}{\tau_1}n_\nu = -2\mathscr{E}^*C$$

or

$$n_\nu = \frac{-2\mathscr{E}^*}{-i\nu + 1/\tau_1}C.$$

Substituting this expression for n_ν in (14) we find that at large values of ν

$$\sigma = \sigma_i(\nu)\exp\left\{-\frac{t}{\tau_2} - \frac{2}{-i\nu + 1/\tau_1}\int_0^t |\mathscr{E}|^2 dt - i\nu t\right\}$$

$$\approx \sigma_i(\nu)\exp\left\{-\frac{t}{\tau_2} - i\nu t\right\}.$$

Here $\sigma_i(\nu)$ - value of σ at $t = 0$.

Thus, for sufficiently large ν, the quantity $\sigma(\nu, t)$ does not depend on f and tends to zero with a characteristic variation time on the order of τ_2. Apparently the most important consequence of such an asymptotic behavior of σ is the possibility of regarding $f(\nu)$ in a finite interval, thus greatly facilitating the approximation problem.

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[1] W. E. Lamb, Phys. Rev. **134**, A1429 (1964).

[2] S. G. Zeiger and E. E. Fradkin, Opt. Spektrosk. **21**, 399 (1966).

[3] Yu. L. Klimantovich, V. N. Kuryatov, and P. S. Landa, Zh. Eksp. Teor. Fiz. **51**, 3 (1966) [Sov. Phys.-JETP **24**, 1 (1967)].

[4] V. V. Korobkin and A. V. Uspenskii, ibid. **45**, 1003 (1963) [**18**, 693 (1964)].

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