TRANSFORMATION OF LASER RADIATION BY STIMULATED RAMAN SCATTERING

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Stimulated Raman scattering (SRS) in resonators with axes perpendicular or parallel to the axis of the primary laser beam is considered. Most attention is paid to the possibility of stabilizing the generation regime of the first Stokes SRS component.

STIMULATED Raman scattering (SRS) arising during passage of a laser beam through a resonator filled with some substance can be used for the frequency transformation of laser radiation, as well as for changing other characteristics of the laser beam—its flux density and divergence. In addition there is the possibility (as was noted for example in [1]) of summing the radiation of several lasers. The process of greatest interest in this connection is evidently SRS in a resonator whose axis is perpendicular to the axis of the primary laser beam, since in this case pumping of the substance in the resonator is most easily accomplished by means of several lasers.

In what follows we shall use the terms "transverse pumping" and "longitudinal pumping" respectively for cases when the axis of the first laser beam is perpendicular and parallel to the resonator axis.

In the present paper we consider some specific features of the SRS process in resonators with transverse pumping. Principal attention is paid to the possibility of stabilizing the generation regime of the first Stokes component of SRS. As will be seen later, this is favorable for obtaining high efficiency of the transformation. In addition, we also briefly discuss the case of longitudinal pumping.

Let the radiation of the primary laser have frequency ω_0, spectral width $\Delta\omega_0$, divergence θ_0, and photon flux density $S_0 = I_0\Delta\omega_0$ (in cm^{-2} sec^{-1}). We shall take the x axis along the axis of the primary laser beam and the y axis along the resonator axis. We also assume that $\theta_0 L \ll 1$, where L is the thickness of the layer of substance in the resonator (length along x axis). In addition, we suppose that the absorption of the substance in the resonator at frequency ω_0 may be neglected and that the threshold for self-focusing is above the threshold for Raman scattering (a similar situation is realized, for example, in gases [2]). In the general case, under the influence of the laser radiation S_0 an entire series of Stokes and anti-Stokes SRS components can arise in the resonator substance.

We shall allow, however, only the generation of the first Stokes component in the resonator, with frequency $\omega_1 = \omega_0 - \Omega$, where Ω is a transition frequency of the scattering molecules, and we shall find the necessary conditions for realizing this kind of regime.

We introduce the following notation: S(x, y) is the flux density of the primary laser beam inside the resonator; $n_1^+(x, y)$, $n_1^-(x, y)$ are the densities of generated photons in the oscillator field (in cm^{-3}) for directions of the wave vector respectively along and opposite to the y axis; σ is the effective cross section for Raman scattering, N is the difference in population of the initial and final levels of the molecules, ΔO is the solid angle of generation, which depends on the resonator parameters, H is the resonator length, r_0 and r_H are the reflection coefficients of the lower (y = 0) and upper (y = H) mirrors of the resonator.

The system of equations for S, n_1^+, n_1^- has the form[1)]

$$\frac{1}{c}\frac{\partial S}{\partial t}+\frac{\partial S}{\partial x}=-SN\sigma_1-SN\sigma_1\frac{\Delta O_1}{4\pi}(n_1^+ + n_1^-),$$
$$a_1\frac{\partial n_1^+}{\partial t}+ca_1\frac{\partial n_1^+}{\partial y}=SN\sigma_1\frac{\Delta O_1}{4\pi}(1+n_1^+),$$
$$a_1\frac{\partial n_1^-}{\partial t}-ca_1\frac{\partial n_1^-}{\partial y}=SN\sigma_1\frac{\Delta O_1}{4\pi}(1+n_1^-). \quad (1)$$

Here c is the speed of light,

$$a_1 = 2\frac{\omega_1^2\Delta\omega\Delta O_1}{(2\pi c)^3}, \quad \Delta\omega=\Delta\omega_0+\Gamma, \quad (2)$$

where Γ is the spectral width for the molecular transition being considered. The solution of the system (1) must satisfy the boundary conditions

$$S(0,y)=S_0(y), \quad n_1^+(x,0)=r_0 n_1^-(x,0), \quad n_1^-(x,H)=r_H n_1^+(x,H). \quad (3)$$

Let us first find under what conditions a steady-state generation regime is possible. This is done most easily in the following way. Divide the second and third equations of (1) respectively by $(1 + n_1^+)$ and $(1 + n_1^-)$, combine them, and then integrate over y between the limits y = 0, y = H. As a result, we obtain

$$a_1\frac{\partial}{\partial t}\int_0^H \ln[(1+n_1^+)(1+n_1^-)]\,dy + ca_1\ln\left[\left(\frac{1+n_1^+}{1+n_1^-}\right)_{y=H}\left(\frac{1+n_1^-}{1+n_1^+}\right)_{y=0}\right]=2R_1\int_0^H S\,dy, \quad (4)$$

where $R_1 = N\sigma_1\Delta O_1/4\pi$. Under conditions of generation we neglect unity compared to n_1^+, n_1^-. Then, using the boundary conditions (3), we have

$$a_1\frac{\partial}{\partial t}\int_0^H \ln(n_1^+ n_1^-)\,dy + ca_1\ln\frac{1}{r_H r_0}=2R_1\int_0^H S\,dy. \quad (5)$$

First of all, we get from (5) the usual threshold condition for establishing generation

1)The possibility of transforming from the equations for the field amplitudes to the equations for intensities in problems of this type is discussed in [3].

$$2R_1 \int_0^H S\,dy \geqslant c a_1 \ln \frac{1}{r_H r_0}. \quad (6)$$

As can be seen from the first equation of the system (1), the left hand side of (6) is a function of x and t; at each point x it decreases with time with growth of n_1^+ and n_1^-, and at any t it is a monotonically decreasing function of x. A stationary generation regime should be attained with establishment of the equality

$$2R_1 \int_0^H S\,dy = c a_1 \ln \frac{1}{r_H r_0}. \quad (7)$$

At the same time, it is not difficult to establish from (6) and (7) that if the right hand part of (7) is independent of x a stationary generation regime is in general impossible. Actually, at the point x = 0, i.e., at the front boundary of the resonator substance, the quantity $R_1 \int_0^H S dy$ is in general independent of n_1^+ or n_1^-, and the magnitude of the incident flux S(0, y) is completely determined. Hence the growth of $n_1^+(0, y)$ or $n_1^-(0, y)$ is not limited by anything. As the process develops the region of generation continually contracts, and the quantities n_1^+ and n_1^- grow within this region. Naturally, all of this is valid only in the considered approximation. In fact, of course, beginning with certain values of n_1^+ and n_1^-, Eqs. (1) become defective, since they do not take into account the existence of various non-linear losses, e.g., generation of the second and other Stokes components of SRS and self-focusing.

First we examine whether it is possible to achieve stabilization of the generation regime and under what conditions of generation of subsequent Stokes SRS components. After this, we investigate a second alternative —the possibility of a stable generation regime of one Stokes component in a resonator whose mirror reflection coefficients r_0 and r_H depend on x. (Remember that all that was said above about the unlimited growth of the quantities $n_1^+(0, y)$ and $n_1^-(0, y)$ was based on the fact that the right hand sides of (6) and (7) are independent of x.)

So, we shall consider the regime of generation of two Stokes components. In the framework of a system of equations like (1), this is done with difficulty. Hence, we shall use a somewhat simplified description. We introduce instead of $n_1^+(x, y)$ and $n_1^-(x, y)$, and S(x, y) the averaged variables

$$n(x) = \frac{1}{H}\int_0^H (n_1^+ + n_1^-)\,dy, \quad S(x) = \frac{1}{H}\int_0^H S(x, y)\,dy. \quad (8)$$

Then instead of system (1) we easily obtain

$$\frac{1}{c}\frac{\partial S}{\partial t} + \frac{\partial S}{\partial x} = -SN\sigma_1 - SR_1 n_1,$$

$$a_1 \frac{\partial n_1}{\partial t} = SR_1(1 + n_1) - \frac{\alpha_1}{H} n_1 - a_1 c n_1 R_2 (1 + n_2),$$

$$a_2 \frac{\partial n_2}{\partial t} = a_1 c n_1 R_2 (1 + n_2) - \frac{\alpha_2}{H} n_2. \quad (9)$$

Here n_1 and n_2 are the average numbers of photons per unit volume in the oscillator field for the first and second Stokes components, respectively, $\omega_1 = \omega_0 - \Omega$, $\omega_2 = \omega_1 - \Omega$. The subscripts 1, 2 on σ, R, a, and α also correspond to the first and second Stokes components; $\alpha_i = a_i c(2 - r_H - r_0 - \delta_H - \delta_0)$; δ is the absorption coefficient of the mirrors; the terms $\alpha_1 n_1/H$ and $\alpha_2 n_2/H$ describe the losses at the exit of photons from the resonator and absorption in the mirrors.[2)]

Note that in the limiting case r_0, $r_H \to 1$, the system of equations (1) and (9), without terms pertaining to generation of the second Stokes component, are completely equivalent. All qualitative features of the process of interest to us even in the general case of arbitrary r_0 and r_H are correctly described already in the approximation (9).

The last term in the second equation of system (9) describes losses of photons of the first Stokes component in generation of the second Stokes component. As n_2 increases these losses increase. If we set $n_2 = 0$, then from (9) it is easy to obtain the earlier conclusion, that a stable regime of generation of one Stokes component is impossible in the presence of x-independent losses α_1.

Setting all time derivatives in (9) equal to zero and neglecting spontaneous scattering, we obtain for the stationary generation regime:

$$n_1 = \frac{1}{a_1 c}\frac{\alpha_2}{HR_2}, \quad n_2 = \frac{1}{a_1 c}\left\{S - \frac{\alpha_1}{HR_1}\right\}\frac{R_1}{R_2}; \quad (10)$$

$$S = S(0)\exp\left[-\frac{1}{a_1 c H}\frac{R_1}{R_2}\int_0^x \alpha_2(x)\,dx\right], \quad S(x) \geqslant \frac{\alpha_1}{HR_1}. \quad (11)$$

As is seen, the relations (10) and (11) can be fulfilled even when r_0 and r_H are independent of x. Thus development of the generation of the second Stokes component leads to stabilization of the regime. From (4) we see that generation arises when the intensity of the incident beam exceeds the threshold value

$$S(0) > S^{\mathrm{T}} = \alpha_1 / HR_1. \quad (12)$$

Since S(x) decreases monotonically with increasing x, at some point l, depending on the condition $S(l) = S^{\mathrm{T}}$, generation ceases. When $x > l$, $S(x) = S(l)$. Thus, the extension of the generation region is

$$l = H\frac{R_2 a_1 c}{R_1 \alpha_2}\ln\left[\frac{S(0)}{S^{\mathrm{T}}}\right]. \quad (13)$$

The densities of the fluxes coming out of the resonator $S_1 = \alpha_1 n_1$ (at frequency ω_1) and $S_2 = \alpha_2 n_2$ (at frequency ω_2) are found from (10). We write out the expressions for the average flux densities in the case $r_0 + \delta_0 = 1$ and $\delta_H \ll 1 - r_H$:

$$\bar{S}_1 = \frac{1}{l}\int_0^l S_1(x)\,dx = S^{\mathrm{T}}(1 - r_H)\frac{\alpha_2 R_1}{\alpha_1 R_2}, \quad (14)$$

$$\bar{S}_2 = \frac{1}{l}\int_0^l S_2(x)\,dx = S(0)(1 - r_H)\left\{\frac{1 - S^{\mathrm{T}}/S(0)}{\ln[S(0)/S^{\mathrm{T}}]} - \frac{S^{\mathrm{T}}}{S(0)}\right\}\frac{R_1}{R_2}\frac{\alpha_2}{\alpha_1}. \quad (15)$$

Let us determine the transformation efficiency κ (with respect to the photon flux):

$$\varkappa = \varkappa_1 + \varkappa_2; \quad (16)$$

$$\varkappa_1 = \frac{1}{S(0)H}\int_0^l S_1(x)\,dx = \frac{S^{\mathrm{T}}}{S(0)}\ln\frac{S(0)}{S^{\mathrm{T}}}, \quad (17)$$

[2)] For simplicity, we assume that the coefficients r and δ are the same for frequencies ω_1 and ω_2. The corresponding generalization does not cause any difficulties.

$$\varkappa_2 = \frac{1}{S(0)H}\int_0 S_2(x)dx = 1 - \frac{S^T}{S(0)} - \frac{S^T}{S(0)}\ln\frac{S(0)}{S_T}. \quad (18)$$

From (17) we see that the magnitude of κ_1 at any excess of the intensity of the incident flux $S(0)$ over the threshold value S^T is small (maximum possible value $\kappa_1 < 0.35$). The magnitude of κ_2 at $S(0)/S^T \gg 1$ is close to unity.

Thus, in the considered regime of generating two Stokes components it is possible to provide a rather high transformation efficiency. The largest exit flux corresponds to the second Stokes component, i.e., to the frequency ω_2.

It follows from (14) and (15) that the output flux densities $\overline{S}_1$ and $\overline{S}_2$ remain less than $S(0)\alpha_1 R_1/\alpha_2 R_2$ at any choice of parameters. The circumstance that $R_1 > R_2$, as a consequence of the frequency dependence of the effective cross sections for Raman scattering, cannot lead to a significant increase in the flux densities $\overline{S}_1$ and $\overline{S}_2$.

In this same scheme it is easy to account for the possibility of generating the third $\omega_3 = \omega_2 - \Omega$ and all subsequent Stokes components. An analogous treatment leads to the following results. In the case of transverse pumping the regime that is established always corresponds to the simultaneous generation of an even number of Stokes components. If the incident flux $S(0)$ is within the limits

$$S^T < S(0) < S^T\left(1 + \frac{\alpha_3}{\alpha_1}\frac{R_2}{R_3}\frac{a_1}{a_2}\right), \quad (19)$$

there is established a regime of generation of the two Stokes components ω_1 and ω_2 described by Eqs. (14)—(18). On increasing $S(0)$, when

$$S^T\left(1 + \frac{\alpha_3}{\alpha_1}\frac{R_2}{R_3}\frac{a_1}{a_2}\right) < S(0)$$

$$< S^T\left(1 + \frac{\alpha_3}{\alpha_1}\frac{a_1}{a_2}\frac{R_2}{R_3} + \frac{\alpha_5}{\alpha_1}\frac{a_1}{a_2}\frac{a_3}{a_4}\frac{R_2}{R_3}\frac{R_4}{R_5}\right), \quad (20)$$

generation of the third and fourth Stokes components ω_3 and ω_4 begins, etc.

We now return to the system (1) and consider the regime established assuming that the reflection coefficients of the resonator mirrors depend on x. From what was said above about Eqs. (6) and (7), it follows that the function $\ln(1/r_0 r_H)$ should monotonically decrease with increasing x.

Setting the time derivatives to zero and leaving out terms pertaining to "spontaneous" scattering, we obtain

$$\frac{\partial S}{\partial x} = -SR_1(n_1^+ + n_1^-), \quad ca_1\frac{\partial n_1^+}{\partial y} = SR_1 n_1^+,$$
$$ca_1\frac{\partial n_1^-}{\partial y} = -SR_1 n_1^-. \quad (21)$$

From the second and third equations of (21) we find

$$\frac{\partial}{\partial y}\ln(n_1^+ n_1^-) = 0, \quad n_1^+ n_1^- = D(x), \quad (22)$$

where $D(x)$ is an arbitrary function of x. Using the boundary conditions (3) we obtain

$$n_1^-(x,0) = \left[\frac{1}{r_0}D(x)\right]^{1/2}, \quad n_1^-(x,H) = [r_H D(x)]^{1/2},$$
$$n_1^+(x,0) = [r_0 D(x)]^{1/2}, \quad n_1^+(x,H) = \left[\frac{1}{r_H}D(x)\right]^{1/2}. \quad (23)$$

From the second equation of system (21) and (23) we find

$$\int_0^H S(x,y)dy = \frac{ca_1}{R_1}\ln\frac{n_1^+(x,H)}{n_1^+(x,0)} = \frac{ca_1}{2R_1}\ln\frac{1}{r_0 r_H}, \quad (24)$$

which agrees with (7).

From system (21) follows the condition of flux conservation

$$\frac{\partial S}{\partial x} = -ca_1\frac{\partial n_1^+}{\partial y} + ca_1\frac{\partial n_1^-}{\partial y}.$$

Integrating this over y and using (23), we obtain

$$\frac{\partial}{\partial x}\int_0^H S(x,y)dy = -ca_1\sqrt{D(x)}\left[\frac{1-r_0}{\sqrt{r_0}} + \frac{1-r_H}{\sqrt{r_H}}\right]. \quad (25)$$

Differentiating (24) with respect to x and equating the result to the right hand side of (25), we can find $(D(x))^{1/2}$ and then get

$$n_1^+(x,H) = \frac{1}{2R_1 r_H\sqrt{r_0}}\frac{1}{[(1-r_0)\sqrt{r_H} + (1-r_H)\sqrt{r_0}]}\frac{d}{dx}(r_0 r_H), \quad (26)$$

$$n_1^-(x,0) = \frac{1}{2R_1 r_0\sqrt{r_H}}\frac{1}{[(1-r_0)\sqrt{r_H} + (1-r_H)\sqrt{r_0}]}\frac{d}{dx}(r_0 r_H). \quad (27)$$

Thus, even though we did not determine the functions $n_1^+(x, y)$ and $n_1^-(x, y)$ completely, we have found boundary values $n_1^+(x, H)$ and $n_1^-(x, 0)$, which is quite sufficient, since Eqs. (26) and (27) contain all the information we need, allowing us to determine the fluxes through both mirrors of the resonator:

$$S^+(x) = a_1 c(1 - r_H - \delta_H)n_1^+(x,H),$$
$$S^-(x) = a_1 c(1 - r_0 - \delta_0)n_1^-(x,0). \quad (28)$$

We can also find the transformation efficiency (with respect to the photon flux)

$$\varkappa = \left\{\int_0^L S^+(x,H)dx + \int_0^L S^-(x,0)dx\right\}\Big/\int_0^H S(0,y)dy. \quad (29)$$

We return to Eqs. (26 and (27)). From these equations it is seen that, in complete correspondence with earlier assertions, $n_1^+(x, H)$ and $n_1^-(x, 0)$ are nonzero only when

$$\frac{d}{dx}(r_0 r_H) \neq 0, \quad (30)$$

and the dependence of $n_1^+(x, H)$ and $n_1^-(x, 0)$ on x is completely determined by the dependence of r_0 and r_H on x. In principle this opens the way to realizing a controlled regime. In particular, for definite values of dr_0/dx, dr_H/dx, and the other resonator parameters one can provide those values of the fluxes $S^+ \propto n_1^+$ and $S^- \propto n_1^-$ for which the phenomena of self-focusing, generation of the second Stokes and anti-Stokes components, etc., do not arise.

We make special note of the limitations associated with the necessity of satisfying condition (8) in the entire generation region. If the intensity of input beam $S(0, y)$ is such that

$$\int_0^H S(0,y)dy > -\frac{ca_1}{2R_1}\ln[r_0(0)r_H(0)], \quad (31)$$

where $r_0(0)$ and $r_H(0)$ are the values of the reflection indices r_0 and r_H at $x = 0$, then condition (24) is not certainly fulfilled and a stable generation regime is impossible. The growth of $n_1^+(0, y)$ and $n_1^-(0, y)$ is not limited by anything in the framework of the approximation considered.

And so it is necessary, in order that $\int_0^H S(0, y)\,dy$ be less than the right hand side of (31) and only for some point $x_1 > 0$, that we have the equality

$$\int_0^H S(0,y)\,dy = -\frac{ca_1}{2R_1}\ln[r_0(x_1)r_H(x_1)]. \qquad (32)$$

(Recall that the right hand side of (24) is a monotonically decreasing function of x.) Then generation develops only in the region $x > x_1$ and in this region the intensity of the primary laser beam satisfies (24). In the region $x < x_1$, $S(x, y) = S(0, y)$.

We consider as an illustration an actual case of a resonator in which the transmission coefficient of one of the mirrors (e.g., the one at $y = 0$) equals zero: $1 - r_0 - \delta_0 = 0$; $1 - r_0 = \delta_0$. We also take the absorption coefficients of both mirrors to be the same, sufficiently small, and independent of x: $\delta_H = \delta_0 = \delta \ll 1$. From (26) one easily obtains

$$S^+ = \frac{a_1 c}{2R_1}\sqrt{1-\delta}\,\frac{1-r_H-\delta}{r_H[\delta\sqrt{r_H}+(1-r_H)\sqrt{1-\delta}]}\,\frac{dr_H}{dx}, \quad x > x_1, \qquad (33)$$

$$\int_{x_1}^{L} S^+(x,H)\,dx = \frac{a_1 c}{2R_1}\left[(1-\delta)\ln\frac{r_H(L)}{r_H(x_1)} - 2\delta\ln\frac{1-\sqrt{1-\delta}\sqrt{r_H(x_1)}}{1-\sqrt{1-\delta}\sqrt{r_H(L)}}\right] \qquad (34)$$

In (34) one can set $1 - \delta \approx 1$. From (29), (32), and (34) we find the transformation efficiency

$$\varkappa \approx \left[\ln\frac{1}{r_H(x_1)}\right]^{-1}\left\{\ln\frac{r_H(L)}{r_H(x_1)} - 2\delta\ln\frac{1-\sqrt{r_H(x_1)}}{1-\sqrt{r_H(L)}}\right\}. \qquad (35)$$

From this expression one sees that κ can be extremely large. For example, with $\delta = 0.03$, $r_H(x_1) = 0.1$, and $r_H(L) = 0.9$, we have $\kappa \approx 0.85$. Averaging (over x), the density of the exit flux S^+ is obviously equal to

$$\overline{S^+(x,H)} = \frac{\varkappa}{l}\int_0^H S(0,y)\,dy = \varkappa\overline{S(0,y)}\,\frac{H}{l}, \qquad (36)$$

where $l = L - x_1$ is the width of the generation region. Since the quantity l is determined by resonator design, i.e., the length in which the necessary change in reflection coefficient $r_H(x)$ is provided, the ratio of the densities of the output $\overline{S}^+$ and incident $\overline{S}$ fluxes can in principle be made greater than unity. Actually the increase in the flux density S^+ is limited, as has already been mentioned, by various non-linear losses—principally by the development of generation at the second Stokes component. In order that generation at the second Stokes component should not develop, one can introduce into the resonator a substance that has large absorption at frequency ω_2. It can be shown that it is sufficient that the absorption coefficient $k(\omega)$ (in cm^{-1}) satisfy the inequalities

$$k(\omega_1) \ll \frac{1}{2H}\ln\left(\frac{1}{r_H}\right),$$
$$k(\omega_2) > \frac{R_2\varkappa S_0}{(L-x_1)(1-r_H)a_2 c} + \frac{1}{2H}\ln r_H. \qquad (37)$$

The above examination shows that SRS in a resonator with transverse pumping has a number of specific features.

Let us now briefly consider SRS in a resonator with longitudinal pumping. In this case the x axis coincides with the resonator axis and the axis of the primary laser beam. The system of equations analogous to (1) takes the form:

$$\frac{1}{c}\frac{\partial S}{\partial t} + \frac{\partial S}{\partial x} = -SN\sigma_1 - SN\sigma_1\frac{\Delta O_1}{4\pi}(n_1^+ + n_1^-),$$
$$a_1\frac{\partial n_1^+}{\partial t} + ca_1\frac{\partial n_1^+}{\partial x} = SN\sigma_1\frac{\Delta O_1}{4\pi}(1+n_1^+),$$
$$a_1\frac{\partial n_1^-}{\partial t} - ca_1\frac{\partial n_1^-}{\partial x} = SN\sigma_1\frac{\Delta O_1}{4\pi}(1+n_1^-), \qquad (38)$$

where n_1^+ and n_1^- are the densities of the generated photons in the oscillator field for directions of the wave vector respectively along and opposite to the x axis. System (38) differs from (1) only by the replacement of $\partial n_1^\pm/\partial y$ by $\partial n_1^\pm/\partial x$. Solutions of system (38) must satisfy the boundary conditions

$$S(0,t) = S_0,\quad n_1^-(H) = r_H n_1^+(H),\quad n_1^+(0) = r_0 n_1^-(0). \qquad (39)$$

We divide the second and third equations of (39) respectively by $(1 + n_1^+)$ and $(1 + n_1^-)$, combine them, and then integrate over x between the limits $x = 0$ and $x = H$. Neglecting the 1 compared to n_1^+ and n_1^-, and using (39), we find:

$$a_1\frac{\partial}{\partial t}\int_0^H \ln(n_1^+ n_1^-)\,dx + ca_1\ln\frac{1}{r_0 r_H} = 2R_1\int_0^H S(x)\,dx. \qquad (40)$$

From the first equation of (38) it is seen that the function $S(x, t)$, and along with it also the right hand side of (40), are monotonically decreasing functions of time. Hence, as is seen from (40), the total number of photons in the resonator increases so long as the equality

$$2R_1\int_0^H S(x)\,dx = ca_1\ln\frac{1}{r_0 r_H} \qquad (41)$$

is not established.

Thus, unlike the case of transverse pumping, in the case of longitudinal pumping a stationary regime is established at any value of the beam intensity of the pump S_0 and any mirror reflection coefficients r_0 and r_H.

Consider the stationary regime. Setting the time derivatives equal to zero and ignoring terms responsible for "spontaneous" scattering, we obtain

$$\frac{dS}{dx} = -SR_1(n_1^+ + n_1^-),\quad ca_1\frac{dn_1^+}{dx} = SR_1 n_1^+,\quad ca_1\frac{dn_1^-}{dx} = -SR_1 n_1^-. \qquad (42)$$

From the last two equations of system (42) we find

$$n_1^+(x)\,n_1^-(x) = A, \qquad (43)$$

where A is an arbitrary constant. Using (43), we get from (39) easily

$$n_1^+(0) = \sqrt{Ar_0},\quad n_1^+(H) = \sqrt{A/r_H}. \qquad (44)$$

From system (42) follows the flux conservation condition

$$\frac{dS}{dx} + ca_1\frac{dn_1^+}{dx} - ca_1\frac{dn_1^-}{dx} = 0. \qquad (45)$$

Integrating (45) between limits 0 to x and using the first equation of (42), as well as (43), (44), and (45), we obtain an equation for the function $n_1^+(x)$:

$$\frac{dn_1^+}{dx} = -R_1(n_1^+)^2 + R_1A + \frac{\sqrt{Ar_0}R_1S_0/a_1c - AR_1(1-r_0)}{\sqrt{Ar_0}}n_1^+, \quad (46)$$

which has to be solved with boundary conditions (44). Finding the general integral of Eq. (46) does not present difficulties:

$$n_1^+(x) = (\sqrt{A+p^2} - p)\frac{1 + BA^{-1}(\sqrt{A+p^2}+p)^2\exp[-2R_1\sqrt{A+p^2}x]}{1 - B\exp[-2R_1\sqrt{A+p^2}x]}$$

$$p = \frac{1}{2}\left\{\sqrt{A}\frac{1-r_0}{\sqrt{r_0}} - \varepsilon\right\}, \quad \varepsilon = \frac{S_0}{a_1c}. \quad (47)$$

The constants A and B are determined from the boundary conditions (44). In the case of large H, when terms containing exponentials in the right hand side of (47) can be neglected when $x = H$, it is easy to find an explicit form for the constants A and B, and

$$n_1^+(x) = \varepsilon\frac{\sqrt{r_0}}{(1-r_H)\sqrt{r_0} + (1-r_0)\sqrt{r_H}}$$
$$\cdot\left\{1 - r_H\frac{1-\sqrt{r_0r_H}}{r_H+\sqrt{r_0r_H}}\exp\left[-\varepsilon R_1\frac{\sqrt{r_0}(1+r_H)}{(1-r_H)\sqrt{r_0}+(1-r_0)\sqrt{r_H}}x\right]\right\}$$
$$\cdot\left\{1 + \frac{1-\sqrt{r_0r_H}}{r_H+\sqrt{r_0r_H}}\exp\left[-\varepsilon R_1\frac{\sqrt{r_0}(1+r_H)}{(1-r_H)\sqrt{r_0}+(1-r_0)\sqrt{r_H}}x\right]\right\}^{-1}. \quad (48)$$

It is seen from (48) that for the validity of the approximation made here it is necessary that the condition

$$\frac{1-\sqrt{r_0r_H}}{r_H+\sqrt{r_0r_H}}\exp\left[-\varepsilon R_1\frac{\sqrt{r_0}(1+r_H)}{(1-r_H)\sqrt{r_0}+(1-r_0)\sqrt{r_H}}H\right] \ll 1. \quad (49)$$

be fulfilled. In fact, fulfillment of this condition implies that the transformation efficiency is close to unity.

In writing Eq. (38) we have assumed that generation of the second Stokes component is not developed. It is obvious that development of the second Stokes component decreases the number of photons $n_1^{\pm}(x)$, which, as is seen from the first equation of (38), leads to decrease in absorption of the pump beam $S(x)$, and consequently also in the transformation efficiency.

Thus, to attain a high transformation efficiency it is necessary to provide fulfillment of condition (49) and the absence of generation at the frequency of the second Stokes component. Numerical estimates show that these requirements are inconsistent. Hence it is of interest to find out what happens when a filter is introduced into the resonator which absorbs radiation at the frequency of the second Stokes component. The transmission coefficient of this filter is symbolized by $d_f(\omega_2)$ ($d_f(\omega_1) = 1$). The role of pump beam for the second Stokes component is played by the sum $a_1c\,[n_1^+(x) + n_1^-(x)]$. If the pumping does not exceed the threshold value

$$2R_2a_1c\int_0^H[n_1^+(x) + n_1^-(x)]\,dx < \ln\frac{1}{r_0r_Hd_f^2}, \quad (50)$$

generation of the second Stokes component does not develop. Using (48) and (43), we get from (50)

$$2\frac{a_1}{a_2}R_2\varepsilon < \frac{(1-r_H)\sqrt{r_0}+(1-r_0)\sqrt{r_H}}{\sqrt{r_0}(1+r_H)}\ln\left\{\left[1 + \frac{1-r_H}{\sqrt{r_H}}\frac{1-\sqrt{r_0r_H}}{r_H+\sqrt{r_0r_H}} - \left(\frac{1-\sqrt{r_0r_H}}{r_H+\sqrt{r_0r_H}}\right)^2\right]\Big/r_0r_H\,d_f^2\right\}. \quad (51)$$

When $d_f(\omega_2) = 0.1$, conditions (49) and (51) are fulfilled simultaneously over a wide range of variations of all initial parameters. It is obvious that in the case of longitudinal pumping the flux density of the transformed radiation does not exceed the flux density of the pump S_0.

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