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EFFECT OF DISLOCATIONS ON THE LINE WIDTH OF UNIFORM FERRO- AND ANTIFERROMAGNETIC RESONANCES

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The Hamiltonian for interaction of spin waves with dislocations in a magnetically ordered crystal is found, and the contribution by processes of scattering of spin waves by dislocations to the line widths γ_{0j} of uniform ferro- and antiferromagnetic resonances is calculated. The dependence of the resonance line widths on the concentration and dimensions of the dislocations is considered. It is shown that if the dislocation concentration ξ is small, then $\gamma_{0j} \propto \xi$; at high dislocation concentrations, $\gamma_{0j} \propto \xi^{-1/2}$. The influence of the specimen shape on the width γ_{0j} is investigated.

INTRODUCTION

A great amount of research has been devoted to the investigation of the line width of ferro- and antiferromagnetic resonances. At present there are many well-studied mechanisms that lead to a finite line width in ferromagnetic resonance (FMR) and antiferromagnetic resonance (AFMR). In particular, there has been a detailed investigation of the temperature dependence of the FMR line width produced by the processes of scattering of spin waves by one another and of spin waves by phonons and conduction electrons. The line width due to these processes approaches zero when the temperature of the body approaches zero.

If there are defects (point, linear, or two-dimensional) in the crystal, then scattering of spin waves by these defects leads to a line width in FMR (AFMR) that is independent of the temperature. These scattering processes can play a decisive role in real crystals at a sufficiently low temperature. Scattering of spin waves from roughnesses of the crystal surface was considered by Clogston et al.[1]

The present work is devoted to the investigation of the contribution to FMR and AFMR line width by processes of scattering of spin waves by linear defects, and specifically on dislocations in the crystal. The dependence of the FMR and AFMR line width on dislocation concentration in the body and on specimen shape is found.

If the FMR frequency is not too close to the minimum frequency of Walker oscillations, and if the characteristic dimensions R of the dislocations (we suppose that the distance between dislocations is also of order R) are not too small, so that the condition $R \gg a(\Theta_C/\mu_B M_0)^{1/2}$ is satisfied, then the FMR line width γ_0 is determined in order of magnitude by the formula $\gamma_0 \approx \omega_0(a^2\xi)$, where ω_0 is the FMR frequency, ξ is the dislocation concentration, a is the lattice constant, Θ_C is the exchange constant (equal in order of magnitude to the Curie temperature), μ_B is the Bohr magneton, and M_0 is the magnetic moment of unit volume. If there are in the body finer dislocations with characteristic distances $R \ll a(\Theta_C/\mu_B M_0)^{1/2}$, then the line width γ_0 approaches zero with increase of the dislocation concentration ξ according to the law $\gamma_0 \propto 1/\sqrt{\xi}$. This is due to the fact that the amplitude of scattering on small-scale dislocations, as on point defects, decreases rapidly with diminution of the wave vector of the spin wave.

The shape of the body, or more accurately the relative sizes of the demagnetizing factors, can exert an appreciable influence on the dependence of FMR line width on dislocation concentration. Thus, for example, if the specimen has the form of a plane-parallel plate and if the characteristic dimensions of the dislocations, R, satisfy the condition $R \gg a(\Theta_C/\mu_B M_0)^{1/2}$, then the FMR line width is inversely proportional to the square root of the dislocation concentration, $\gamma_0 \sim \omega_0(\mu_B M_0/\Theta_C)^{3/2} \times (\theta^3/a\sqrt{\xi})$, in the case in which the external magnetic field H_0 is oriented almost perpendicular to the surface of the plate (θ is the angle between the normal to the plate and the external magnetic field), and $\gamma_0 \sim \omega_0(\mu_B M_0/\Theta_C)^{1/2}\, a\sqrt{\xi}$ when H_0 is oriented along the surface of the plate. In the rest of the angular range, $\gamma_0 \approx \omega_0 a^2\xi$. Investigation of the angular dependence of the size of γ_0 gives, in principle, still another possibility for the experimental determination of the characteristic dimensions of dislocations in a body.

In this paper it is also shown that γ_1, the half-width of the AFMR line corresponding to the higher frequency, is practically independent of the size of the magnetic field H_0, whereas γ_2, the half-width of the AFMR line corresponding to the lower of the frequencies, approaches zero both in the weak-field region and in the field region close to the turning-over field. The magnitude of γ_2 is proportional to the square root of the dislocation concentration ξ; the dependence of the magnitude of γ_1 is described by two terms, proportional to ξ and to $\sqrt{\xi}$.

The dependence of the half-widths of the AFMR lines, γ_{0j}, on specimen shape has been investigated.

1. HAMILTONIAN FOR THE INTERACTION OF SPIN WAVES WITH DISLOCATIONS

In order to determine the Hamiltonian that describes the interaction of spin waves with dislocations, we shall start from the general expression for the energy of a ferromagnet. This energy, as is well known, depends on the magnetic-moment density and the deformations in the body; the magnetization and the displacements occur in the energy expression in the form of combinations that are invariant with respect to rotations and translations of the body as a whole.[2] We have

$$\mathscr{H} = \int \left\{ \rho F(K_i, K_{il}, I_{lm}) + \frac{h^2}{8\pi} \right\} dV, \tag{1.1}$$

where

$$K_i = \mu_l \frac{\partial x_l}{\partial \eta_i}, \quad K_{il} \equiv \frac{\partial \mu_s}{\partial \eta_i} \frac{\partial x_s}{\partial \eta_l}, \quad I_{lm} = \frac{\partial x_s}{\partial \eta_l} \frac{\partial x_s}{\partial \eta_m},$$

ρ is the density of the material, **h** is the alternating magnetic field, μ is the magnetic moment of unit mass, and x_i and η_i are the Eulerian and Lagrangian coordinates (we recall that $x_i = \eta_i + u_i(\eta)$, where **u** is the displacement vector).

We shall designate by μ_0 the equilibrium value of the magnetic moment, and we shall suppose that the Lagrangian coordinates η_i define the coordinates of the points of the body in the equilibrium state. The small deviations of the magnetic moment from the equilibrium value we shall designate by **m**: $\mathbf{m} = \mu - \mu_0$; the small deformation fields in the body we shall describe by the distortion tensor $\partial u_i/\partial x_k$. For later work it is convenient to expand the function F, which occurs in formula (1.1), in a series in the quantities m_i, $\partial m_i/\partial x_k$, and $\partial u_l/\partial x_k$:

$$F = F_0 + F_2 + F_3 + \ldots,$$

$$\begin{aligned}
F_2 = {} & \frac{1}{2}\left(\frac{\partial^2 F}{\partial K_i \partial K_k}\right)_0 \left(m_i + \mu_{0l}\frac{\partial u_l}{\partial x_i}\right)\left(m_k + \mu_{0s}\frac{\partial u_s}{\partial x_k}\right) \\
& + \frac{1}{2}\left(\frac{\partial^2 F}{\partial K_{ij}\partial K_{lm}}\right)_0 \frac{\partial m_i}{\partial x_j}\frac{\partial m_l}{\partial x_m} + \frac{1}{2}\left(\frac{\partial^2 F}{\partial I_{ij}\partial K_l}\right)_0 \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left(m_l + \mu_{0s}\frac{\partial u_s}{\partial x_i}\right) \\
& + \frac{1}{2}\left(\frac{\partial^2 F}{\partial I_{ij}\partial I_{lm}}\right)_0 \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right), \\
F_3 = {} & \left(\frac{\partial^2 F}{\partial K_i \partial K_l}\right)_0 m_i m_s \frac{\partial u_s}{\partial x_l} + \left(\frac{\partial^2 F}{\partial K_{ij}\partial K_{lm}}\right)_0 \frac{\partial m_i}{\partial x_j}\frac{\partial m_s}{\partial x_l}\frac{\partial u_s}{\partial x_m} \\
& + \frac{1}{2}\left(\frac{\partial^3 F}{\partial I_{ij}\partial K_k \partial K_l}\right)_0 \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) m_k m_l \\
& + \frac{1}{2}\left(\frac{\partial^3 F}{\partial K_{ij}\partial K_{lm}\partial I_{ns}}\right)_0 \left(\frac{\partial u_n}{\partial x_s} + \frac{\partial u_s}{\partial x_n}\right)\frac{\partial m_i}{\partial x_j}\frac{\partial m_l}{\partial x_m} \\
& + \frac{1}{2}\left(\frac{\partial^2 F}{\partial K_{ij}\partial K_{lm}\partial K_n}\right)_0 \frac{\partial m_i}{\partial x_j}\frac{\partial m_l}{\partial x_m}\left(m_n + \mu_{0s}\frac{\partial u_s}{\partial x_n}\right) \\
& + \frac{1}{3!}\left(\frac{\partial^3 F}{\partial K_i \partial K_j \partial K_l}\right)_0 \left(m_i m_j m_l + 3 m_i m_j \mu_{0s}\frac{\partial u_s}{\partial x_l}\right) \\
& + \frac{1}{3!}\left(\frac{\partial^3 F}{\partial K_{ij}\partial K_{lm}\partial K_{sp}}\right)_0 \frac{\partial m_i}{\partial x_j}\frac{\partial m_l}{\partial x_m}\frac{\partial m_s}{\partial x_p} \quad \ldots
\end{aligned} \tag{1.2}$$

The index zero in these formulas designates F and the derivatives of F in the equilibrium state.

Since we shall be interested hereafter only in the interaction of spin waves with dislocations, we have, in the expression for F_3, limited ourselves to terms linear in the distortion tensor $\partial u_i/\partial x_k$ and have omitted terms of higher order in $\partial u_i/\partial u_k$. We have further supposed that the crystal possesses a center of inversion, and therefore the series for F contains no terms of the type $m_l \,\partial m_i/\partial x_k$. The absence of linear terms in the expansion (1.2) is due to the fact that the Hamiltonian (1.1) has a minimum for $m_i = \partial u_i/\partial x_k = 0$.

Presence of dislocations in the body leads to deformations of it, and these in turn lead to a nonuniform deviation $\mathbf{m}_d$ of the magnetic moment from its equilibrium value. In order to determine these deformations and the magnetic-moment deviations caused by them, it is necessary to solve simultaneously the equations of elasticity and the equations that determine the magnetization. Since a given magnetization deviation produces in a ferromagnet, thanks to magnetostriction, deformations of the order of [2],

$$\frac{\partial u_i}{\partial x_k} \sim \frac{m}{\mu_0}\frac{M_0^2}{\rho s^2} \approx 10^{-4} \text{ to } 10^{-6}\frac{m}{\mu_0},$$

where $M_0 = \rho_0 \mu_0$ is the magnetic moment of unit volume and s is the speed of sound, while given deformations lead to a magnetic-moment deviation $m/\mu_0 \sim \partial u_i/\partial x_k$, we may in the solution of the equations of elasticity neglect the magnetic properties of the medium; and we may, in the determination of the magnetic-moment deviation $\mathbf{m}_d$ caused by the dislocations, consider the tensor $\partial u_i/\partial u_k$ given. The magnetization $\mathbf{m}_d$ and the magnetic field $\mathbf{h}_d$ corresponding to it are determined by the equations

$$\begin{gathered}
\operatorname{rot} \mathbf{h}_d = 0, \qquad \operatorname{div}(\mathbf{h}_d + 4\pi\rho_1\mu_0 + 4\pi\rho_0\mathbf{m}_d) = 0, \\
\mathbf{H}_{eff} = 0, \qquad \rho_1 = -\rho_0 \partial u_i / \partial x_i,
\end{gathered} \tag{1.3}$$

where $\mathbf{H}_{eff}$ is the effective magnetic field acting on the magnetic moment of unit mass μ:[2]

$$\mathbf{H}_{eff} = \mathbf{h} - \frac{\partial F}{\partial \mu} + \frac{1}{\rho}\frac{\partial}{\partial x_k}\rho\frac{\partial F}{\partial(\partial \mu/\partial x_k)}.$$

We shall suppose that the crystal is uniaxial. By use of formula (1.2), one can obtain the following expression for $\mathbf{H}_{eff}$:

$$\begin{aligned}
\mathbf{H}_{eff} = {} & \mathbf{h}_d - \rho_0\beta\mathbf{m}_d + \rho_0\alpha\Delta\mathbf{m}_d - \rho_0\mu_0[(f+\beta)\nabla(\mathbf{nu}) + f(\mathbf{n}\nabla)\mathbf{u}] \\
& - \mathbf{n}\rho_0[b(\mathbf{nm}_d) + c(\nabla\mathbf{u}) + (b + \mu_0^2 d)(\mathbf{n}\nabla)(\mathbf{nu})],
\end{aligned} \tag{1.4}$$

where α is the exchange constant, β is the magnetic-anisotropy constant, f, c, and d are magnetostriction constants, and **n** is a unit vector along the anisotropy axis. They are defined in accordance with the formulas

$$\left(\frac{\partial^2 F}{\partial K_i\, \partial K_j}\right)_0 = \rho_0^2[\beta\delta_{ij} + b n_i n_j],$$

$$\left(\frac{\partial^2 F}{\partial I_{lm}\, \partial K_s}\right)_0 = \rho_0^2[f\mu_0(\delta_{ms}n_l + \delta_{ls}n_m) + c\mu_0\delta_{lm}n_s + d\mu_0^3 n_l n_m n_s].$$

On using expression (1.4) for $\mathbf{H}_{eff}$ and on going over to the Fourier components

$$\mathbf{m}_d(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} \mathbf{m}_d(\mathbf{k})e^{i\mathbf{kr}},$$

we get from (1.3):

$$\begin{aligned}
(m_d)_x = {} & \mu_0\frac{gM_0}{\varepsilon_k^2}\Big\{[[\varepsilon_k^2 + (2\pi g M_0 \sin^2\theta)^2]^{1/2} - 2\pi g M_0 \sin^2\theta] \cdot \\
& \cdot [\beta\omega_{yz} - (\beta + 2f)u_{yz} - 2\pi \sin 2\theta \sin\psi u_{ll}] - \frac{2\pi g M_0}{\varepsilon_k} f\omega_{xz}\sin 2\theta\cos\psi\Big\}, \\
(m_d)_y = {} & \mu_0\frac{gM_0}{\varepsilon_k^2}\Big\{[[\varepsilon_k^2 + (2\pi g M_0 \sin^2\theta)^2]^{1/2} - 2\pi g M_0 \sin^2\theta] \cdot \\
& \cdot [\beta\omega_{xz} - (\beta + 2f)u_{xz} - 2\pi \sin 2\theta \cos\psi u_{ll}] + \frac{2\pi g M_0}{\varepsilon_k} f\omega_{yz}\sin 2\theta\sin\psi\Big\}, \\
\mathbf{h}_d = {} & -4\pi\rho_0\mathbf{k}(\mathbf{km}_d - i\mu_0 u_{ll})k^{-2}, \qquad u_{lk} = i(u_l k_k + u_k k_l), \\
& \omega_{lk} = i(u_l k_k - u_k k_l),
\end{aligned} \tag{1.5}$$

where $\varepsilon_k = gM_0[(\alpha k^2 + \beta)(\alpha k^2 + \beta + 4\pi \sin^2\theta)]^{1/2}$, the z axis is directed along **n**, θ and ψ are the polar and azimuthal angles of the wave vector **k**, and g is the gyromagnetic ratio. These formulas determine the nonuniform magnetization and the magnetic field that arise because of dislocations.

We further represent the deviation of the magnetic moment from its equilibrium value μ_0 in the form

$$\mathbf{m} = \mathbf{m}_d + \mathbf{m}_s, \tag{1.6}$$

where $\mathbf{m}_s$ is the deviation caused by spin waves and connected with the Holstein-Primakoff operators by the formulas[3]

$$m_s^-(\mathbf{k}) = m_s^x(\mathbf{k}) - im_s^y(\mathbf{k}) \approx \frac{\sqrt{2\mu_B M_0}}{\rho_0} a_{-k}^+,$$

$$m_s^+(\mathbf{k}) = m_s^x(\mathbf{k}) + im_s^y(\mathbf{k}) \approx \frac{\sqrt{2\mu_B M_0}}{\rho_0} a_k,$$

$$m_s^z(\mathbf{k}) = -\frac{\mu_B}{\rho_0}\sum_{k_1k_2} a_{\mathbf{k}_1}^+ a_{\mathbf{k}_2} \Delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}). \tag{1.7}$$

We get the Hamiltonian for interaction of spin waves with dislocations if we substitute (1.6) into formulas (1.2) and pick out the terms linear in $\partial u_i/\partial x_k$, $\mathbf{m}_d$, and $\mathbf{h}_d$ and quadratic in $\mathbf{m}_s$:

$$\begin{aligned}\mathcal{H}_{sd} = \int \Big\{ & m_z M_0[(c - a_1)u_{ii} + (f + d + \beta + b - a_3)u_{zz}] \\ & + \rho_0\left(\beta + \frac{a_2}{2}\right) m_i m_k u_{ik} + \frac{\rho_0}{2}\frac{\partial m_i}{\partial x_k}\frac{\partial m_i}{\partial x_k}(\alpha_1 u_{ll} + \alpha_2 u_{zz})\Big\}\rho dV \\ & + \int \left[\frac{1}{4\pi}\mathbf{h}_s\mathbf{h}_d + \frac{1}{8\pi}\frac{\rho}{\rho_0}\mathbf{h}_s^2\right] dV,\end{aligned} \tag{1.8}$$

where the quantities a_i and α are connected with the values of derivatives of the function F:

$$\begin{aligned}\left(\frac{\partial^3 F}{\partial I_{ik}\partial K_l \partial K_m}\right)_0 = \frac{\rho_0}{2}\Big[& a_1\delta_{ik}\delta_{lm} + \frac{1}{2}a_2(\delta_{il}\delta_{km} + \delta_{im}\delta_{kl}) \\ & + a_3\delta_{lm}n_i n_k + a_4\delta_{ik}n_l n_m + a_5(\delta_{il}n_k n_m + \delta_{im}n_k n_l \\ & + \delta_{kl}n_i n_m + \delta_{km}n_i n_l) + a_6 n_i n_k n_l n_m\Big],\end{aligned}$$

$$\left(\frac{\partial^3 F}{\partial K_{ij}\partial K_{kl}\partial I_{mn}}\right)_0 = \frac{1}{2}\rho_0\alpha_1\delta_{mn}\delta_{ik}\delta_{lj} - \alpha\rho_0\delta_{im}\delta_{kn}\delta_{jl},$$

$$\left(\frac{\partial^3 F}{\partial K_{ij}\partial K_{kl}\partial K_m}\right)_0 = \frac{\rho_0}{\mu_0}\alpha_2\delta_{ik}\delta_{jl}n_m.$$

The constants a_i have the meaning of constants of the relativistic magnetostrictive interaction, the constant α of exchange magnetostrictive interaction. The appearance in the expression for $(\partial^3 F/\partial K_{ij}\partial K_{kl}\partial I_{mn})_0$ of the second term, with a constant α that coincides with the exchange constant in the expansion of $(\partial^2 F/\partial K_{ij}\partial K_{kl})_0$, is due to the fact that the exchange energy, because of its invariance with respect to rotations in the spin system, depends not on the invariants K_{ij}, K_i, and I_{lm} separately, but on their combinations $K_{ij}I_{jl}^{-1}K_{ls}$ and $K_i I_{ij}^{-1}K_l$, which are invariant with respect to rotations of the spins (that is, of μ).

On going over to Fourier components in the expression (1.8) and on using formulas (1.5) and (1.7), we get

$$\mathcal{H}_{sd} = \sum_{1,2}\Phi(1,2)a_1^+ a_2 + \mathrm{h.c.} \tag{1.9}$$

where the scattering amplitude of the spin waves is[1)]

$$\begin{aligned}\Phi(1,2) = gM_0\{ & \alpha_1 k_1 k_2 u_{ll}(\mathbf{q}) + \beta[u_{ll}(\mathbf{q}) - u_{zz}(\mathbf{q})] + (a_1 - c)u_{ll}(\mathbf{q}) \\ & - (f + d + \beta + b - a_3)u_{zz}(\mathbf{q}) - 4\pi q_z q^{-2}(\mathbf{q}, \mathbf{m}_d - \mu_0 u_{ll}(\mathbf{q}))\};\end{aligned}$$

$\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$. The Fourier components of the deformation tensor determined by the dislocations,[5] which occur in this formula, are

$$u_{ij}(\mathbf{q}) = -\frac{i}{qV}\varphi_{iklm}(\mathbf{q}^0)\sum_\nu b_l^{(\nu)}T_m^{(\nu)}(\mathbf{q})e^{-i\mathbf{q}\mathbf{r}^{(\nu)}},$$

$$\begin{aligned}\varphi_{iklm}(\mathbf{q}^0) = {} & 2(\zeta^2 - 1)q_i^0 q_k^0 q_n^0 \varepsilon_{lmn} \\ & + {}^1/_2(\varepsilon_{iml}q_k^0 + \varepsilon_{kml}q_i^0 + \varepsilon_{kmn}q_n^0\delta_{il} + \varepsilon_{inm}q_n^0\delta_{kl}),\end{aligned}$$

$$T_m^{(\nu)}(\mathbf{q}) = \oint_{C_\nu} \tau_m e^{-i\mathbf{q}\mathbf{l}}\,dl,$$

where $\mathbf{b}^{(\nu)}$ is the Burgers vector of the νth dislocation, C_ν is the contour of the νth dislocation and τ is a unit vector tangent to the dislocation line, dl is an element of length along the dislocation line, $\mathbf{q}^0 = \mathbf{q}q^{-1}$, and $\zeta^2 = \eta/(\lambda + 2\eta)$ (λ and η are the Lamé coefficients).

It is convenient to represent the amplitude of scattering of spin waves on dislocations in the form

$$\Phi(1,2) = \frac{\mu_B M_0}{qV} bR\varphi_{nm}(\varkappa_1, \varkappa_2)\sum_\nu \frac{b_n^{(\nu)}}{b} t_m^{(\nu)}(\mathbf{q})\, t^{i\mathbf{q}\mathbf{r}^{(\nu)}}, \tag{1.10}$$

where b is the mean magnitude of the Burgers vector over the dislocations of the body, R is the mean dimensions of the dislocation, $\varphi_{nm}(\kappa_1, \kappa_2)$ is a certain function of the order of unity, dependent on the directions of the wave vectors κ_1 and κ_2, and

$$t_m^{(\nu)}(\mathbf{q}) = \frac{1}{R}T_m^{(\nu)}(\mathbf{q}) = \frac{1}{R}\oint_{C_\nu} \tau_m e^{-i\mathbf{q}\mathbf{l}}\,dl. \tag{1.11}$$

2. WIDTH OF THE FERROMAGNETIC RESONANCE LINE

Knowing the amplitude of scattering of spin waves on dislocations, we can find the collision integral for a spin wave with wave vector $\mathbf{k} = 0$:

$$\mathcal{L}_0\{n\} = 2\pi\sum_{\mathbf{k}}\overline{|\Phi(0,\mathbf{k})|^2}(n_k - n_0)\delta(\varepsilon_0 - \varepsilon_k), \tag{2.1}$$

where the bar denotes an average over the random distribution of dislocations in the body, and where n_k is the number of spin waves with wave vector $\mathbf{k}$. As in [4], we assume that the free path length of a quasiparticle is appreciably larger than the mean distance R between dislocations.

Hereafter we shall only estimate the value of the relaxation time τ_0 of spin waves with $\mathbf{k} = 0$ due to the scattering by the dislocations. In accordance with this, in the exact formula that determines τ_0 for $n_0 \gg n_k$,

$$\gamma_0 \equiv \frac{1}{\tau_0} = 2\pi\sum_{\mathbf{k}}\overline{|\Phi(0,\mathbf{k})|^2}\delta(\varepsilon_0 - \varepsilon_k),$$

we shall set $\varphi(\kappa_1, \kappa_2) = 1$. Then by use of formulas (1.10) and (1.11) we get

$$\gamma_0 = \pi(gM_0)^2\frac{(bR)^2}{V}\sum_\nu\frac{(b^{(\nu)}R^{(\nu)})^2}{(bR)^2 R^{(\nu)}}\int\frac{dk}{k^3}\int_0^{2kR^{(\nu)}} J_2(x)\delta(\varepsilon_0 - \varepsilon_k)\,dx, \tag{2.2}$$

where J_2 is the second-order Bessel function and $R^{(\nu)}$ is the radius of the νth dislocation, which for simplicity we consider circular.

Since, in general, $\varepsilon_k = \varepsilon_0$[1] when $k \sim a^{-1}(\mu_B M_0/\Theta_C)^{1/2}$ (a is the lattice constant), it follows from formula (2.2) that γ_0 is determined in order of magnitude as follows.

If $R \gg a(\Theta_C/\mu_B M_0)^{1/2}$, then

$$\gamma_0 \approx gM_0(b/R)^2 \approx gM_0 a^2\xi. \tag{2.3}$$

In obtaining this formula, we have taken into account that

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1)We note that the relativistic interaction of spin waves with dislocations was written down incompletely in [4], since the change of magnetization caused by the dislocations was not taken into account.

$$\int_0^\infty J_2(x)\,dx \sim 1, \quad \int \frac{dk}{k^3}\delta(\varepsilon_0-\varepsilon_k) \sim \frac{1}{\varepsilon_0},$$

and we have supposed that the sum over all dislocations $\Sigma_\nu[(b^{(\nu)}R^{(\nu)})^2/(bR)^2R^{(\nu)}]$ is approximately equal to $/R$, where is the number of dislocations in the body, equal in order of magnitude to V/R^3.

If, however, $R \ll a(\Theta_C/\mu_B M_0)^{1/2}$, then

$$\gamma_0 \approx gM_0(\mu_B M_0/\Theta_C)^{3/2}(a^2\xi)^{-1/2}. \tag{2.4}$$

In obtaining this formula, we have taken into account that

$$\int_0^{2kR^{(\nu)}} J_2(x)\,dx \sim (kR^{(\nu)})^3.$$

From (2.4) we see that at small dislocation dimensions the time of relaxation of spin waves on them diminishes. This is due to the fact that at small dislocation dimensions, when $kR \ll 1$, scattering on them becomes of "Rayleigh" type, and its cross section decreases with decrease of the dimension R of the scattering centers.

Thus the mean frequency for collisions of spin waves with $\mathbf{k} = 0$ with dislocations decreases both for small and for large dislocation dimensions R and attains its greatest value when $R \approx a(\Theta_C/\mu_B M_0)^{1/2}$:

$$(\gamma_0)_{max} \approx gM_0(\mu_B M_0/\Theta_C)^{1/2} \approx 10^8 \ \text{sec}^{-1}$$

The dislocation concentration in the body $\xi \propto R^{-2}$; in this case it is equal in order of magnitude to $(\mu M_0/\Theta_C)a^{-2} \approx 10^{12}\ \text{cm}^{-2}$, if we suppose that $M_0 \sim 10^3$G, $\Theta_C \approx 10^3\,^\circ$K, and the lattice constant $a \approx 10^{-8}$ cm. Such dislocation concentrations are often encountered; the estimates show that these can give an appreciable contribution to the line width and that it would be very desirable to observe the diminution of this "residual" collision frequency both on increase and on decrease of the number of dislocations in the body.

Formulas (2.3) and (2.4) were obtained on the assumption that the distance L between dislocations is of the same order as the dimensions R of the dislocations. If the dislocations in the body are distributed sparsely, so that $L \gg R$, then by use of formula (2.2) we can obtain the following expression for the quantity γ_0:

$$\gamma_0 \approx \begin{cases} gM_0\dfrac{b^2R}{L^3}, & R \gg a\left(\dfrac{\Theta_C}{\mu_B M_0}\right)^{1/2}, \\ gM_0\left(\dfrac{\mu_B M_0}{\Theta_C}\right)^{3/2}\dfrac{b^2R^4}{a^3L^3}, & R \ll a\left(\dfrac{\Theta_C}{\mu_B M_0}\right)^{1/2}. \end{cases} \tag{2.5}$$

We shall now consider, using the example of the plane-parallel plate, the dependence of the frequency γ_0 of collision of spin waves with dislocations on the demagnetizing factors.[2)] The frequency of uniform resonance in such a plate can be represented in the form

$$\varepsilon_0 = g[(H+\beta M_0+4\pi M_0\sin\vartheta)(H+\beta M_0)]^{1/2}, \tag{2.6}$$

where H is the magnetic field intensity in the body and β is the magnetic-anisotropy constant ($\sin\theta$ describes the orientation of the external magnetic field with respect to the surface of the plate).

By substituting the expression (2.6) for the frequency ε_0 into formula (2.2), we can find the following expression for the value of γ_0 when $(R/a)(\mu_B M_0/\Theta_C)^{1/2} \gg 1$:

$$\gamma_0 \approx \begin{cases} gM_0\left(\dfrac{\mu_B M_0}{\Theta_C}\right)^{3/2}\dfrac{\vartheta^3}{a\sqrt{\xi}}, & \vartheta\dfrac{R}{a}\left(\dfrac{\mu_B M_0}{\Theta_C}\right)^{1/2} \ll 1 \\ gM_0a^2\xi\ln\left[\dfrac{\sin\vartheta}{a\sqrt{\xi}}\left(\dfrac{\mu_B M_0}{\Theta_C}\right)^{1/2}\right], & \sin\vartheta \lessapprox 1. \\ gM_0a\sqrt{\xi}\left(\dfrac{\mu_B M_0}{\Theta_C}\right)^{1/2}, & \left|\dfrac{\pi}{2}-\vartheta\right| \ll \dfrac{a}{R}\left(\dfrac{\Theta_C}{\mu_B M_0}\right)^{1/2} \end{cases} \tag{2.7}$$

The first of these formulas describes the FMR line width in the case when the external magnetic field is perpendicular, the last when it is parallel, to the surface of the plate. The middle formula relates to "intermediate" angles. We see that with change of the inclination angle θ there is an appreciable change in the dependence of the FMR line width on the dislocation concentration ξ.

3. WIDTH OF THE ANTIFERROMAGNETIC RESONANCE LINE

We now consider scattering of spin waves by dislocations in antiferromagnets whose ground state is determined by two compensated sublattices. We consider only uniaxial antiferromagnets with magnetic anisotropy of the "easy axis" type. The Hamiltonian for interaction of spin waves with dislocations in such an antiferromagnet can be deduced in a manner similar to that in which we found the Hamiltonian for interaction of spin waves with dislocations in ferromagnets. The independent invariants from which the Hamiltonian of an antiferromagnet is constructed have the form[6]

$$\mu_{\alpha s}\frac{\partial x_s}{\partial \eta_l}, \quad \frac{\partial \mu_{\alpha s}}{\partial \eta_l}\frac{\partial x_s}{\partial \eta_l}, \quad \frac{\partial x_s}{\partial \eta_i}\frac{\partial x_s}{\partial \eta_l}, \quad \alpha = 1, 2.$$

We shall not present all the calculations here, but shall present only the final result for the Hamiltonian that describes the scattering of spin waves by dislocations:[3)]

$$\mathscr{H} = \sum_{\mathbf{k}j,\mathbf{k}'j'} \Psi_{jj'}(\mathbf{k},\mathbf{k}')a_{\mathbf{k}j}^+a_{\mathbf{k}'j'} + \mathbf{h.c.} \tag{3.1}$$

where

$$\Psi_{11}(\mathbf{k},\mathbf{k}') = \Psi_{22}(\mathbf{k},\mathbf{k}') = \Theta_N(M_0/H_{AE})(f_1u_{ll}+f_2u_{zz}),$$

$$\Psi_{12}(\mathbf{k},\mathbf{k}') = \Psi_{21}^*(\mathbf{k},\mathbf{k}') = \Theta_N\frac{M_0}{\sqrt{H_{AE}(H_{AE}+2H_0)}}$$

$$\times[f_3(u_{xx}-u_{yy}+2iu_{xy})+f_4(u_{xx}-u_{yy}-2iu_{xy})],$$

$$u_{ik} \equiv u_{ik}(\mathbf{q}) = i(u_iq_k+u_kq_i), \qquad \mathbf{q} = \mathbf{k}_1-\mathbf{k}_2,$$

M_0 is the magnetic moment of a sublattice, H_{AE} is the field that determines the activation of spin waves in antiferromagnets, Θ_N is the exchange constant, equal in order of magnitude to the Néel temperature, and the quantities f_j (of order unity) describe the magnetostriction in antiferromagnets.

We write the collision integral of spin waves with dislocations in the form

$$\mathscr{L}_{\mathbf{k}j} = 2\pi\sum_{\mathbf{k}'j'}\overline{|\Psi_{jj'}(\mathbf{k},\mathbf{k}')|^2}(n_{\mathbf{k}'j'}-n_{\mathbf{k}j})\delta(\varepsilon_j(\mathbf{k})-\varepsilon_{j'}(\mathbf{k}')), \tag{3.2}$$

whence we get the following expression for the AFMR line width:

$$\gamma_{j0} = 2\pi\sum_{\mathbf{k}'j'}\overline{|\Psi_{jj'}(0,\mathbf{k}')|^2}\delta(\varepsilon_j(0)-\varepsilon_{j'}(\mathbf{k}')). \tag{3.3}$$

In order to find the values of γ_{j0} in explicit form,

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2)In the case of a sphere or of a not too anisotropic ellipsoid, formulas (2.3) to (2.5) are valid but naturally differ by coefficients of order unity for specimens of different shape.

3)We note that in finding this Hamiltonian, we may disregard the magnetic field produced by the nonuniformity of the magnetic moments, since the static magnetic susceptibility of antiferromagnets is small.

besides knowing the spin-wave scattering amplitudes $\Psi_{jj'}(\mathbf{k}, \mathbf{k}')$, we must use the expressions for the frequencies of spin waves in antiferromagnets and for the AFMR frequencies. Since transition of a spin wave of type j, with wave vector $\mathbf{k} = 0$, into a spin wave of the same type, but with wave vector $\mathbf{k} \neq 0$, is possible only if, in the spin-wave spectrum, account is taken of the contribution from the magnetic dipole interaction, we shall quote here these expressions for the spin-wave frequencies[7] or for the AFMR frequencies:

$$\varepsilon_{1,2}(\mathbf{k}) = \omega(k)\left\{1 + \left(\frac{\omega_H}{\omega(k)}\right)^2 + 2\pi\chi_0\cos^2\theta \pm 2\left[\pi^2\chi_0^2\cos^4\theta + \left(\frac{\omega_H}{\omega(k)}\right)^2(1 + 2\pi\chi_0\cos^2\theta)\right]^{1/2}\right\}^{1/2},$$

$$\varepsilon_{1,2}(0) = \omega(0)\left\{1 + \left(\frac{\omega_H}{\omega(0)}\right)^2 + 2\pi\chi_0\cos^2\theta_0 \pm 2\left[\pi^2\chi_0^2\cos^4\theta_0 + \left(\frac{\omega_H}{\omega(0)}\right)^2(1 + 2\pi\chi_0\cos^2\theta_0)\right]^{1/2}\right\}^{1/2}.$$

where $\omega(0) = gH_{AE} = g\sqrt{H_A H_E}$, $\omega^2(k) = \omega^2(0) + \Theta_N^2(ak)^2$, $\omega_H = gH_0$ (θ_0 is the polar angle when $\mathbf{k} = 0$), and H_A and H_E are the anisotropy and exchange fields, respectively. The angle θ_0 serves to describe the demagnetizing factors ($\theta_0 = \pi/2$ if $\mathbf{H}_0$ is parallel to the surface of the plate, and $\theta_0 = 0$ if it is perpendicular to the surface of the plate).

By using these formulas and the expressions (3.1) for $\Psi_{jj'}(\mathbf{k}, \mathbf{k}')$ and by carrying out the integration in formula (3.3), we can obtain the following expressions for γ_{10} and γ_{20}:

$$\gamma_{10} \approx \frac{\Theta_N^2}{\mu_B H_{AE}} \frac{H_{AE} + H_0}{H_{AE}} a^2\xi \ln\left[\left(\frac{\mu_B M_0}{\Theta_N}\right)^2 \frac{1}{a^2\xi}\right],$$

$$\gamma_{20} \approx \frac{\Theta_N^2}{\mu_B H_{AE}} \frac{H_{AE} - H_0}{H_{AE}} a^2\xi \ln\left[\frac{\mu_B M_0}{\Theta_N} \frac{1}{a^2\xi}\right]. \tag{3.4}$$

These formulas are valid if $R \gg a(\Theta_N/\mu_B M_0)^{1/2}$, the external magnetic field is large enough so that $H_0 > \pi\chi_0 H_{AE}$, where χ_0 is the static magnetic susceptibility of the antiferromagnets, and $\cos\theta_0 \sim 1$. We note that these formulas determine, in order of magnitude, the AFMR line width also in the case of an ellipsoid.

If the magnetic field $\mathbf{H}_0$ is almost perpendicular to the surface of the plate, so that $\theta_0 R \ll a(\Theta_N/\mu_B M_0)^{1/2}$, then

$$\gamma_{10} \approx gM_0 \frac{\mu_B M_0}{\Theta_N} \sqrt{\chi_0} \frac{\theta_0}{a\sqrt{\xi}},$$

$$\gamma_{20} \approx gM_0 \frac{\mu_B M_0}{\Theta_N} \sqrt{\frac{\chi_0 H_{AE}}{H_{AE} - H_0}} \frac{\theta_0}{a\sqrt{\xi}}, \quad \frac{H_{AE} - H_0}{H_{AE}} \gg \frac{a}{R}. \tag{3.5}$$

On comparing these formulas with formulas (3.4), we see that the AFMR line width decreases abruptly in a narrow range of small angles.

We quote, finally, the formulas that determine the width γ_{j0} in the case in which the magnetic field is oriented along the surface of the plate:

$$\gamma_{10} \approx gM_0\left(\frac{\Theta_N}{\mu_B M_0}\right)^{1/2}\left(a^2\xi \frac{H_{AE} + H_0}{\beta^2 H_{AE}}\right)^{1/2} + \Theta_N a^2\xi \frac{H_{AE} M_0}{\beta H_0(H_0 + H_{AE})} \tag{3.6}$$

$$\gamma_{20} \approx gH_{AE}\left(a^2\xi \frac{H_{AE} - H_0}{\beta^3 H_{AE}}\right)^{1/2}.$$

From a comparison of these formulas with formulas (3.4) it is evident that the AFMR line widths may increase on change of the angle of inclination of the magnetic field from a value $\theta_0 \approx 1$ to $|\theta - \pi/2| \ll 1$.

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184