CALCULATION OF THE CRITICAL CURRENT OF EXTRANEOUS PARTICLES IN A BOSE SYSTEM

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We calculate the dependence of the current of extraneous particles interacting with a Bose system at absolute zero. The main mechanism of the dissipation is assumed to be the decay of the excitation into two. It is shown that the current has a power-law singularity in the case of weak fields, and can either increase or decrease with increasing field.

As is well known, extraneous particles can move in a superfluid liquid without experiencing any resistance if their velocity is lower than at certain critical values. At finite temperatures, this effect does not become manifest, owing to collisions with excitations in the Bose system. However, in the case of finite accelerating fields and very low temperatures, we have practically the case of zero temperature, since the particles reach critical velocity before they collide with the excitations. The well known experiments of Rayfield and Reif[1] with ions moving in a liquid He II were performed precisely under such conditions. The purpose of the present paper is to calculate the dependence of the particle current on the electric field under these conditions (at absolute zero). We note that the usual approximation, linear in the field, is not applicable here. In this paper we consider this problem at absolute zero.

The process limiting the current at low particle density is the decay of the excitation corresponding to the particle, which we shall call ionic excitation, into an ionic excitation with a smaller momentum and into an excitation in the Bose system, which we shall call arbitrarily a phonon. Such a situation can be realized, generally speaking, not only in He II, but in a semiconductor at sufficiently low carrier density and in the absence of impurities.

We consider here the general theory for a weak electric field, without describing the system concretely, since the answer contains only the properties of the spectrum of the ionic excitations $\epsilon(p)$ and the Bose-system excitations $\omega(q)$. From the point of view of real systems, such an approach is in part phenomenological, since we assume both spectra to be specified. In particular, there exists the possibility that the decay conditions are not satisfied for any momentum of the ionic excitation and the momentum of the ion in a constant homogeneous field **E** will increase without limit. We assume below that the decay is impossible when the ionic excitation momentum $p < p_c$ and is first realizable at $p = p_c$, and furthermore with emission of only one phonon.

The singularity of the single-particle Green's function at the decay point in the absence of an external field, and the classification of the different cases of decay, were clarified by Pitaevskiĭ[2]. We confine ourselves here to the case of decay into excitations with parallel momenta not equal to zero. The case of decay with emission of an acoustic phonon is less interesting from the theoretical point of view, owing to the weak singularity in the spectrum and, apparently, can be considered in accordance with perturbation theory. As to the decay into excitations with non-parallel momenta, in this case the momenta after the decay should correspond to the minima of the excitation energy. With the exception of the random coincidences, this case can be realized only for sufficiently complicated spectra of ionic excitations, having more than one extremal point $\partial\epsilon/\partial p = 0$, for in the opposite case the entire momentum of the ionic excitation should be carried away by the phonon (one energy minimum of the ionic excitations is known: $p = 0$). This leads to the very stringent condition $\omega(p_c) = \epsilon(p_c)$, without any free parameters at all.

An earlier paper by the author[3] considered the analog of the kinetic equation for particles with low concentration interacting with phonons. It was shown that in the stationary state in the absence of a field, it is possible to introduce the distribution function of the quasiparticles corresponding to the ionic excitations, from which it is easy to calculate the density and the current of the particles. Far from p_c, this function satisfies the usual kinetic equation. At absolute zero, however, even in the presence of an arbitrarily weak external field **E**, the vicinity of $p = p_c$ plays an essential role, in which connection the results of[3] have in the present case only limited applicability. To solve the problem we must consider the equations (2) and (3) of that paper, which are accurate within the limit of low ion density.

A similar situation was considered within the framework of the kinetic equation for semiconductor carriers that emit optical phonons without dispersion in a paper by Shockley[4]. Such an analysis is valid only in the case of a weak interaction and sufficiently strong fields, when it is possible to neglect the band in which the singularity obtained by Pitaevskiĭ[2] is significant.

1. SINGLE-PARTICLE GREEN'S FUNCTION IN A WEAK FIELD

Proceeding to solve the problem, we consider certain changes in the diagram technique in the momentum representation in the presence of a constant electric field. It is easy to show that in the interaction representation the ion creation and annihilation operators assume the form

$$a_{\mathbf{p}}(t) = a(\mathbf{p} - e\mathbf{E}t)\exp\left\{-i\int_0^t \varepsilon_0(\mathbf{p} - e\mathbf{E}\tau)\,d\tau\right\}, \quad (1.1)$$

$$a_{\mathbf{p}}^+(t) = a^+(\mathbf{p} - e\mathbf{E}t)\exp\left\{i\int_0^t \varepsilon_0(\mathbf{p} - e\mathbf{E}\tau)\,d\tau\right\},$$

where $a(\mathbf{p})$, $a^+(\mathbf{p})$—operators in the Schrödinger representation. It follows therefore that the momenta corresponding to the start and end of the ion line are different: $\mathbf{p}' - \mathbf{p} = e\mathbf{E}(t' - t)$. By virtue of the momentum conservation at the vertices, allowance for the interaction with the phonons does not change this circumstance.

We shall use a mixed representation, with a Fourier transformation with respect to the spatial variables but not with respect to time. In this case, two equivalent representations are possible for the Green's functions, in which the independent argument is either the momentum $\mathbf{p}$ of the end of the line, or the momentum $\mathbf{p}'$ of the start of the line and the time difference $t' - t$.

We are interested in calculating the ion current. The current is determined by the formula

$$\mathbf{J} = -\frac{i}{m}\nabla_{\mathbf{x}}G^+(\mathbf{x}, \mathbf{x}')\,|_{\mathbf{x}'=\mathbf{x}},$$

where $G^+(\mathbf{x}, \mathbf{x}') = \langle\psi^+(\mathbf{x}')\psi(\mathbf{x})\rangle$, and the averaging is carried out over the stationary state of the system containing one ion in an electric field (the case of low ion density). The equations for G^+ (see[3]) contain the single-particle Green's functions G and $\widetilde{G}$, which are determined independently of the G^+ and have a singularity connected with the decay point.

The determination of this singularity in a weak electric field is of interest in itself, and is in addition, a component part of the determination of the singularities of G^+. We shall consider in detail only the function

$$G(\mathbf{x}', \mathbf{x}) = -i\langle T\psi(\mathbf{x}')\psi^+(\mathbf{x})\rangle_{vac}, \quad (1.2)$$

Since

$$\widetilde{G} = -i\langle\widetilde{T}\psi(\mathbf{x}')\psi^+(\mathbf{x})\rangle_{vac} = -G^*(\mathbf{x}, \mathbf{x}') \quad (1.3)$$

($\widetilde{T}$—anti-T-product).

Going over to the Fourier transformation with respect to the spatial coordinates and retaining the time dependence (this form is the most convenient for an analysis of the singularity), we obtain the Dyson equation in differential form (see, for example,[5]):

$$i\frac{\partial G(\mathbf{p}, t' - t)}{\partial t'} - \varepsilon_0(\mathbf{p} + e\mathbf{E}(t' - t))G(\mathbf{p}, t' - t)$$
$$= \int_t^{t'} d\tau'\Sigma(\mathbf{p} + e\mathbf{E}(\tau' - t), t' - \tau')G(\mathbf{p}, \tau' - t) + \delta(t' - t), \quad (1.4)$$

where we choose as the independent argument the momentum of the end of the line p (corresponding to the instant of time t).

We are interested in the behavior of $G(\mathbf{p}, t' - t)$ at large $t' - t$ (at small $t' - t$ the influence of the field is exceedingly small owing to the smallness of $\mathbf{E}$, and the Green's function does not differ from the Green's function without the field). We shall call this part of G, which does not vanish at large $t' - t$, the pole part. The δ-function in the right side of (1.4) then drops out, and the lower limit in the integral can be set equal to $-\infty$, since only the region near the upper limit with $\tau' \sim t'$ is important, owing to the decrease of Σ.

As usual in the quasi-classical approach, we should take into account in (1.4) terms of order E. To this end we note that the running momentum in $\Sigma(\mathbf{p} + e\mathbf{E}(\tau' - t), t' - \tau')$ varies from $\mathbf{p} + e\mathbf{E}(\tau' - t)$ to $\mathbf{p} + e\mathbf{E}(t' - t)$ and it can be assumed, accurate to terms quadratic in E, that it remains unchanged, and is taken at the midpoint

$$\mathbf{p} + e\mathbf{E}\frac{t' + \tau' - 2t}{2}$$

We shall seek the solution for G in the form

$$G(\mathbf{p}, t' - t) = A(\mathbf{p}, t' - t)\exp\left\{-i\int_0^{t'-t}\varepsilon(\mathbf{p} + e\mathbf{E}\tau)\,d\tau\right\},$$

where A is a slowly varying function, such that its derivative is on the order of E. Then, expanding the integral term with accuracy to terms of first order in E, we can easily obtain the following equations:

$$\varepsilon(p') - \varepsilon_0(p') - \Sigma_F(\mathbf{p}', \varepsilon(p')) = 0, \quad \mathbf{p}' = \mathbf{p} + e\mathbf{E}(t' - t), \quad (1.5)$$

$$\frac{\partial\ln A}{\partial t'} = -\frac{1}{2}\frac{\partial}{\partial t'}\ln\left[1 - \frac{\partial\Sigma_F(\mathbf{p}', \varepsilon(p'))}{\partial\varepsilon}\right], \quad (1.6)$$

where $\Sigma_F(\mathbf{p}', \omega)$ is the Fourier transform of $\Sigma(\mathbf{p}', t')$ with respect to the variable t' at $E = 0$. It follows therefore that $\epsilon(\mathbf{p}')$ coincides with the pole of the Green's function $G(\mathbf{p}', \omega)$ without a pole. Equation (1.6) can be readily integrated, and the arbitrary constant is determined in this case from the condition that the solution must be continuous with the asymptotic form of G at times that are sufficiently large, but much shorter than 1/E. The final result takes the form

$$G(\mathbf{p}, t' - t) = (-i)\left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\mathbf{p}}^{-1/2}\left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\mathbf{p}'}^{-1/2}\exp\left[-i\int_{p_x}^{p_x'}\frac{\varepsilon(\eta, \mathbf{p}_\perp)}{eE}d\eta\right]$$
$$(\mathbf{p}_x \parallel \mathbf{E}, \quad \mathbf{p}_\perp \perp \mathbf{E}). \quad (1.7)$$

This asymptotic form is true when $t' - t < t_c$, where $t_c = (p_c - p)/eE$, for only in this case are the expansions in terms of the small field E, used to transform the integral (1.4) into the differential equation (1.6) for A, valid. Near $t' - t \approx t_c$ the quantity $\partial\Sigma_F/\partial\epsilon|_{p'}$ becomes infinite like $1/(p' - p_c)$ (see[2]), and consequently, the terms that were assumed to be small in the derivation of (1.6) are actually not small. Therefore formula (1.7), according to which $G \to 0$ as $p' \to p_c$, does not yield the correct behavior of G.

To determine G in the region $p' \sim p_c$, it is necessary to solve Eq. (1.4) directly; however, it is necessary to take into account in the integral term only the singular part of ($\partial\Sigma_{F,\,sing}/\partial\epsilon \sim 1/(p - p_c)$), whereas for the regular part the old differential approximation, which led to (1.5) and (1.6), remains valid. The singularity in the integral term, as shown by Pitaevskiĭ[2] is connected with the decay loop given by the expression

$$-ig^2\int_{-\infty}^{t'}d\tau'\int\frac{d^3q}{(2\pi)^3}\left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\mathbf{p}+e\mathbf{E}(\tau'-t)-\mathbf{q}}^{-1/2}\left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\mathbf{p}'-\mathbf{q}}^{-1/2}\exp\left\{-i\int_{\tau'}^{t'}[\varepsilon(\mathbf{p} - \mathbf{q} + e\mathbf{E}(\tau - t)) + \omega(q)]\,d\tau\right\} \quad (1.8)$$

(we note that in[2] and formula (3) the sign in front of the entire expression is incorrect). We have retained here only the pole part of G, and the phonons are regarded as an ideal gas. The singularity is connected with the fact that near $p' = p_c$ the effective region of integration with respect to τ becomes large because the oscillating exponential factors cancel each other out, so that

$$\omega(q) + \varepsilon(\mathbf{p}' - \mathbf{q}) \approx \varepsilon(p');$$

This occurs first near the minimum of the left side of $\tilde{\epsilon}$, when $\epsilon(p') \approx \tilde{\epsilon}(p')$, $p' \approx p_c$.

Expanding near the minimum, we get

$$\omega(q) + \varepsilon(\mathbf{p}' - \mathbf{q}) = \tilde{\varepsilon}(p') + M(\mathbf{q} - \mathbf{q}_m(p'))^2 + L\frac{(\mathbf{q} - \mathbf{q}_m(p'), \mathbf{p}_c)^2}{p^2} \tag{1.9}$$

where, according to[2],

$$M = \frac{v_c p_c}{2q_m p_m}, \qquad L = \frac{1}{2}\left\{\frac{\partial^2\varepsilon}{\partial p^2}\bigg|_{p_m} + \frac{\partial^2\omega}{\partial q^2}\bigg|_{q_m} - \frac{v_c p_c}{q_m p_m}\right\}$$

$$v_c = \frac{\partial\varepsilon}{\partial p}\bigg|_{p_c} = \frac{\partial\varepsilon}{\partial p}\bigg|_{p_m} = \frac{\partial\omega}{\partial q}\bigg|_{q_m}, \qquad q_m = q_m(p_c), \qquad p_m = p_c - q_m,$$

$$M > 0, \quad M + L > 0.$$

The inequalities follow from the fact that $\tilde{\epsilon}$ is a minimum of the right side of (1.9).

Near $p' = p_c$, using the condition at the minimum $\partial\epsilon(\mathbf{p}' - \mathbf{q})/\partial\mathbf{p} = \partial\omega/\partial\mathbf{q}$, we can obtain the relation

$$\mathbf{q}_m(p') - \mathbf{q}_m = \frac{q_m}{p_c}(\mathbf{p}' - \mathbf{p}_c) + \lambda\frac{(\mathbf{p}_c, \mathbf{p}' - \mathbf{p}_c)}{p_c^2}\mathbf{p}_c, \tag{1.10}$$

where

$$\lambda = \left[\frac{p_m}{p_c}\frac{\partial^2\varepsilon}{\partial p^2}\bigg|_{p_m} - \frac{q_m}{p_c}\frac{\partial^2\omega}{\partial q^2}\bigg|_{q_m}\right]\left[\frac{\partial^2\varepsilon}{\partial p^2}\bigg|_{p_m} + \frac{\partial^2\omega}{\partial q^2}\bigg|_{q_m}\right]^{-1}.$$

Thus, in considering the singular part of Σ we should consider only the small region of integration with respect to $\mathbf{q}$ near $\mathbf{q}_m$ and, in addition $\tau - t$ should be still sufficiently close to $t' - t \sim t_c$ (we shall show later that in the essential region of integration $\tau' - t - t_c \sim E^{-2/3}$, which is much less than $t_c \sim 1/E$, and the changes of the momentum $\mathbf{p}'$ are small in this case). In the regular parts, the essential region of integration does not depend on E at all, and its order of magnitude is $1/\epsilon_c$, which is much smaller than the region of integration in the singular part.

Allowance for these circumstances results in the fact that, when considering an arbitrary diagram with a large number of loops (1.8), the singular contribution in each loop can be separated, and the remaining loops can be regarded in this case as non-singular (see[5]). Summing the non-singular parts on the right and on the left of the separated loop, we find, just as in the case when E = 0 (see[5]), that g goes over into a certain non-singular real vertex $\Lambda(\mathbf{p}', \mathbf{q})$. Recognizing that only a narrow region with respect to $\mathbf{p}'$ and $\mathbf{q}$ is significant, we can take Λ and $\partial\Sigma_F/\partial\epsilon$ at $\mathbf{p}' = \mathbf{p}_c$ and $\mathbf{q} = \mathbf{q}_m$. Then the integral with respect to $\mathbf{q}$ can be calculated by using (1.10) and (1.9). Leaving out the intermediate steps, we obtain for the singular part of the integral term

$$i\Lambda^2\frac{\Gamma(-1/2)e^{i\pi/4}}{(1 - \partial\Sigma_F/\partial\varepsilon)|_{p_m}}(2\pi\mu)^{3/2}\int_{-\infty}^{t'} d\tau'\frac{\exp[-i\alpha^2(eE)^2(t' - \tau')^3/24\mu]}{(t' - \tau')^{3/2}\Gamma(-1/2)}$$
$$\times\left[\exp i\int_{\tau'-t}^{t'-t}\tilde{\varepsilon}(\mathbf{p} + e\mathbf{E}\theta)\,d\theta\right]G(\mathbf{p}, \tau' - t),$$

$$\mu = (2M\sqrt{2M + 2L})^{-2/3}, \quad \frac{\alpha^2}{\mu} = 2(M + L)\left(\frac{q_m}{p_c} + \lambda\right)\frac{p_{cx}^2}{p_c^2} + 2M\frac{q_m}{p_c}\frac{p_{c\perp}^2}{p_c^2} \tag{1.11}$$

Here $\Gamma(-1/2)$ is the Euler gamma function, introduced to normalize the Fourier transform of the function $t^{-3/2}$. It is seen from this expression that the effective region of integration with respect to τ' near $p' = p_c$ is of the order of $E^{-2/3}$, and the entire singular region is in general $t' - t - t_c \sim E^{-2/3}$.

The integrand has a non-integrable singularity at $\tau' = t'$. However, this singularity is the result of the divergence in the region of large q, where we did not take into account the dependence of Λ on q. This region is of no importance in the determination of the singularities of the Green's function, and shall exclude it, assuming that the integral in (1.11) is regularized, and $(t' - \tau')^{3/2}$ must be understood as the appropriate generalized function (see[6]).

For the function

$$\tilde{A}(\mathbf{p}, t' - t) = \left[\exp i\int_0^{t'-t}\tilde{\varepsilon}(\mathbf{p} + e\mathbf{E}\tau)\,d\tau\right]G(\mathbf{p}, t' - t) \tag{1.12}$$

it is easy to obtain from (1.4) now in the singular region (we use the old differential approximation for the regular part of Σ and formula (1.11) for the singular part) the following equation:

$$-av_c(p_x' - p_{cx})\tilde{A} = -i\int_{-\infty}^{t'} d\tau' R(t' - \tau')\tilde{A}(\tau' - t), \tag{1.13}$$

where

$$R(t) = \frac{e^{i\pi/4}\exp[-i\alpha^2(eE)^2t^3/24\mu]}{t^{3/2}\Gamma(-1/2)}g^2b\mu^{3/2},$$

$$g^2b = -\frac{\Lambda^2\Gamma(-1/2)(2\pi)^{3/2}}{(1 - \partial\Sigma_F/\partial\varepsilon)_{p_m}} > 0, \qquad a > 0.$$

We have used here the fact that Eq. (1.5) has a solution $\epsilon(p')$ for $p' \le p_c$, and we expanded the regular part of Σ in the vicinity of $p' = p_c$ (see[2]), so that the dimensionless constant a is determined by the following expression:

$$-av_c = \frac{\partial}{\partial p_x'}[\tilde{\varepsilon}(p') - \varepsilon_0(p') - \Sigma_{F|\mathrm{reg}}(\mathbf{p}', \tilde{\varepsilon}(p'))]|_{p'=p_c}.$$

In addition, recognizing that in the region $\tilde{A} = \tilde{A}(E^{2/3}(t' - t - t_c))$, we neglect the terms with derivatives $\partial\tilde{A}/\partial t'$ as being of order $E^{1/3}$ relative to the retained terms.

If we introduce the dimensionless variable

$$x = \frac{(eE\alpha)^{2/3}}{(24\mu)^{1/3}}(t' - t - t_c) = \left(\frac{\alpha^2}{24\mu}\right)^{1/3}\frac{p' - p_c}{(eE)^{1/3}}, \tag{1.14}$$

then (1.13) takes the form

$$x\tilde{A}(x) = i\gamma\int_{-\infty}^{x} dx' K(x - x')\tilde{A}(x'), \tag{1.15}$$

where

$$\gamma = \frac{g^2b\mu}{v_c a}\left(\frac{\alpha^2}{24}\right)^{1/2}, \qquad K(x) = \frac{e^{i\pi/4}e^{-ix^3}}{\Gamma(-1/2)x^{3/2}}.$$

For the complex-conjugate quantity $\tilde{A}^*$ the corresponding equation is the complex conjugate of (1.14), with

$$K^*(x - x') = K(x' - x) \qquad (x > x'). \tag{1.16}$$

The function $\tilde{A}(x)$ is obtained from Eq. (1.14) accurate to an arbitrary constant. This constant can be determined from the condition that (1.12) must be continuous with formula (1.7) at $x \ll -1$. We present here only the final result:

$$\tilde{A}(x) = \left[\frac{2av_c}{(g^2b)^2}\left(\frac{24}{\alpha^2}\right)^{1/3}\mu^{-1/3}\right]^{1/2}(eE)^{1/6}\exp\left\{-i\int_0^{t_c}(\varepsilon - \tilde{\varepsilon})_{\mathbf{p}+e\mathbf{E}\tau}\,d\tau\right\}F(x)\cdot$$
$$\cdot(1 - \partial\Sigma_F/\partial\varepsilon)_{\mathbf{p}}^{-1/2}, \tag{1.17}$$

where F(x) is the solution of (1.15), with asymptotic value

$$F(x)=\sqrt{-x}\exp\left(i\frac{8v_c^2a^2}{\mu^2(g^2b)^2}x^3\right),\qquad x\ll-1.$$

The most important fact here is that the single-particle Green's turns out to be of the order of $E^{1/6}$ in the singular region.

Equation (1.15) can be solved with the aid of a Fourier transformation:

$$F(x)=\int\frac{d\omega}{2\pi}F(\omega)e^{-i\omega x},\qquad F(\omega)=C\left[\exp-\gamma\int_0^\omega K(\omega)\,d\omega\right],\quad (1.18)$$

where

$$K(\omega)=\frac{e^{i\pi/4}}{\Gamma(-{}^1/_2)}\int_0^\infty\frac{e^{i\omega x}e^{-ix^3}}{x^{3/2}}dx,$$

and the constant C is determined from the asymptotic form of (1.17). It can be shown that when $x\gg1$ the quantity $\widetilde{A}$ attenuates:

$$\widetilde{A}\sim\exp[-\rho x\ln x],\qquad\rho>0,\qquad(1.19)$$

for which purpose it is necessary to find the asymptotic value of $K(\omega)$ at large values of $|\omega|$ in the lower half plane, and to use the saddle-point method to determine $\widetilde{A}$. At large values of $|\omega|$ and for real values of ω we have

$$K(\omega)\approx\sqrt{\omega}.\qquad(1.20)$$

2. SOLUTION OF THE EQUATIONS FOR THE TWO-PARTICLE GREEN'S FUNCTION G^+

We now proceed to consider the equations for G^+. Using the same representation as for the function G, we can obtain from Eq. (2) of[3]

$$i\frac{\partial G^+(\mathbf{p},t'-t)}{\partial t'}-\varepsilon_0(\mathbf{p}+e\mathbf{E}(t'-t))G^+(\mathbf{p},t'-t)$$

$$-\Lambda^2\int_{-\infty}^{t'}d\tau'\int\frac{d^3q}{(2\pi)^3}e^{-i\omega(q)(t'-\tau')}G(\mathbf{p}-\mathbf{q}+e\mathbf{E}(\tau'-t),t'-\tau')G^+(\mathbf{p},\tau'-t)$$

$$=-\Lambda^2\int_{-\infty}^{t}d\tau\int\frac{d^3q}{(2\pi)^3}e^{i\omega(q)(t'-\tau)}G^+(\mathbf{p}+\mathbf{q}+e\mathbf{E}(\tau-t),t'-\tau)\widetilde{G}(\mathbf{p},\tau-t)\qquad(2.1)$$

and analogously, but using the end-point momentum p′ (corresponding to the instant t′), we get from (3) of[3]

$$i\frac{\partial G_*^+(\mathbf{p}',t'-t)}{\partial t}+\varepsilon_0(\mathbf{p}'+e\mathbf{E}(t-t'))G_*^+(\mathbf{p}',t'-t)$$

$$-\Lambda^2\int_{-\infty}^{t}d\tau\int\frac{d^3q}{(2\pi)^3}e^{-i\omega(q)(t-\tau)}\widetilde{G}(\mathbf{p}'-\mathbf{q}+e\mathbf{E}(\tau-t),t'-\tau)G_*^+(\mathbf{p}',t'-\tau)$$

$$=-\Lambda^2\int_{-\infty}^{t'}d\tau'\int\frac{d^3q}{(2\pi)^3}e^{-i\omega(q)(t-\tau')}G(\mathbf{p}',t'-\tau')$$

$$\times G_*^+(\mathbf{p}+\mathbf{q}+e\mathbf{E}(\tau'-t'),\tau'-t).\qquad(2.2)$$

In these equations we have confined ourselves, to simplify the notation, only to the singular parts of Σ and γ (γ is the irreducible four-point diagram, see[3]), which lead to the decay singularity, assuming, in accordance with the preceding section, that the vertices reduce to the constant Λ. To reconcile the results it is necessary to assume here that the Green's function G and $\widetilde{G}$ are determined by the Dyson equations with the same Σ and $\widetilde{\Sigma}$. This, however, does not limit the applicability of the results in the general case, since the detailed structure of Σ and γ is significant in what follows only in the singular regions, where they coincide with those used in (2.1) and (2.2).

Just as in[3], G^+ can be separated into a pole or slowly-decreasing part and a "background," that decreases within times on the order of $1/\epsilon_c$. In finding the pole part of G^+, we shall first neglect the term describing the arrival, since, according to[3] there is no arrival with encounter of the pole of G anywhere except in the vicinity of the point $p = p_m$. We shall improve this result somewhat in the presence of a weak field E. The equation for G^+, obtained upon discarding the right side of (2.1), which describes the arrival, coincides with Eq. (1.4) at large $t'-t$, when the lower limit can set equal to $-\infty$.

The solutions of this equation are given by formulas (1.7), (1.12), and (1.13). Now, however, the constant factors, which depend only on the end-point momentum, are of no interest to us, since the solutions for G^+ are determined accurate to an arbitrary factor that depends on p_x and $p_\perp$ for Eq. (2.1) and on p_x' and $p_\perp$ for (2.2). Taking this circumstance into account, we can write for the pole part

$$G_n{}^+(\mathbf{p},\mathbf{p}')=G_n(\mathbf{p}')\Phi(p_x,\mathbf{p}_\perp),\quad G_{n*}{}^+(\mathbf{p},\mathbf{p}')=G_n{}^*(\mathbf{p})\Phi_*(p_x',\mathbf{p}_\perp).\ (2.3)$$

But when the same arguments are used we have $G^+=G_*^+$, whence

$$\Phi(p_x,\mathbf{p}_\perp)=G_n{}^*(\mathbf{p})n_0(\mathbf{p}_\perp),\qquad\Phi_*(p_x',\mathbf{p}_\perp)=G_n(\mathbf{p}')n_0(\mathbf{p}_\perp),$$
$$G_n{}^+(\mathbf{p},\mathbf{p}')=G_n(\mathbf{p}')G_n{}^*(\mathbf{p})n_0(\mathbf{p}_\perp),\qquad(2.4)$$

where $n_0(p_\perp)$ is a certain still arbitrary function.

The function $G_n(p')$ (G_n^* is obtained by complex conjugation) coincides, apart from a factor, with the expression for the pole part of G, and is given by

$$G_n(\mathbf{p}')=\left(1-\frac{\partial\Sigma_\Phi}{\partial\varepsilon}\right)_{\mathbf{p}'}^{-1/2}\exp\left\{-i\int_0^{p_x'}\frac{\varepsilon(x,\mathbf{p}_\perp)\,dx}{eE}\right\}\qquad(2.5)$$

when $\mathbf{p}'$ is far from the singular region $\mathbf{p}'-\mathbf{p}_c\sim E^{1/3}$. In the vicinity of p_c we have, according to (1.12) and (1.17),

$$G_n(\mathbf{p}')=(eE)^{1/6}\left[\frac{2av_c}{(g^2b)^2}\left(\frac{24}{\alpha^2}\right)^{1/3}\mu^{-5/6}\right]^{1/2}$$

$$\times\exp\left[-i\int_0^{p_{cx}}\frac{\widetilde{\varepsilon}(x)-\varepsilon(x)}{eE}dx\right]\exp\left[-i\int_0^{p_x'}\frac{\widetilde{\varepsilon}(\eta)\,d\eta}{eE}\right]F(x),\qquad(2.6)$$

where F(x) is a suitably normalized solution of (1.14).

It follows from (2.6) that the pole part of $G^+(p,t'-t)$ begins to decrease rapidly when $t'-t>(p_c-p)/eE>0$. On the other hand, if $p>p_c$, then the pole part is small like $\exp[-\rho x\ln x]$ for arbitrary $t'-t$ (according to (1.19)), where x(p) is given by (1.4).

As regards negative $t'-t$ (when $p<p_c$), it appears at first glance that formulas (2.4) and (2.5) are valid without any limitations in this case, too. However, sooner or later we arrive at an instant of time when $\mathbf{p}'=\mathbf{p}_m$. In this case great importance attaches to the term describing the arrival from the states with $\mathbf{p}\approx\mathbf{p}_c$, which can be regarded in (2.4) as a specified inhomogeneous term, assuming that G^+ is determined in it by formulas (2.5) and (2.6). When finding the pole part of G^+ in the vicinity of $t'-t=(p_m-p)/eE$, we should invert the left side of (2.1) with the aid of the single-particle Green's function G, assuming that G^+ vanishes as $t'-t\to-\infty$.

Let us write out the corresponding expression:

$$G_1^-(\mathbf{p}, t'-t) = \Lambda^2 \int_{-\infty}^{t'} d\tau' \int_{-\infty}^{t} d\tau \int \frac{d^3q}{(2\pi)^3} G_n^*(\boldsymbol{\pi}+\mathbf{q}) G_n(\boldsymbol{\pi}'+\mathbf{q}) n_0(\mathbf{p}_\perp + \mathbf{q}_\perp)$$
$$\times e^{-i\omega(q)(\tau-\tau')} \exp\left(-i\int_{p_x}^{p_x'} \frac{\varepsilon(x)\,dx}{eE}\right) \exp\left(i\int_{\pi_x}^{\pi_x'} \frac{\varepsilon(y)\,dy}{eE}\right)$$
$$\times\left(1-\frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\mathbf{p}}^{-1/2}\left(1-\frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\mathbf{p}'}^{-1/2}\left(1-\frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\boldsymbol{\pi}}^{-1/2}\left(1-\frac{\partial\Sigma_F}{\partial\varepsilon}\right)_{\boldsymbol{\pi}'}^{-1/2}, \quad (2.7)$$

where $\boldsymbol{\pi}' = \mathbf{p} + e\mathbf{E}(\tau' - t)$ and $\boldsymbol{\pi} = \mathbf{p} + e\mathbf{E}(\tau - t)$. We have retained in G and $\tilde{G}$ only the pole parts (formula (1.7)), bearing in mind that all the singularities are connected just with these parts (it can be shown, in addition, that at large values of $t' - t$ the non-pole parts of G and $\tilde{G}$ make in general a very small contribution to G_1).

Since actually only $\boldsymbol{\pi}' \approx \boldsymbol{\pi} \approx \mathbf{p}_m$, $|\boldsymbol{\pi} + \mathbf{q}| \approx p_c$, and $\mathbf{q} \approx \mathbf{q}_m$ are significant under the integral sign, we can use a quadratic dispersion law for ϵ and ω, bearing in mind serious expansions about the corresponding points. The expression (2.7) yields the pole part of G^+ in the vicinity of $t' - t = (p_m - p)/eE$, and to reconcile the solution formula (2.7) should then go over into formula (2.4) when $t' - t \gg (p_m - p)/eE$, when the arrival is negligibly small. This yields an integral equation for n_0:

$$n_0(p_\perp) = \frac{\Lambda^2}{(1-\partial\Sigma_F/\partial\varepsilon)_{p_m}} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\pi_x'}{eE} \int_{-\infty}^{\infty} \frac{d\pi_x}{eE} G_n(\boldsymbol{\pi}') G_n^*(\boldsymbol{\pi})$$
$$\times\left[\exp i \int_{\pi_x - q_x}^{\pi_x' - q_x} \frac{\varepsilon(y)+\omega(q)}{eE} dy\right] n_0(\mathbf{p}_\perp + \mathbf{q}_\perp). \quad (2.8)$$

We have replaced here $\partial\Sigma_F/\partial\epsilon|_{\pi-q}$ by $\partial\Sigma_F/\partial\epsilon|_{p_m}$, bearing in mind that only a small vicinity of $\mathbf{p}_m$ is significant, and we set the upper integration limit equal to $+\infty$, which is approximately valid for $t' - t \gg (p_m - p)/eE$. The small vicinity of $\boldsymbol{\pi}' = \mathbf{p}_c$ makes an appreciable contribution here, because its region coincides with the region of the stationarity of the rapidly oscillating exponentials. When $t' - t \ll (p_m - p)/eE$, there is no such region, and consequently the pole part of G^+ vanishes.

The solvability of (2.8) follows from an identity that has the meaning of a conservation law: the integral of the left side of (2.8) with respect to $d^2p_\perp$ is equal to the integral of the right side for an arbitrary function $n_0(\mathbf{p}_\perp)$. A proof is given in the appendix.

We are not interested in the concrete form of $n_0(\mathbf{p}_\perp)$, since it is of no importance in the determination of the current per particle. What is important in this case is that the width of the distribution with respect to $\mathbf{p}_\perp$ (about $\mathbf{p}_\perp = 0$) is of the order of $E^{1/3}$, since the quantities $q - q_m$ and $p_{ex} - p_x$ are of the same order in the singular region. We shall assume henceforth that $n_0(\mathbf{p}_\perp)$ is so normalized that the integral of this function with respect to $(2\pi)^{-2}d^2p_\perp$ is equal to unity.

Expression (2.7) yields also the singular part of $G^+(\mathbf{p}, t' - t)$ when $p - p_m \sim E^{1/3}$ and $t' = t$. We note that when $p < p_m$ the quantity G_1^+ consists exclusively of a "short-lived" (non-pole) part, connected with the contribution made from the region near the upper limits of integration with respect to τ and τ', whereas when $p > p_m$ the quantity G^+ consists both of a pole part connected with the stationarity points of the arguments of the exponential (with respect to τ and τ'), and also a short-lived part.

We now proceed to construct the complete solution of Eqs. (2.1) and (2.2). We write these equations in symbolic form, introducing the irreducible four-point diagram γ (see[31]):

$$G^{-1}G^+ = \tilde{G}\gamma G^+, \quad \tilde{G}^{-1}G^+ = G\gamma G^+. \quad (2.9)$$

We shall solve these equations by iteration, starting with

$$G^+ = G_1^+ = G\tilde{G}\gamma G_{nn}^+, \quad (2.10)$$

where $G_{nn}^+(\mathbf{p}, t' - t)$ is the pole part of G^+, given by formulas (2.4), (2.5), and (2.6) at $t_x \geq p_x^* > p_{mx}$, and equal to zero when $p_x < p_x'$. Formula (2.10) differs from formula (2.7), for G_1^+ in the fact that it includes the nonsingular parts of the irreducible four-point diagram γ, which were omitted for simplicity from (2.1) and (2.2). This, of course, does not change the fact that the singular part of G_1^+ is connected only with the singular part of γ, and is given by formula (2.7). It is easy to show, by using (2.8) (we assume that $n_0(\mathbf{p}_\perp)$ in G_{nn}^+ satisfies this equation) that when $p_x > p_{mx}$ the pole part of G_1^+ coincides with the pole part given by formulas (2.4), (2.5), and (2.6). The iteration solution of (2.9) is of the form

$$G^+ = G_1^+ + \sum_{s=1}^{\infty} (G\tilde{G}\gamma)^{s-1}(G\tilde{G}\gamma G_1^+ - G_1^+)$$
$$= G_1^+ + \sum_{s=1}^{\infty} (G\tilde{G}\gamma)^s (G_1^+ - G_{nn}^+). \quad (2.11)$$

We see that the iterations do not contain the pole part of G^+ at $p_x > p_x^*$, since the pole parts of G_1^+ and G_{nn}^+ coincide in this case.

All the integrations in (2.11) at specified G_1^+ reduce to integrals of the type (2.7), and in no case do their arise any stationarity points of the oscillating components (this is connected with the fact that when $p < p_c$ the decay is impossible, and the polar parts in the vicinity of p_c, as already noted, have been excluded). Because of this, the integrations with respect to the times are limited to a band of the order of $1/\epsilon_c$ near the upper limit. This makes it possible to neglect the influence of the electric field in the operators $G\tilde{G}\gamma$ with accuracy to terms that are linear in the field. As we shall show, the first non-vanishing term at the current, which depends on the field, is of the order of $E^{1/3}$, and we shall assume that in (2.11) the field influences only the distribution of $G_1^+(\mathbf{p}, 0)$ with respect to the momenta, neglecting the remaining effects.

In order to separate the field-independent part of G^+, we introduce the following functions:

$$G_{n0}^+ = \begin{cases} 0, & p_x > p_{cx},\ p_x < p_{mx} \\ \dfrac{n_0(\mathbf{p}_\perp)\exp[-i\varepsilon(p)(t'-t)]}{(1-\partial\Sigma_F/\partial\varepsilon)_{\mathbf{p}}}, & p_{mx} \leqslant p_x \leqslant p_{cx} \end{cases} \quad (2.12)$$

$$G_{10}^+ = G_0\tilde{G}_0\gamma_0 G_{nn0}^+, \quad G_{nn0}^+ = \begin{cases} G_{n0}^+, & p_x > p_x^* \\ 0, & p_x < p_x^* \end{cases}, \quad (2.13)$$

The zero subscript denotes that the designated function is taken in the absence of the electric field.

Then (2.11), in accord with the foregoing, can be written in the form

$$G^+(\mathbf{p},0) = G_{n0}^+ + G_{10}^+ + \sum_{s=1}^{\infty} (G_0\tilde{G}_0\gamma_0)^s (G_{n0}^+ + G_{10}^+ - G_{nn0}^+)$$
$$+ G_1^+ - G_{n0}^+ - G_{10}^+ + \sum_{s=1}^{\infty} (G_0\tilde{G}_0\gamma_0)^s (G_1^+ - G_{nn}^+ - G_{10}^+ + G_{nn0}^+ - G_{n0}^+)$$
$$= G^+(\mathbf{p},0)|_{E=0} + \sum_{s=0}^{\infty} (G_0\tilde{G}_0\gamma_0)^s (G_1^+ - G_{nn}^+ - G_{10}^+ - G_{n0}^+ + G_{nn0}^+). \quad (2.14)$$

Here $G^+(p, 0)|_{E=0}$ is the function G^+ in the absence of the field, constructed from the quasi-particle distribution given by (2.12), and in addition we have neglected the difference $G^+_{nn} - G^+_{nn0}$, which differs from zero only in the vicinity of $|p - p_c| \sim E^{1/3}$, where it itself is a quantity of the order of $E^{1/3}$, in accord with (2.6), giving a contribution of the order of $E^{2/3}$, which is insignificant in the calculation of the integral quantities.

3. CALCULATION OF THE CURRENT

The electric current and the number of particles per unit volume are determined by the following formulas:

$$\mathbf{J} = e\int \frac{\mathbf{p}}{m} G^+(\mathbf{p}, 0)\frac{d^3p}{(2\pi)^3}, \quad \frac{N}{V} = \int G^+(\mathbf{p}, 0)\frac{d^3p}{(2\pi)^3}. \tag{3.1}$$

As shown in[3], the sum of all the iterations from the pole term in $G^+(p, 0)|_{E=0}$ (formula (2.14)) reduces simply to the appearance of the factors

$$\left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)\frac{m}{p}\frac{\partial\varepsilon}{\partial p}, \quad \left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)$$

in front of the pole term in the current and in the density, respectively. In the second term of (2.14), the sum of all the iterations reduces to the appearance of the same factors in front of $\delta G^+ = G^+_1 - G^+_{nn} - G^+_{10} - G^+_{n0} + G^+_{nn0}$. Thus,

$$\frac{N}{V} = \int n_0(\mathbf{p}_\perp)\frac{d^2p_\perp}{(2\pi)^2}\int\limits_{p_{mx}(\mathbf{p}_\perp)}^{p_{cx}(\mathbf{p}_\perp)}\frac{dp_x}{2\pi} + \int\left(1 - \frac{\partial\Sigma_F}{\partial\xi}\right)_{\mathbf{p}}\delta G^+(\mathbf{p}, 0)\frac{d^3p}{(2\pi)^3}. \tag{3.2}$$

We do not write out the analogous expression for the current, which differs by a factor $e\partial\epsilon/\partial\mathbf{p}$ under the integral sign. As already noted, the width of the $n_0(\mathbf{p}_\perp)$ distribution is of the order of $E^{1/3}$, so that the difference between $p_{cx}(\mathbf{p}_\perp)$, $p_{mx}(\mathbf{p}_\perp)$, and the values of these functions at $\mathbf{p}_\perp = 0$ can be neglected, since these functions are even, accurate to $E^{2/3}$. Recognizing further that the integrand in the terms containing δG^+ is concentrated near $p = p_m$, we obtain from (3.2) and from the analogous formula for the current the following expression for the current per particle:

$$j = j_x = \frac{JV}{N} = \frac{\varepsilon(p_c) - \varepsilon(p_m)}{p_c - p_m} + \left[\frac{\partial\varepsilon}{\partial p}\bigg|_{p_c} - \frac{\varepsilon(p_c) - \varepsilon(p_m)}{p_c - n}\right]\nu(E). \tag{3.3}$$

The value of $\nu(E)$ is given by the integral:

$$\nu(E) = \left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)\int\limits_{p_m}(G_1{}^+ - G_{nn}{}^+ - G_{10}{}^+ - G_{n0}{}^+ + G^+_{nn0})\frac{d^3p}{(2\pi)^3}. \tag{3.4}$$

The terms subtracted from G^+_1 have an entirely different nature. $G^+_{nn} + G^+_{n0} - G^+_{nn0}$ represents the pole term corresponding to the distribution of the quasiparticles with respect to formula (2.12) up to $p = p_m$ (we could neglect in (3.4) the difference $G^+_{nn} \sim G^+_{nn0} \sim E^{1/3}$ when $|p - p_c| \sim E^{1/3}$), where the separation of G^+_1 into a pole and a short-lived part becomes arbitrary. The term G^+_{10} describes the iteration of first order of the pole part of G^+_{nn0}, calculated in the absence of the field (i.e., the short-lived part of $G^+|_{E=0}$, which has a singularity $p = p_m$).

In the calculation of (3.4), only the singular parts of G^+_{10} and G^+_1 are important, and these are obtained by taking the singular part of γ and the pole parts of G and $\widetilde{G}$. The non-singular parts of G^+_1 and G^+_{10} cancel each other, since they are obtained by identical pole parts (actually, as just noted, the pole parts differ when $|p - p_c| \sim E^{1/3}$ by an amount on the order of $E^{1/3}$, which leads to a value of the order of $E^{2/3}$ in the integral). Thus, we can use in the calculation of the integral (3.4) formula (2.7) for G^+_1 and an analogous formula for G^+_{10}.

In order to perform some of the integrations in (3.4), it is convenient to proceed as follows: we cut off the integration with respect to p_x by the segment $(-L, L)$, after which we interchange the order of integration with respect to p and q (see (2.7)), and make the change of variables $\mathbf{p} \to \mathbf{p} - \mathbf{q}$. Recognizing that we should confine ourselves only to the singular region $q - q_m \sim E^{1/3}$ in the integration with respect to q, we obtain, taking the limit as $L \to \infty$,

$$\begin{aligned}\nu(E) = {}& \Lambda^2\int\limits_{-\infty}^{\infty}\frac{dp_x}{2\pi}\int\frac{d^2p_\perp}{(2\pi)^2}\int\frac{d^3q}{(2\pi)^3}\int\limits_{-\infty}^{p_x}\frac{d\pi_x'}{eE}\int\limits_{-\infty}^{p_x}\frac{d\pi_x}{eE}G_n(\boldsymbol{\pi}')G_n{}^*(\boldsymbol{\pi}) \\ &\times\left\{\left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)^{-1}_{p_m}\exp\left[i\int\limits_{\pi_x}^{\pi_x'}\frac{\varepsilon(x - q_x, \mathbf{p}_\perp - \mathbf{q}_\perp) + \omega(q)}{eE}dx\right] - 1\right\}n_0(\mathbf{p}_\perp) \\ &+ \lim_{L\to\infty}\Lambda^2\int\frac{d^3q}{(2\pi)^3}\int\frac{d^2p_\perp}{(2\pi)^2}\int\limits_{L-p_c+q_m}^{L-p_c+q_x}\frac{dp_x}{2\pi}\int\limits_{-\infty}^{p_x}\frac{d\pi_x'}{eE}\int\limits_{-\infty}^{p_x}\frac{d\pi_x}{eE}G_n(\boldsymbol{\pi}')G_n{}^*(\boldsymbol{\pi}) \\ &\times\left(1 - \frac{\partial\Sigma_F}{\partial\varepsilon}\right)^{-1}_{p_m}\exp\left[i\int\limits_{\pi_x}^{\pi_x'}\frac{\varepsilon(x - q_x, \mathbf{p}_\perp - \mathbf{q}_\perp) + \omega(q)}{eE}dx\right]n_0(\mathbf{p}_\perp).\end{aligned} \tag{3.5}$$

Since only $\mathbf{q}$ close to $\mathbf{q}_m$ play an important role in the integrals, the argument of the exponential under the integral signs can be expanded in the vicinity of $\mathbf{q} = \mathbf{q}_m(\mathbf{p})$, in accordance with (1.9), and the remainder of the procedure is analogous to that of obtaining the singular part of the integral term in the equation for G in Sec. 1 (formula (1.11)). The resultant expression in the curly brackets coincides with the expression for $Q - 1$ and, as shown in the appendix, vanishes (see (A.6)). Thus, there remains only the second term. As $L \to \infty$, the expression in the square brackets does not depend on p_x, and we can carry out the integration with respect to p_x. Carrying out, in addition, the integration with respect to $\mathbf{q}$ (after expanding the argument of the exponential) and going over to the dimensionless variables $\xi = x(p)$ and $\xi' = x(p')$, where $x(p)$ is given by formula (1.14) we obtain, using (2.6) and (1.13):

$$\begin{aligned}\nu(E) = {}& -(eE)^{1/3}\left(\frac{24\mu}{\alpha^2}\right)^{5/6} \cdot \\ &\times\frac{av_c}{bg^2}\int\limits_{-\infty}^{\infty}d\xi'\int\limits_{-\infty}^{\infty}d\xi\frac{q_{mx}((\mathbf{p} + \mathbf{p}')/2) - q_m(p_c)}{(eE)^{1/3}(24\mu/\alpha^2)^{1/3}}F(\xi')F^*(\xi)K(\xi - \xi'),\end{aligned} \tag{3.6}$$

where $F(\xi)$ is the solution of (1.15) with the asymptotic form (1.17). We have purposefully retained here under the integral sign the quantity $q_{mx}((\mathbf{p} + \mathbf{p}')/2) - q_m(p_c)$, in order to show that $\nu(E)$ is actually corrected with the average momentum lost by the ion to the phonon emission. Using the Fourier transformation and formulas (1.15), (1.18), and (1.10), we obtain the final expression

$$\begin{aligned}\nu(E) = {}& (eE)^{1/3}\left(\frac{24\mu}{\alpha^2}\right)^{5/6}\frac{\beta av_c}{bg^2}\gamma\int\limits_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{K^2(\omega) - K^{*2}(\omega)}{i} \\ &\times\exp\left\{-\gamma\int\limits_{-\infty}^{\omega}d\omega\left[K(\omega) + K^*(\omega)\right]\right\}, \\ \beta = {}& \frac{\partial^2\varepsilon}{\partial p^2}\bigg|_{p_m}\bigg/\left(\frac{\partial^2\varepsilon}{\partial p^2}\bigg|_{p_m} + \frac{\partial^2\omega}{\partial q^2}\bigg|_{q_m}\right).\end{aligned} \tag{3.7}$$

The integral with respect to ω can be determined only numerically. The results of the calculations with an electronic computer for different values of γ will be presented later. The most important fact is that this integral is positive.

We see therefore that the current per particle is given by the formula

$$j_x = e\frac{\omega}{q}\Big|_{q_m} + e\left[\frac{\partial\omega}{\partial q}\Big|_{q_m} - \frac{\omega(q)}{q}\Big|_{q_m}\right]\beta\sigma E^{1/3};$$

$$\sigma > 0,\quad \beta = \frac{\partial^2\varepsilon}{\partial p^2}\Big|_{p_m}\Big/\left(\frac{\partial^2\varepsilon}{\partial p^2}\Big|_{p_m} + \frac{\partial^2\omega}{\partial q^2}\Big|_{q_m}\right),\quad \frac{\partial^2\varepsilon}{\partial p^2}\Big|_{p_m} + \frac{\partial^2\omega}{\partial q^2}\Big|_{q_m} > 0, \tag{3.8}$$

if we use in (3.3) the conservation laws and the equations relating the group velocities at the decay point[2]. This result differs from that given by the elementary kinetic equation (see[4]) in that there is a stronger singularity in the field-dependent term (exponent 1/3 in lieu of 2/3). In addition, the sign of this term is determined uniquely by the signs of $\partial^2\epsilon/\partial p^2|_{pm}$ and $(\partial\omega/\partial q - \omega/q)_{q_m}$. If we apply (3.8) to the emission of an optical phonon without dispersion,

$$\frac{\partial^2\varepsilon}{\partial p^2}\Big|_{p_m} > 0,\quad \left(\frac{\partial\omega}{\partial q} - \frac{\omega}{q}\right)_{q_m} = -\frac{\omega_0}{q_m} < 0,$$

and we find that the current decreases with increasing field, unlike the result obtained by Shockley[4] in accordance with the kinetic equation. This difference is connected with the fact that the calculation with the aid of the kinetic equation is valid only in the case of weak interaction in sufficiently strong fields. The particle momentum distribution then appreciably overlaps the region near p_c in which the singularity obtained by Pitaevskiĭ is effective, and the following condition is satisfied

$$\frac{\partial\varepsilon}{\partial p}\Big|_{p_c} \approx \frac{\partial\varepsilon}{\partial p}\Big|_{p_m}$$

(this relation is not satisfied in the region where the kinetic equation is applicable). Thus, in the case of weak interaction, upon emission of an optical phonon, the current first decreases (in very weak fields) with increasing field, and begins to increase only in a sufficiently strong field.

For ions in He II, if we assume that the decay is effected with emission of a vortex ring, for which $\omega/q > \partial\omega/\partial q$ and $\partial^2\omega/\partial q^2 < 0$, and consequently $\partial^2\epsilon/\partial p^2|_{pm} > 0$, then the current will also decrease with increasing field. This result agrees qualitatively with the experimental data[7].

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APPENDIX

The proof of the fact that the integral of the left side of (2.8) with respect to $d^2p_\perp$ is equal to the integral of the right side for arbitrary $n_0(\mathbf{p}_\perp)$ calls for calculation of the integral

$$P = \Lambda^2 \int \frac{d^2p_\perp}{(2\pi)^2} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\pi_x'}{eE} \int_{-\infty}^{\infty} \frac{d\pi_x}{eE} G_n(\pi') G_n^*(\pi)$$

$$\times\left(1 - \frac{\partial\Sigma_\Phi}{\partial\varepsilon}\right)_{p_m}^{-1} \exp\left\{i \int_{\pi_x - q_x}^{\pi_x' - q_x} \frac{\varepsilon(y, \mathbf{p}_\perp) + \omega(q)}{eE} dy\right\} n_0(\mathbf{p}_\perp + \mathbf{q}_\perp). \tag{A.1}$$

The integrals with respect to π_x and π_x' should be understood in the sense that contours of integration are shifted somewhat away from the real axis as $\pi_x', \pi_x \to -\infty$, so as to obtain convergence, and that the essential contribution is made only by the vicinity of $\pi' = \pi = p_c$. Instead of this, it is more convenient to introduce the convergence factors $\exp(\delta\pi_x)\exp(\delta\pi_x')$ $(\delta \to +0)$ and assume that the integrals are taken along the real axis. Making the change of variables $\mathbf{p}_\perp \to \mathbf{p}_\perp - \mathbf{q}_\perp$, expanding the argument of the exponential in $\mathbf{q}$ near $\mathbf{q} = \mathbf{q}_m(\mathbf{p})$, in accord with (1.9), and then calculating the integral with respect to d^3q, we get

$$P = \int \frac{d^2p_\perp}{(2\pi)^2}\left\{\int_{-\infty}^{\infty} e^{\delta\pi_x'} \frac{d\pi_x'}{eE} \int_{-\infty}^{\infty} e^{\delta\pi_x} \frac{d\pi_x}{eE} \exp\left[i\int_{\pi_x}^{\pi_x'} \tilde{\varepsilon}(x, \mathbf{p}_\perp)\frac{dx}{eE}\right] \times G_n(\pi') G_n^*(\pi) R\left(\frac{\pi_x - \pi_x'}{eE}\right) n_0(\mathbf{p}_\perp)\right\}, \tag{A.2}$$

where R(t) is the same as in formula (1.13).

Let us consider now the integral in the curly brackets:

$$Q = \lim_{\delta\to+0} \int_{-\infty}^{\infty} e^{\delta\pi_x'} \frac{d\pi_x'}{eE} \int_{-\infty}^{\infty} e^{\delta\pi_x} \frac{d\pi_x}{eE} \tilde{A}^*(\pi_x)\tilde{A}(\pi_x') R\left(\frac{\pi_x - \pi_x'}{eE}\right), \tag{A.3}$$

where, according to (1.12), $\tilde{A}(\pi_x)$ and $\tilde{A}^*(\pi_x)$ satisfy the integral equations

$$-av_c\pi_x\tilde{A}(\pi_x) = -i\int_{-\infty}^{\pi_x} R\left(\frac{\pi_x - \pi_x'}{eE}\right)\tilde{A}(\pi_x')\frac{d\pi_x'}{eE}, \tag{A.4}$$

$$-av_c\pi_x\tilde{A}^*(\pi_x) = i\int_{-\infty}^{\pi_x} R\left(\frac{\pi_x' - \pi_x}{eE}\right)\tilde{A}^*(\pi_x')\frac{d\pi_x'}{eE}.$$

The integral Q is best represented by introducing a certain π_0, in the form

$$Q = \lim_{\delta\to+0}\left\{\int_{\pi_0}^{\infty}\frac{d\pi_x'}{eE}\int_{-\infty}^{\pi_x'}\frac{d\pi_x}{eE}Z + \int_{\pi_0}^{\infty}\frac{d\pi_x}{eE}\int_{-\infty}^{\pi_x}\frac{d\pi_x'}{eE}Z\right\}$$

$$+ \lim_{\delta\to+0}\left\{\int_{-\infty}^{\pi_0}\frac{d\pi_x'}{eE}\int_{-\infty}^{\pi_x'}\frac{d\pi_x}{eE}Z + \int_{-\infty}^{\pi_0}\frac{d\pi_x}{eE}\int_{-\infty}^{\pi_x}\frac{d\pi_x'}{eE}Z\right\}. \tag{A.5}$$

In the first two terms it is then possible to go to the limit as $\delta \to +0$ under the integral sign, owing to the uniform integrability of the integrand (the dangerous region $\pi_x = \pi_x' \to -\infty$ is excluded). It is then possible to show, using (A.4), that the sum of the corresponding two integrals is equal to zero. Thus, for any π_0 the integral Q is equal to the last term. By choosing π_0 negative, with a sufficiently large modulus, it is possible to use the asymptotic form (1.17) for $\tilde{A}$ and $\tilde{A}^*$, and to calculate the corresponding integral for finite δ. The corresponding derivations are elementary and we present only the final result:

$$Q = \lim_{\delta\to+0}\int_{-\infty}^{\pi_0} e^{\delta\pi_x'}\frac{d\pi_x'}{eE}\int_{-\infty}^{\pi_0} e^{\delta\pi_x}\frac{d\pi_x}{eE}\tilde{A}(\pi_x')\tilde{A}^*(\pi_x)R\left(\frac{\pi_x - \pi_x'}{eE}\right) = \lim_{\delta\to+0} e^{2\delta\pi_0} = 1. \tag{A.6}$$

Substituting (A.5) in (A.2) we obtain the required identity

$$P = \int \frac{d^2p_\perp}{(2\pi)^2} n_0(\mathbf{p}_\perp). \tag{A.7}$$

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