CURRENT COLUMN IN A SEMICONDUCTOR WITH S-SHAPED CURRENT VOLTAGE CHARACTERISTIC

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The complex differential impedance $Z_d(\omega)$ of a semiconductor with a nonuniform current distribution in the shape of a column produced as a result of thermal instability is calculated. The case of a thick column whose radius is much greater than its wall thickness is considered. At low frequencies $Z_d(\omega)$ is determined by the oscillations (or at $\omega = 0$, by the shift) of the column wall and is sensitive to the frequency ω. The contribution of the wall motion to $Z_d(\omega)$ vanishes already at frequencies which are small with respect to the reciprocal relaxation time of the energy in a homogeneous semiconductor plasma. The shape of the static current-voltage characteristic is determined.

1. INTRODUCTION

In semiconductors in which the current-voltage characteristic would be S-shaped if the current were uniformly distributed over the cross section, stationary states with negative differential conductivity ($\sigma_d < 0$) are unstable (we are referring to conductors with sufficiently large cross sections). As a result of this instability, an inhomogeneous distribution is produced in the conductor, and the current becomes "pinched"[1,2] (if the conductor carries a specified total current). There are many known mechanisms producing an S-shaped characteristic. A mechanism of interest in semiconductor physics is the superheat mechanism, in which the ambiguity of the characteristic is due to the dependence of the electron mobility on their temperature at a fixed electron density. Under conditions when the electron-temperature approximation is applicable and the electron gas can be regarded as incompressible, this mechanism is perhaps the simplest model for theoretical analysis of the resultant inhomogeneous current distribution.

Volkov and the author[3] considered (in connection with the superheat mechanism) different stationary inhomogeneous transverse distributions of the current and their stability. The distribution of a current with axial asymmetry, which depends monotonically on the radius ρ (current-pinch column) is of interest because it is stable against perturbations that do not change the total resistance of the sample (as distinguished, for example, from current distributions in the form of flat layers of finite thickness). The energy conservation equation which determines the electron temperature $T(\rho, t)$, has the same form as the equation of heat conduction with a nonlinear source

$$-nc_e\frac{\partial T}{\partial t}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\varkappa\frac{\partial T}{\partial\rho}\right)+\sigma(T)E_z^2-P(T)=0. \qquad (1)$$

Here ρ is the distance from the axis of the sample, which for simplicity is assumed to be in the form of a cylinder of radius R; n is the electron density; $\sigma(T)$, $\kappa(T)$, and nc_e are the specific electric conductivity, thermal conductivity, and heat capacity of the plasma; $P(T)$ is the power transferred to the lattice vibrations per unit volume; E_z is the field component parallel to the current. The condition on the lateral boundary of the sample can be stipulated in the form $\partial T/\partial\rho = 0$. This is equivalent to assuming that there are no specific surface mechanism for the energy transfer from the electron gas to the lattice (other than the volume mechanisms defining $P(T)$).

The equation for the stationary distributions can be represented in the form

$$d^2\Theta / d\rho^2 + \rho^{-1}d\Theta / d\rho + dU / d\Theta = 0,$$

$$\Theta = \int^T dT'\varkappa(T'), \quad U(\Theta) = \int^\Theta d\Theta'[\sigma(\Theta')E_z^2 - P(\Theta')]. \qquad (2a)$$

In order to find the static current-voltage characteristic of the sample with inhomogeneous current distribution, it would be necessary to integrate Eq. (2) (in the case of alternating fields—(1)), find $T(\rho)$ and the current density $j_z(\rho) = \sigma(T(\rho))E_z$. It is known that Eq. (2) can in general not be integrated without resorting to numerical calculations. On the other hand, when the temperature varies only in one direction, the equation for the temperature (which differs from (2) in the absence of a term with $d\Theta/d\rho$), can be easily integrated[3]. It is therefore natural to investigate a definite limiting case of axisymmetric distribution, close to uniform distribution, namely a current pinch of large radius ρ_0.

In the case of one-dimensional distribution, there exists a solution in the form of two homogeneous stable phases—cold and hot—with a narrow transition layer between them. We define as the phase a region in the cross section of the sample having a practically uniform temperature distribution, equal to one of the three roots of the equation $dU/d\Theta = 0$ possible for a given field E_z (the temperature Θ_1 of the cold phase is equal to the smallest root, and that of the hot phase Θ_3 to the largest root). The thickness of the transition layer (wall) l_W is of the order of the energy scattering length, $l_W \sim (\kappa\tau/n)^{1/2}$, where τ is the energy scattering time. The hot and cold phases coexist in the case of a flat boundary only at a definite value of the field E_0, at which $U(\Theta_1) = U(\Theta_3)$[3].

Such a one-dimensional temperature distribution corresponds to a distribution with cylindrical symmetry $T(\rho)$ in the form of a thick column of one stable phase, surrounded by another stable phase, for example a hot pinch column in a cold plasma. In the limiting case of a very large column radius and negligible curvature of its surface, the properties of the indicated two distributions of the temperature and the current coincide.

In the present paper we find the form of the static current-voltage characteristic and the complex differential impedance $Z_d(\omega)$ of a conductor in which superheat instability has produced a current pinch with a wall located at a large distance from both the axis of the sample ($\rho_0 \gg l_W$) and from its boundary ($R - \rho_0 \gg l_W$). It is important that the radius of the pinch is assumed to be large but finite. This makes it possible to trace the variation of the static characteristic and of the dynamic properties of the conductor with increasing ρ_0 and "compression" of the pinch wall.

Obviously, the current pinch of large radius ($\rho_0 \gg l_W$) exists in fields E_Z close to E_0 (see above), and the static current-voltage characteristic $I(E_Z)$ approaches the vertical $E_Z = E_0$, which is characteristic of a flat boundary between phases, with increasing total current I. We shall find later the manner in which this approach takes place.

The main change of the electron temperature in a sample with a large-radius pinch occurs in the narrow region of the wall. On going away from the wall to the interior of the pinch or to the interior of the cold plasma, $\Theta(\rho)$ approaches exponentially its values $\Theta(0)$ and $\Theta(R)$, and, as follows from (2), the characteristic lengths of exponential variation of $\Theta(\rho)$ are equal respectively to $l_3 = (-U''(\Theta_3))^{-1/2}$ and $l_1 = (-U''(\Theta_1))^{-1/2}$. The differences $\Theta_3 - \Theta(0)$ and $\Theta(R) - \Theta_1$ are therefore exponentially small quantities. Namely,

$$(\Theta_3 - \Theta(0)) / \Theta_3 \sim \exp(-\rho_0 / l_3),$$
$$(\Theta(R) - \Theta_1) / \Theta_1 \sim \exp[-(R - \rho_0) / l_1]. \qquad (3)$$

We shall neglect exponentially small quantities, but we shall retain in the calculation of the impedance the relatively large quantities of the order of l_W/ρ_0 and $(l_W/\rho_0)^2$. It is easy to understand that the latter are the results of the curvature of the surface of the pinch. It is precisely the curvature, and not the inhomogeneity of $T(\rho)$ deep inside the pinch or far from the wall, which determines the difference between the properties of distributions of the temperature and the current in the form of a column of finite radius, and the properties of distributions in the form of two phases with a flat boundary between them. This effect recalls the influence of the curvature of the surface of a liquid on its equilibrium with vapor.

We shall find that in the static case and at low frequencies the electric conductivity of the sample is determined by the displacement (oscillations) of the pinch wall. It is therefore easy to see the analogy between our problem and problems of domain-wall motions in the magnetic susceptibility of a ferromagnet[4] or the dielectric susceptibility of a ferroelectric[5], and also the problem of the impedance of a sample with a Gunn domain[6].

Since we are considering in this article only the distributions of the current and of the temperature, which do not depend on z, all the results are valid not only in the absence of the magnetic field, but also in a longitudinal field. In the latter case $\sigma = \sigma_{zz}$ and $\kappa = \kappa_{xx}$.

2. DIFFERENTIAL COMPLEX IMPEDANCE

Assume that besides the constant field E_Z there is applied to the sample a small alternating field $\delta E_Z(\omega)\exp(-i\omega t)$. We denote the amplitudes of the ac components of the current density and of the temperature by $\delta j_Z(\rho, \omega)$ and $\delta T(\rho, \omega)$. The differential electric conductivity of the sample is (l_Z—length of the sample):

$$Z_d^{-1}(\omega) = \frac{2\pi}{l_z}\int_0^R d\rho\rho\,\frac{\delta j_z(\rho,\omega)}{\partial E_z(\omega)} = \frac{2\pi}{l_z}\int_0^R d\rho\rho\left\{\sigma(T) + E_z\frac{d\sigma}{dT}\frac{\delta T(\rho,\omega)}{\delta E_z(\omega)}\right\}. \qquad (4)$$

It is necessary to substitute here in lieu of T the stationary distribution $T(\rho)$. The equation for $\delta T(\rho, \omega)$ is obtained by linearization of (1). After making the following change of variable and of the unknown function

$$r = \int_0^\rho d\rho\,(nc_e/\varkappa)^{1/2},$$
$$\eta = \varkappa(d\ln r / d\ln\rho)^{1/2}\delta T(\rho, \omega), \qquad (5)$$

we get

$$(\hat{H} - i\omega)\eta(r) = F(r). \qquad (6)$$

Here

$$\hat{H} = -\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) + V(r), \qquad (7)$$

$$V(r) = -\frac{\varkappa}{nc_e}\frac{d^2U}{d\Theta^2} - \quad \frac{\varkappa}{nc_e}\Big)^{3/4}(r\rho)^{-1/2}\frac{d}{d\rho}\rho\frac{d}{d\rho}\left(\frac{d\ln r}{d\ln\rho}\right)^{-1/2}, \qquad (8)$$

$$F(r) = 2\sigma E_z(\rho/r)^{1/2}(\varkappa/nc_e)^{3/4}\delta E_z(\omega). \qquad (9)$$

The "potential" V(r) has the form of a well at $\rho \approx \rho_0$, with sufficiently abrupt edges (Fig. 1). Outside the wall $Z(r) > 0$ and is constant with exponential accuracy.

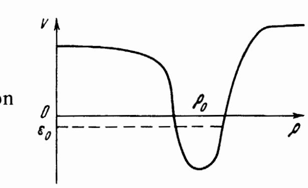

FIG. 1. Schematic representation of the "potential" V(ρ).

We denote by η_n and ϵ_n the eigenfunctions and the eigenvalues of $\hat{H}$, and assume that η_n satisfy the same conditions as $\eta(r)$ (and also T(r)). The functions η_n constitute a system of "elementary perturbations" of the stationary distribution of the temperature in the column. The ground state function $\eta_0(r)$, which does not reverse sign and is concentrated in the potential well V(r), corresponds to a change of the radius of the column, i.e., to a motion of its wall. We separate in the response $\eta(r)$ the part connected with this motion:

$$\frac{\eta_0(r)}{\varepsilon_0 - i\omega}\,\frac{\int dr\,r\eta_0 F}{\int dr\,r\eta_0^2} \qquad (10)$$

To calculate η_0 and ϵ_0, we compare the equation for η_0

$$(\hat{H} - \varepsilon_0)\eta_0 = 0 \qquad (11)$$

with the equation for the function

$$\psi_0 = \varkappa(d\ln r / d\ln\rho)^{1/2}(dT / d\rho), \qquad (12)$$

which is of the form[1)]

$$(\hat{H} + \varkappa / nc_e\rho^2)\psi_0 = 0. \quad (13)$$

Eq. (13) can be easily obtained by differentiating (2) with respect to ρ. By virtue of the hermiticity of H and of the smallness of $d\psi_0/dr$ and η_0 on the boundary $\rho = R$, we get from (11) and (13)

$$\varepsilon_0 = -\frac{\int dr\, r\eta_0(\varkappa/nc_e\rho^2)\psi_0}{\int dr\, r\psi_0\eta_0}. \quad (14)$$

Since ψ_0 and η_0 do not reverse sign, we get $\epsilon_0 < 0$ (including, in particular, the case of a thin pinch with radius $\sim l_W$).

In the case of a thick pinch, the main change of T takes place in the region of the wall, accordingly the function ψ_0 has the form of a peak localized near $\rho = \rho_0$, and attenuates exponentially with increasing distance from the wall. Therefore the main contribution to the integrals in (14) is made by the region of the wall and $\epsilon_0 \propto \rho_0^{-2}$. The functions η_0 and ψ_0 behave differently near the center of the column, but are both exponentially small there. When $\rho \sim \rho_0$ the "perturbation" $\kappa/nc_e\rho^2$ in (13) is small, and with this accuracy we have $\eta_0 = \psi_0$ and

$$\varepsilon_0 = -\rho_0^{-2}\left(\int_{\Theta_1}^{\Theta_3} d\Theta\,|d\Theta/d\rho|\right)\left(\int_{\Theta_1}^{\Theta_3} d\Theta\,(nc_e/\varkappa)\,|d\Theta/d\rho|\right)^{-1}. \quad (15)$$

It is possible to substitute in the integrands the derivative $d\Theta/d\rho$ of the stationary distribution, calculated for the case of a flat boundary between phases [3]:

$$|d\Theta / d\rho| = \{2[U(\Theta_3) - U(\Theta)]\}^{1/2}. \quad (15a)$$

In order of magnitude $|\epsilon_0| \sim (l_W/\rho_0)^2 V(0)$ (Fig. 1). We cannot expect at least one discrete eigenvalue ϵ_n (if it exists at all) to be small. Therefore, in the remaining part of the response $\eta(r)$,

$$\sum_{n(\neq 0)} \frac{\eta_n(r)}{\varepsilon_n - i\omega}\frac{\int dr\, r\eta_n F}{\int dr\, r\eta_n^2}, \quad (16)$$

we can neglect at low frequencies ($\ll V(0)$) the contribution of the discrete levels compared with (10), inasmuch as the corresponding frequency denominators are not small.

For the same reason, the function (16) is of the same order both inside and outside the wall. Since the latter one is narrow, this gives a small ($\sim l_W/\rho_0$) contribution to the interval (4). The value of (16) outside the walls is significant, but here $V(r) \approx$ const and $\delta T/\delta E_z$ can be determined in elementary fashion. The contribution from (16) to $Z_d^{-1}(\omega)$ is equal to the sum of the differential electric conductivities of the column $Z_{d3}^{-1}(\omega)$ and the surrounding part of the sample $Z_{d1}^{-1}(\omega)$ (with "cold" plasma), regarded as homogeneous phases with cross sections $S_3 = \pi\rho_0^2$ and S_1. The corresponding specific differential electric conductivities are equal to (i = 1, 3)

$$\sigma_{di}(\omega) = \sigma\{1 + 2E_0^2[(d\sigma/dT)/nc_e](\tau^{-1} - i\omega)^{-1}\}_{T=T_i}, \quad (17)$$

where $\tau(T_i) = nc_e/\kappa l_i^2$ is the energy scattering time.

After substituting (10) in (4) we get finally

$$Z_d^{-1}(\omega) = Z_{d1}^{-1}(\omega) + Z_{d3}^{-1}(\omega) + R_c^{-1}(1 + i\omega/|\varepsilon_0|)^{-1}, \quad (18)$$

where

$$R_c^{-1} = -l_z^{-1}[\sigma(T_3) - \sigma(T_1)]2\pi\rho_0^3/l, \quad (19)$$

$$l \equiv \left(\int_{\Theta_1}^{\Theta_3} d\Theta\,|d\Theta/d\rho|\right) \Big/ 2E_0^2\int_{\Theta_1}^{\Theta_3} d\Theta\,\sigma(\Theta). \quad (20)$$

The length l is of the order of thickness of the column wall.

Thus, the equivalent circuit of a sample with a current column is made up of the parallel-connected impedances of the column (Z_3), the surrounding plasma (Z_1), and the "impedance of the wall." The latter is made up of the resistance $R_W < 0$ and the inductance $L_W = R_W/\epsilon_0$.

The radius of the pinch can be connected with the current I, using the fact that the density of the current is almost homogeneous both inside the pinch (j_3) and outside (j_1):

$$I = j_1(E_z)\pi R^2 + [j_3(E_z) - j_1(E_z)]\pi\rho_0^2. \quad (21)$$

3. CONCLUSIONS

Differentiating (21) with respect to E_z and comparing with $Z^{-1}(0)$, we get $d\rho_0/dE_z$, and from it the dependence of E_z on ρ_0:

$$(E_z - E_0)/E_0 = l/\rho_0. \quad (22)$$

The same result can be obtained also directly from Eq. (2) for the stationary distribution $T(\rho)$.

The static differential electric admittance $Z^{-1}(0)$ of a sample with a column goes through zero when

$$(E_z - E_0)/E_0 = l/\rho_0 = [2(\sigma(T_3) - \sigma(T_1))l^2/\sigma_{d1}R^2]^{1/3} \quad (23)$$

(the equality holds true if the right-hand side is small compared with unity). At smaller values of $E_z - E_0$, the admittance $Z^{-1}(0) < 0$ (the drooping section of the characteristic), and for large values $Z^{-1}(0) > 0$. It follows from (18) that $Z^{-1}(\omega)$ has no zeroes in the upper half-plane of ω, and consequently the column is stable in the specified-current regime in the case when $Z^{-1}(0) < 0$, i.e., on the drooping part of the current voltage curve. It is unstable regardless of load if $Z^{-1}(0) > 0$.

When ρ_0 exceeds the value given by (23), i.e.,

$$I/\pi R^2 - j_1(E_0) \gg (3/2^{2/3})E_0[\sigma(T_3) - \sigma(T_1)]^{1/3}\sigma_{d1}^{2/3}(l/R)^{2/3}, \quad (24)$$

the absolute magnitude of the "wall" conductivity $|R_W^{-1}| \gg Z_1^{-1}(0) + Z_3^{-1}(0)$. In this case the static characteristic of the sample with the current column is practically vertical: an increase of the current takes place as a result of an increase of the column radius.

Owing to the small value of the characteristic frequency $|\epsilon_0| \sim (l/\rho_0)^2\tau^{-1}$ (here τ is of the order of the relaxation time of the energy of the electrons in the homogeneous plasma), the impedance $Z(\omega)$ experiences dispersion even at low frequencies $\omega \ll \tau^{-1}$. Figure 2 shows schematically $\mathrm{Re}Z^{-1}(\omega)$. It becomes positive at the frequency

$$\omega_0 = |\varepsilon_0|\,|Z^{-1}(0)|^{1/2}(Z_1^{-1}(0) + Z_3^{-1}(0))^{-1/2} \sim (l/\rho_0)^{3/2}\tau^{-1} \times[(\sigma_3 - \sigma_1)/(\sigma_{d1}R^2/\rho_0^2 + \sigma_{d3} - \sigma_{d1})]^{1/2}, \quad (25)$$

which is proportional to the small parameter $(l/\rho_0)^{3/2}$.

[1)] The idea of using the equation for the derivative of the stationary distribution dates back, apparently, to the paper by Zel'dovich and Barenblatt on the stability of a flat flame front [7].

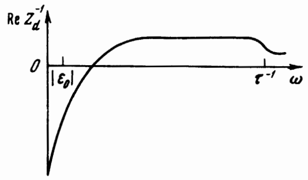

FIG. 2. Frequency dependence of the real part of the differential conductivity of a sample with a current pinch.

With increasing frequency ω, owing to the "inertia" of the column wall, which determines the inductance L_W, the oscillations of the wall become negligible. The contribution of the oscillations of the column wall to $\operatorname{Im} Z^{-1}(\omega)$ vanishes when

$$\omega > (l/\rho_0)\tau^{-1}[(\sigma_3 - \sigma_1)/(\sigma_{d1}R^2/\rho_0^2 + \sigma_{d3} - \sigma_{d1})].$$

At higher frequencies, the differential electric conductivity of the sample is equal simply to $Z_1^{-1} + Z_3^{-1}$ (it experiences dispersion when $\omega \sim \tau^{-1}$).

Vertical or near-vertical static current-voltage characteristics, connected with the pinching of the current, were observed in a large number of semiconductors (n-InAs at helium temperatures, in compensated germanium and silicon at low-temperature impurity breakdown). But investigations with an alternating signal, insofar as we know, have not been carried out under these conditions.

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