

A CALCULATION OF THE EFFECTIVE CROSS SECTION FOR LOSS OF K-ELECTRONS
 BY FAST HELIUM-LIKE IONS COLLIDING WITH HYDROGEN AND HELIUM ATOMS

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The cross section for K-electron loss by fast helium-like H^- , He^0 , Li^+ , and Be^{2+} ions and also by Be atoms colliding with hydrogen or helium atoms is calculated in the nonrelativistic Born approximation for particle velocities lying between 0.6×10^8 and 60×10^8 cm/sec. Simple approximate formulas are derived for the cross section for K-electron loss by arbitrary particles. The results are compared with similar ones for hydrogen-like particles. A theoretical explanation is given of the experimentally observed identity of the mean cross section for electron loss on the external K- and L-shells for ions with equal ionization potentials.

1. INTRODUCTION

UNTIL recently, the cross section for electron loss by fast ions with an arbitrary nuclear charge z colliding with neutral atoms were calculated for a wide range of relative velocities v of the colliding particles only for the hydrogen-like atoms colliding with hydrogen and helium atoms^[1]. In the present paper are given results of similar calculations of the cross section for the electron loss by the helium-like particles H^- , He^0 , Li^+ , and Be^{2+} .

The computations were done using the Born approximation. The hydrogen-like wave functions for electrons in a Coulomb field of an effective charge $Z^*e = (Z - 0.312)e$ ^[2] were used as the wave functions for the initial and final states of the electron. The difference between the actual binding energy I of the electron and the binding energy $mv_0^2 Z^{*2}/2$ in a Coulomb field of a charge Z^*e was taken into account, as usual, by introducing a parameter of external screening $\theta = 2I/mv_0^2 Z^{*2}$ ^[2,3] where m is the electron mass and $v_0 = e^2/\hbar$ is the atomic unit of velocity.

The possibility of using the Born approximation in the calculations of the cross section for the electron loss by ions colliding with neutral atoms was discussed in^[1]. In the case of electron loss by helium-like particles colliding with neutral atoms having a nuclear charge Z^*e , the Born approximation is correct only for particles with an effective charge $Z^* \leq Z_m$ and with $v > Z_m v_0$, and for ions with $Z^* \gg Z_m$ and $v \gtrsim (0.1 \text{ to } 0.2) Z v_0$.

2. GENERAL FORMULAS THE K-ELECTRON LOSS CROSS SECTIONS

In a manner similar to that used in^[1], we shall apply to the electron loss processes the known expression for the inelastic-scattering cross sections in the Born approximation using the sum rule for the total cross section for K-electron loss by ions with two K-electrons,^[4] and obtain the following expression:

$$\sigma = 16\pi a_0^2 \left\{ \left(\frac{Z_m v_0}{Z^* v} \right)^2 \int_{q_{min}}^{q_{max}} (1-F)^2 \frac{dq}{q^3} \int_{k_{min}}^{k_{max}} \epsilon_{kq}^2 dk + \frac{Z_m^2 v_0^2}{Z^{*2} v^2} \int_{q_{min}}^{q_{max}} (1-I^2) \frac{dq}{q^3} \int_{k_{min}}^{k_{max}} \epsilon_{kq}^2 dk \right\}, \quad (1)$$

where $a_0 = \hbar^2/me^2$ is the atomic unit of length; $q = Qa_0/Z^*$, Q is the change of the ion wave vector as result of the collision; $k = Ka_0/Z^*$, where K is the wave vector of the final-state electron at a large distance from the ion; F is the atomic form factor. For hydrogen and helium atoms $F = [1 + (Qa_0/2Z_m^*)^2]^{-2}$, where Z_m^*e is the effective nuclear charge (for hydrogen atoms $Z_m^* = 1$, and for helium atoms $Z_m^* = 1.688$).

An equation for ϵ_{kq}^2 was derived in^[5,6] in a description of the unbound (final) electron state by means of a non-relativistic Coulomb function

$$\epsilon_{kq}^2 dk = \frac{256q^2(q^2 + 1/3 k^2 + 1/3)}{(1 - e^{-2\pi/\hbar})[(q^2 - k^2 + 1)^2 + 4k^2]^3} \exp \left[-\frac{2}{k} \arctg \frac{2k}{q^2 - k^2 + 1} \right] k dk. \quad (2)$$

The limits of the integration over q and k in (1) are determined by the laws of the energy and the momentum conservation. The change of the relative particle velocity in the considered collisions is much smaller than its absolute value ($\Delta v \ll v$), and, therefore we have for the limits of the integration

$$q_{min} = \frac{\Delta E a_0}{Z^* v \hbar}, \quad q_{max} = \frac{2M v a_0}{Z^* \hbar}, \quad (3)$$

where ΔE is the change of the internal energy of the colliding particles and M is the reduced mass of the system. The cross section for the electron loss is practically independent of q , and therefore one can assume that $q_{max} = \infty$.

According to Arthurs and Bates^[7,8] one should assume for the ionization of any atomic particle that $k_{min} = 0$. Therefore k_{max} can be determined from the equation

$$q \frac{v}{Z^* v_0} = \frac{\Delta E}{Z^{*2} m v_0^2} = \frac{k_{max}^2}{2} + \frac{\theta}{2} + \frac{\Delta E_c}{Z^{*2} m v_0^2}, \quad (4)$$

where ΔE_m is the internal energy change of the atoms of the medium.

The first of the two terms in (1) corresponds to pseudoeelastic collisions that leave the atoms of the medium unchanged, and the second corresponds to pseudoinelastic collisions that excite or ionize the atoms of the medium. Hence for the pseudoeelastic part of the cross section σ_{el} , for which $\Delta E_m = 0$, we have

$$\sigma_{el} = 16\pi a_0^2 \left(\frac{Z_m}{Z^{*2}s} \right)^2 \int_{\theta/2s}^{\infty} (1-F)^2 \frac{dq}{q^3} \int_0^{(2qs-\theta)^{1/2}} \epsilon_{hq^2} dk, \quad (5)$$

where $s = v/Z^*v_0$.

For $s \ll \theta$, the main contribution to σ_{el} is made by the collisions with $q \approx \theta/2s$ and $k \approx 0$, for which $\epsilon_{kg}^2 = 2^8/kg^{-8}$. Therefore for $s \ll 0$ and $s < \theta Z^*/4Z_m^*$, when the values of $(1-F)^2$ are close to unity for $q \approx \theta/2s$, the limiting expression obtained for σ_{el} agrees with the limiting equation for the cross section for the knockout of the K-electron by the nucleus^[2]:

$$\sigma_{el} = \frac{2^{20}}{45} \pi a_0^2 Z_m^2 s^8 / Z^{*4} \theta^9. \quad (6)$$

Taking into account the properties of the quantity ϵ_{kq}^2 , in analogy with the procedure used in^[1], we get for the value of σ_{el} in the high velocity region $s \geq 3$

$$\sigma_{el} = 16\pi a_0^2 \left(\frac{Z_m}{Z^{*2}s} \right)^2 \left\{ \int_1^{2s} (1-F)^2 \frac{dq}{q^3} + 0,28 \int_{(1/4+\theta)/2s}^1 (1-F)^2 \frac{dq}{q} \right\}. \quad (7)$$

For $Z^* = Z$ and $\theta = 1$, formula (7) agrees with the corresponding expression in^[1] for the doubled cross section for K-electron loss by hydrogen-like ions.

From (7), recognizing that $(1-F)^2$ is close to $Z^*q^2/2Z_m^*$ when $q < Z_m^*/Z$ and is almost unity, when $q > 2Z_m^*/Z$, we get that for $s > 3$ the following relations hold with an accuracy of 15%: for ions with $Z^* \leq 2/3Z_m^*$

$$\sigma_{el} = 4\pi a_0^2 \frac{Z_m^2}{Z_m^{*2}Z^{*2}s^2} (1 - Z_m^*/2s^2 Z^{*2}); \quad (8a)$$

for ions with $Z^* = (2/3 - 3/3)Z_m^*$

$$\sigma_{el} = 3\pi a_0^2 \frac{Z_m^2}{Z_m^* Z^{*3} s^2} (1 - 2Z_m^*/3s^2 Z^{*2}); \quad (8b)$$

for ions with $Z^* \geq 8/3 Z_m^*$

$$\sigma_{el} = 8\pi a_0^2 \left(\frac{Z_m}{Z^{*2}s} \right)^2 \left\{ 1 - \frac{1}{4s^2} + 0,56 \ln A \right\}. \quad (8c)$$

where A is equal to the smaller of two quantities $Z^*/2Z_m^*$ or $8s/(1+4\theta)$.

For the pseudoinelastic part of the cross section we have:

$$\sigma_{inel} = 16\pi a_0^2 \frac{Z_m}{(Z^{*2}s)^2} \int_{q_{min}}^{\infty} (1-F^2) \frac{dq}{q^3} \int_0^{k_{max}} \epsilon_{hq^2} dk, \quad (9)$$

where $k_{max} = (2qs - \theta - 2\Delta E_m / Z^* m v_0^2)^{1/2}$, and q_{min} is determined from the equation $k_{max} = 0$. At the same time $\Delta E_m = I_m + \hbar^2 k^2 / 2m$, where I_m is the electron binding energy in the atoms of the medium, and k is the wave number of an electron emitted by the atom of the medium. For $\hbar Q > Z_m^* v_0$, i.e., for $q > Z_m^*/Z^*$, we have according to^[1] $\overline{k_m^2} = Q^2 = (qZ^*/a_0)^2$ and $q_{min} = s - (s^2 - \theta - I_m/Z^* I_0)^{1/2}$, where $I_0 = m v_0^2 / 2$.

Equation (9) holds for $s \geq [3(\theta - I_m/Z^* I_0)]^{1/2}$, when according to^[1] k_{max} exceeds q in a wide range of q , from q_1 to q_2 , such that $q_2/q_1 \geq 3$. At lower velocities, Eq. (9) with $\overline{k_m^2} = Q^2$ gives too low values of σ_{inel} , as shown in^[1]. For $s < 1$, the contribution of

σ_{inel} to the total cross section for electron loss $\sigma_{el} + \sigma_{inel}$ is negligible, since in that velocity region $\sigma_{inel} \ll \sigma_{el}$. In the same way as (7) results from (5), we get from (9) for $s \geq 3$

$$\sigma_{inel} = 16\pi a_0^2 \frac{Z_m}{(Z^{*2}s)^2} \left\{ \int_1^s (1-F^2) \frac{dq}{q^3} + 0,28 \int_{q'}^1 (1-F^2) \frac{dq}{q} \right\}, \quad (10)$$

where $q' = (1/4 + \theta + I_m/Z^* I_0)/2s$.

Taking it into account that for $q > Z_m^*/Z^*$ quantity $(1-F^2)$ is close to unity and for $q \ll Z_m^*/Z^*$ it is close to $(Z_m^*q/Z_m^*)^2$, Eq. (10) yields for $s > 3$, with an accuracy 15%, for ions with $Z^* = (0.1-0.6) \cdot 2Z_m^*$

$$\sigma_{inel} = 6,4\pi a_0^2 \frac{Z_m}{Z^{*3} Z_m^* s^2} \left\{ 1 - \frac{5}{4} \frac{Z_m^*}{Z^{*2} s^2} \right\}, \quad (11a)$$

and for ions with $Z^* \geq 0,6 \cdot 2Z_m^*$

$$\sigma_{inel} = 8\pi a_0^2 \frac{Z_m}{(Z^{*2}s)^2} \{1 - s^{-2} + 0,56 \ln B\}, \quad (11b)$$

where B is the smaller of the quantities Z^*/Z_m^* and $2s/(\theta + 1/4 + I_m/Z^* I_0)$.

Since $\theta \leq 1$, and since decreasing θ widens the range of the possible values for k and q , which are the variables of the integration in the general expression for the K-electron loss cross section, it follows that for any velocity the K-electron loss cross section ($\sigma(m, Z^*, \theta, v)$) for ions with two K electrons ($m = 2$) is not smaller than the doubled cross section for electron loss by hydrogen-like atoms ($m = 1, \theta = 1$) with a nuclear charge equal to the effective charge of the considered ions, i.e., $\sigma(2, Z^*, \theta, v) \geq 2\sigma(1, Z^*, \theta, v)$. For high velocities, as follows from (8) and (10), the quantity $\sigma(2, Z^*, \theta, v)$ practically coincides with the doubled value of $\sigma(1, Z^*, 1, v)$. Therefore for high velocities we have

$$\sigma(2, Z^*, \theta, v) < 2\sigma(1, Z^* \sqrt{\theta}, 1, v),$$

i.e., the considered cross sections are smaller than the doubled cross sections $\sigma(1, Z^* \sqrt{\theta}, 1, v)$ for hydrogen-like particles with charge $Z^* \sqrt{\theta}$, the electron binding energy of which agrees with the electron binding energy of the considered ions. From the approximate relation for the ionization cross section in the collisions with the unscreened nucleus with $s < 1$, given in the review article by Merzbacher and Lewis^[2], it follows that in the velocity range where σ_{el} is proportional to v^k , we have $\sigma_{el}(2, Z^*, \theta, v) = 2\theta^\alpha \sigma(1, Z^* \sqrt{\theta}, 1, v)$, where $\alpha = 1 - k/2$. Therefore for very small velocities, when $k = 8$, just as it was obtained from (6), we get $\alpha = -3$. In the high-velocity region the inverse dependence of cross sections on θ is observed at fixed values of the binding energies of the lost electron: for ions with large charges Z^* , according to (8c) and (11b), the exponent α is close to unity, and for ions with small charges Z^* , according to Eqs. (8a) and (11a), we have $0 < \alpha < 0,5$, i.e., $0 < \alpha < 1$ for all ions.

3. DISCUSSION OF THE RESULTS

Using (5) and (9), we calculated the cross sections for K-electron loss by helium-like ions H^-, He^0, Li^+, Be^{2+} in collisions with the atoms of hydrogen and helium, and the K-electron loss cross section for atoms of beryllium scattered by helium atoms. In these cases

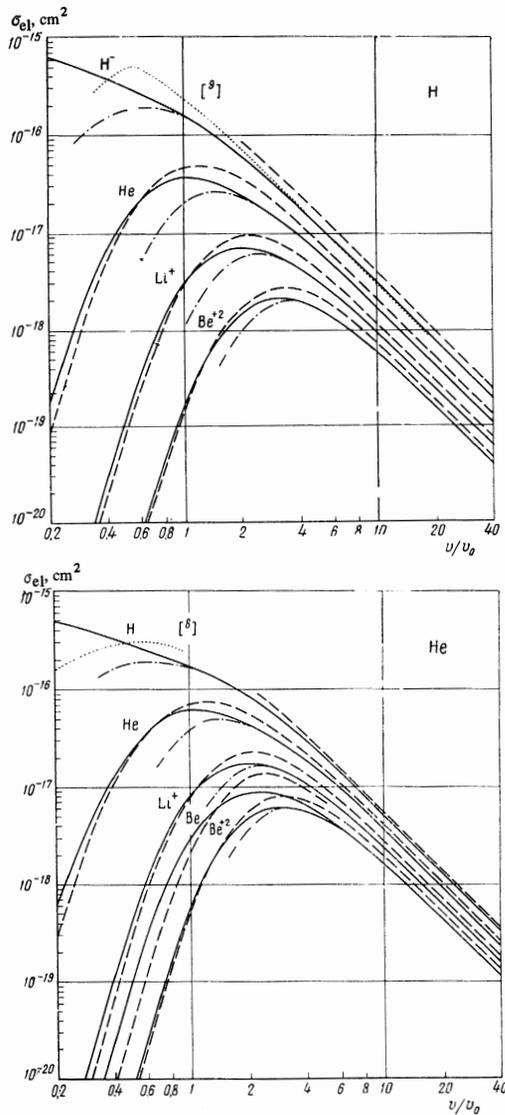


FIG. 1. $\sigma_{el}(2, Z^*, \theta; v)$ vs. v/v_0 in hydrogen and helium. The ions are indicated next to the curves. The dashed line shows the values of $2\sigma_{el}(1, Z^*, 1, v)$. Shown are also the results of the calculations by Sida [8] and McDowell and Peach [9].

$Z^* = Z - 0.312$, and the values of θ were correspondingly 0.117, 0.64, 0.78, 0.85, and 0.61. The obtained results for σ_{el} are shown in the Fig. 1. For comparison, the doubled cross section for the hydrogen-like ions with the nuclear charge equal to Z^* and $Z^*\sqrt{\theta}$ are also shown there. The latter were obtained by interpolation of the results for the cross section of ions with even nuclear charge Z , given in [1].

In the region $v \geq 1.5Z^*v_0$, as it is evident from the Fig. 1, the obtained values of $\sigma_{el}(2, Z^*, \theta, v)$ practically agree with the values of $2\sigma_{el}(1, Z^*, 1, v)$. For ions with $Z^* \gtrsim 1.5$ and $v \approx 0.4Z^*v_0$ the cross sections $\sigma_{el}(2, Z^*, \theta, v)$ are close to $2\sigma_{el}(1, Z^*, \sqrt{\theta}, 1, v)$ and in the low-velocity region they approach $2\theta^\alpha \sigma(1, Z^*, \sqrt{\theta}, 1, v)$ with $\alpha = 1 - k/2$, where $k = d \log \sigma(1, Z^*\sqrt{\theta}, 1, v)/d \log v$.

The values of $\sigma_{inel}(1, Z^*, \theta, v)$ obtained using Eq. (9) for $v > 3Z^*v_0$ practically coincide, in accordance with Eqs. (11a) and (11b), with the values of

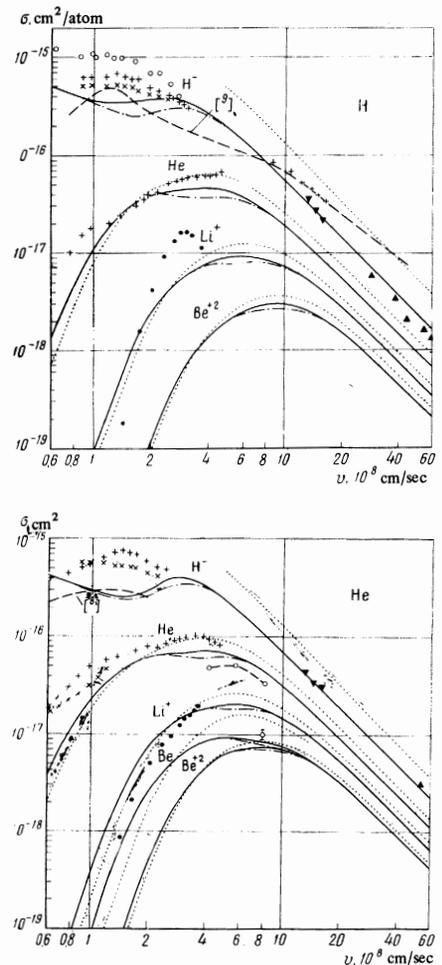


FIG. 2. The total cross section for K-electron loss vs. velocity for ions with two K-electrons colliding with hydrogen and helium atoms. The ions are indicated next to curves. The dot-dash line indicates the lower limit for the possible values of σ . The dotted curve shows the values of $2\sigma(1, Z^*\sqrt{\theta}, 1, v)$. Shown also are the results of the calculation by Sida [8] and McDowell and Peach [9]. The experimental results for σ are indicated by: + and X for H^- and He in H_2 and He (from the review article [10]); v and Δ for H^- in H_2 and He ([11] and [12]); O is for H^- in H [13]; Φ is for Li^+ and Be^{2+} in He [15].

$2\sigma_{inel}(1, Z^*, 1, v)$. For $v \leq 3Z^*v_0$, where (9) gives too low a value for σ_{inel} , the cross sections σ_{inel} were calculated assuming that

$$\sigma_{inel}(2, Z^*, \theta, v) = 2\sigma_{inel}(1, Z^*, 1, v) \frac{\sigma_Y(2, Z^*, \theta, v/2)}{2\sigma_Y(1, Z^*, 1, v/2)}$$

The values of $\sigma_{inel}(1, Z^*, 1, v)$ were obtained by interpolating the results given in [1] for $\sigma_{inel}(1, Z, 1, v)$, with $Z = 1, 2, 3, 5$ and 10.

The obtained cross sections for K-electron loss $\sigma = \sigma_{el} + \sigma_{inel}$, are shown in Fig. 2 (continuous lines). The dash-dot line marks the lower limit of the possible values of σ , corresponding to the cross sections $\sigma_{inel}(2, Z^*, \theta, v) = 2\sigma_{inel}(1, Z^*, 1, v)$. The difference between the values of σ shown by the continuous and dash-dot curves gives an idea of the possible error in the value of σ . This difference, as is clear from Fig. 2, is limited by the velocity region where the cross section is close to the maximum value and, in general, does not exceed 15%; only for He atoms in hydrogen

with $v = (3-5) \times 10^8$ cm/sec and H^- ions in hydrogen with $v = (1.5-2.0) \times 10^8$ cm/sec does it amount to 25 and 40% correspondingly.

In Figs. 1 and 2 are shown the results for the cross sections for H^- ions calculated in the Born approximation by Sida^[8] and McDowell and Peach^[9]. The former calculated the cross section for electron loss by H^- ions in the collisions with helium atoms as a result of which the hydrogen and helium atoms were left in the ground states. This cross section is the main component of σ_{el} ; the expression for it differs from the general equation only by the presence of the factor $2^6(Z^*)^3(Z/(Z+Z^*))^6$, which for the H^- ions is equal to 0.904. In^[9] were calculated the partial cross sections for electron loss by the H^- ions scattered from the hydrogen atoms, as a result of which the fast and slow atoms in states 1s and 2p were produced. Figure 1 shows the sum of two cross sections calculated in^[9], which correspond to the production of fast hydrogen atoms in states 1s and 1p when the atoms of the target were left in 1s state. This sum gives a value close to σ_{el} . In Fig. 2, the sum of all four partial cross sections is presented.

The initial state of the H^- ions was described in^[8,9] by the same type of function as in the present work. To describe the electron with the continuous spectrum, they used a variational function with three parameters in^[8] and a plane wave in^[9]. From Fig. 1 one can see that for $v \geq 3v_0$ the value of σ_{el} obtained in the present work practically coincides with the results of^[9]. The use of different wave functions in the description of the final state of the electron leads to an appreciable difference in the value of σ_e for $v \lesssim v_0$, where the Born approximation is not quite adequate. The sum of two partial cross sections evaluated in^[9], corresponding to collisions with the transition of the target atoms to the 2p state, happens to be considerably higher for $v > 4v_0$, than the value of σ_{el} obtained by us. Therefore for $v > 10v_0$, as is evident from Fig. 2, the sum of the four partial cross sections in^[9] is 2 to 2.5 times larger than the total cross section for electron loss obtained in our work and in the experiment^[11,12].

The cross sections for K-electron loss calculated in the Born approximation for hydrogenlike ions, as mentioned in^[1], agree with the experiment for $v \gtrsim 2Z_m^*v_0$. In the same velocity region one should observe agreement between the theoretical and experimental values of the cross sections also for the considered particles. From Fig. 2 it is clear that for H^- ions the calculated values of the cross sections differ from the experimental ones^[11-13] by no more than 20% when $v \gtrsim 3 \times 10^8$ cm/sec¹⁾. Comparing the calculated and the experimental results for other helium-like particles, one should keep in mind that the latter could be higher, owing to the presence in the ion beam of the excited particles in long-lived metastable states. The

¹⁾The evaluated cross sections, according to^[16], should be compared with the experimental values for $\sigma_{2-2, z-1} + 2\sigma_{2-2, z}$ (first index of σ indicates the initial charge of the ion and the second one for final one). However, for most of the experimental cross sections shown in Fig. 2, the corresponding values of $\sigma_{2-2, z}$ are unknown, and the biggest ones of the known ones (for H^- ions) do not exceed the value of $\sigma_{2-2, z-1}$ by 5 to 9%. Accordingly, only the experimental results for $\sigma_{2-2, z-1}$ are shown in Fig. 2.

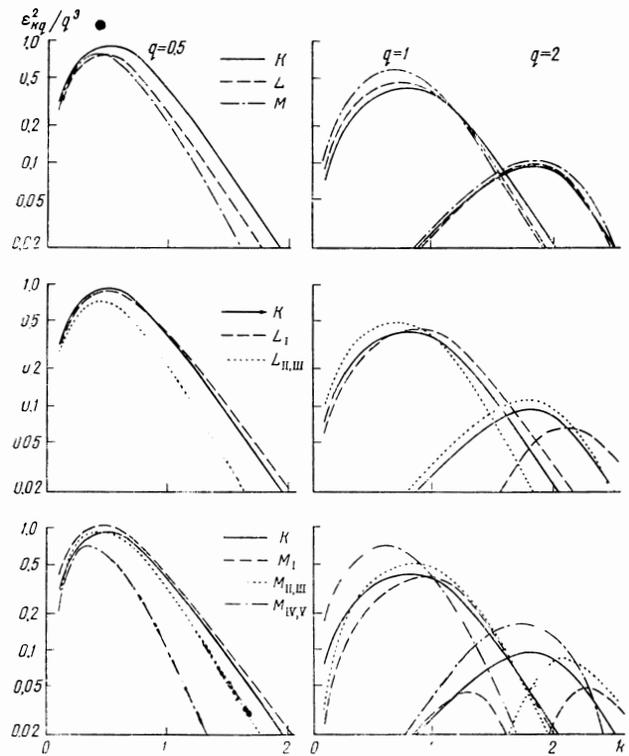


FIG. 3. ϵ_{kq}^2/q^3 vs. $k = Ka_0/(Z^*/n)$, for three fixed values of $q = Qa_0/(Z^*/n)$. The upper, middle and lower graphs show values of ϵ_{kq}^2/q^3 for totally filled K-, L-, and M-shells respectively, for L_I -, L_{II} - and L_{III} -subshells and M_I -, $M_{II,III}$ - and $M_{IV,V}$ -subshells as compared with the K-shell.

experimental values of the cross sections for He atoms with $v < 5 \times 10^8$ cm/sec and Li^+ ions with $v = (4-8) \times 10^8$ cm/sec, shown in Fig. 3, were obtained by sending them through hydrogen and celluloid targets respectively. This is the reason why some of those particles were in a metastable excited state. The experimental values for $v \gtrsim 3 \times 10^8$ cm/sec are therefore higher than the calculated ones. For Li^+ ions with $v < 4 \times 10^8$ cm/sec, the experimental results for σ were obtained in the experiment with ion beams obtained directly from a thermo-ionic source, where probability for the creation of metastable states is low. For $v < 4 \times 10^8$ cm/sec these values of the cross section were close to the calculated ones. The experimental value of σ for Be^{2+} ions with $v = 8 \times 10^8$ cm/sec was obtained from experiments the ion beams from an arc cyclotron source. These values exceeded the results of the calculations by 30%.

The results of the present work can also be used to estimate the cross section for electron loss from other atomic shells. Comparison of the square of the matrix element ϵ_{kq}^2 for K-electrons with known values of ϵ_{kq}^2 ^[17,18] for L- and M-shell electrons with principal quantum numbers $n = 2$ and 3 shows (Fig. 3) that in the range of values $q = Qa_0/(Z^*/n)$ and $k = Ka_0/(Z^*/n)$ that are significant for the calculations of the electron loss cross sections at $v \gtrsim (Z^*/n)v_0$, the value of ϵ_{kq}^2 for L-electrons differs, in general, by no more than 20% from ϵ_{kq}^2 for K-electrons with the same values of q and k . For the M-shell electrons a larger difference between values of ϵ_{kq}^2 was observed. Hence, for the

same values of Z^*/n and θ (or I and θ), the cross sections for the K- and L-electrons should not differ by more than 20%.

According to the known experimental data^[16], the mean values for the electron loss cross sections for one of the electrons from the outer K- or L-shells of ions depend, with a precision of 20%, on one parameter that characterizes the initial states of electrons - the electron binding energy I . For ions with $v \sim 8 \times 10^8$ cm/sec for which the Born approximation gives cross sections that agree with the experimental result, there exist among the ions with close values of I some having θ from 1 to 0.6. For these ions $I \sim 50$ eV and $Z^*/n = (I/\theta I_0)^{1/2} \sim 2.5$. In this case, according to the results of our work, for fixed values of I , the cross sections should be approximately proportional to θ^α , with $\alpha \approx 1$, so that for ions with $\theta = 0.6$ the value of the cross section would be 40% lower than for the ions with $\theta = 1$ at the same binding energy I . In the region $v \sim (2-4) \times 10^8$ cm/sec where the Born approximation yields cross sections 1.5 to 2 times the experimental values, the experimental results for the cross section are known for ions with θ from 1 to 0.3-0.5 and I from 25 to 100 eV. The exponent α in this case is $\sim 1/3$, so that for ions with the same values of I the cross sections for electron loss should differ by 20 to 40%.

Therefore, there is a theoretical basis to the observed agreement between the average values of the cross sections for the electron loss from outer K- and L-shells by ions with the same ionization potential; in the region of small velocities it is qualitative, and in the high velocities region it is quantitative.

Note added in proof (March 12, 1968). We have received a short time ago some new experimental data for the electron loss cross sections for hydrogen and helium atoms, obtained by sending He^+ ions through the helium target with $v \sim (2 \text{ to } 4) \cdot 10^8$ cm/sec (A. B. Wittkower, G. Levy, and H. B. Gilbody, Proc. Phys. Soc., **91**, 862, 1967). These cross sections are 15 to 20% lower than the ones previously known, and they practically agree with the results of the present work.

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