CYCLOTRON WAVES IN A DEGENERATE ELECTRON FLUID

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The spectra of some electromagnetic waves that can propagate in a degenerate electron Fermi liquid in a magnetic field are considered in detail. Dispersion equations are obtained for the ordinary, extraordinary, and plasma waves propagated across the magnetic field. It is shown that a strong interaction of oscillations with frequencies close to the limiting frequencies of excitation of the homogeneous field in the electron fluid is possible even at long wavelengths. Some possibilities of determining experimentally the quantities characterizing the correlation of electrons and responsible for the difference between the electron fluid and a gas are discussed.

FOLLOWING the experimental observation of spin waves in alkali metals[1,2], intense experimental investigations were initiated in a large number of laboratories throughout the world[3], aimed at observing effects due to the fact that the conduction electrons of metals constitute a degenerate Fermi liquid[4,5]. It is appropriate in this connection to call attention to the possibility of revealing Fermi-liquid effects in the already employed experimental setup. Namely, we shall speak of the so-called electron cyclotron waves.

The possibility of propagation of electromagnetic waves with frequency close to the gyroscopic frequency Ω of the electron in an electron gas of high density was pointed out by Drummond[6]. The theory of such waves was developed also by others. The most complete exposition can be found in the paper by Stepanov[7], where the necessary references are also given. In metallic potassium, electron cyclotron waves were observed by Walsh and Platzman[8,9]. Their experimental results are of particular interest to us, since they contain information regarding long-wave cyclotron waves. Namely, in the region of large wavelengths, the natural frequencies of the oscillations of the electron liquid can differ from the ordinary gyroscopic frequency of the electrons and its harmonics[10,11].

According to the Landau theory of the Fermi liquid, the energy of the quasiparticle (electron) depends on the distribution function, as is manifest in the following manner in the relation

$$\delta\hat{\varepsilon} = \mathrm{Sp}_{\sigma'} \int d\mathbf{p}' \{\varphi(\mathbf{p}, \mathbf{p}') + (\hat{\sigma}\hat{\sigma}')\psi(\mathbf{p}, \mathbf{p}')\} \delta\hat{n}(p'), \quad (1)$$

which connects the change of the energy operator $\delta\hat{\varepsilon}$ with the change of the electron density spin matrix $\delta\hat{n}$. In formula (1) $\hat{\sigma}$ is the spin operator, and the trace is taken over the spin states of the electron. The functions φ and ψ cause the difference between the electron liquid and a gas. With this, experiments aimed at the study of spin waves[1,2] afford a possibility of measuring the function ψ (see[11,12]). The function φ has so far not been determined experimentally. We shall show that information concerning the function φ can be obtained by studying cyclotron waves and, in particular, attempts can be made to obtain such information from the results of Walsh and Platzman[8,9].

When speaking of electron cyclotron waves, one usually has in mind electromagnetic waves propagating transversely to a constant magnetic field. Orienting the wave vector **k** along the x axis and the constant magnetic field **B** along the z axis, it is possible to write down for an isotropic electron liquid the dielectric tensor in the form

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}. \quad (2)$$

In accordance with (2), the dispersion equation relating the frequency ω and the wave vector k of the electromagnetic waves breaks up into two equations:

$$k^2c^2 / \omega^2 = \varepsilon_{zz}, \quad (3)$$

$$(k^2c^2 / \omega^2)\, \varepsilon_{xx} = \varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}\varepsilon_{yx}. \quad (4)$$

Equation (3) describes the so-called ordinary waves, and Eq. (4) the extraordinary and plasma waves. In order to understand the possibility of observing the Fermi-liquid effects in the study of cyclotron waves in metals, it is necessary to consider the solutions of the dispersion equations (3) and (4). Inasmuch as for short-wave perturbations with wavelength much smaller than the Larmor radius of the electron the difference between the Fermi liquid and a gas is negligibly small, we turn to the opposite limit, that of long waves. In this case, say for ϵ_{zz}, we can write, neglecting collisions, the following approximate equation:

$$\varepsilon_{zz} = 1 - \frac{\omega_p^2}{\omega^2}\left\{1 + \frac{k^2v^2}{10\omega}\, \frac{(1+\alpha_1)(1+\alpha_2)}{\omega - \omega_{1,2}(k)}\right\}, \quad (5)$$

where

$$\omega_{1,2}(k) = (1+\alpha_2)\left\{\Omega + \frac{k^2v^2}{7}\left[\frac{7(1+\alpha_1)+3(1+\alpha_3)}{10\omega} + \frac{1+\alpha_3}{\omega - 2(1+\alpha_3)\Omega}\right]\right\}. \quad (6)$$

Here $\Omega = eBv/pc$ is the Larmor (gyroscopic) frequency of the electron, $\omega_p = \sqrt{4\pi e^2 N(1+\alpha_1)/m^*}$ is the plasma frequency of the electron liquid, N is the number of electrons per cubic centimeter, and $m^* = p/v$ is the effective mass of the electron on the Fermi sphere. Finally, α_n are the coefficients of the following expansion:

$$\frac{p^2}{\pi^2\hbar^3 v}\varphi(\mathbf{p}, \mathbf{p}') = \sum_{n=0}^{\infty} (2n+1)\alpha_n P_n(\cos\Theta), \quad (7)$$

where Θ is the angle between the vectors p and p′, lying on the Fermi sphere, and $P_n(\cos\Theta)$ are Legendre polynomials.

Formula (5) makes it possible to obtain readily the spectrum of the ordinary waves in the vicinity of

$$\omega = \Omega(1+\alpha_2) \equiv \omega_{1,2}(0). \tag{8}$$

Under the conditions of the experiments of [8,9], the following inequalities are satisfied:

$$1 \ll k^2c^2/\omega^2 \ll \omega_p^2/\omega^2. \tag{9}$$

Expression (8) represents in this case the limiting frequency of the ordinary wave, corresponding to the limit $k = 0$. Owing to the influence of the Fermi-liquid effects, this limiting frequency differs from the Larmor frequency of the electron Ω 1). This fact is contained in the experimental data of [8]. If we relate it with the theory of the predictions of an electron liquid, then it is possible to obtain the following estimate: $\alpha_2 \approx -(0.02 - 0.03)$. Presumably a reliable estimate of α_2 would call for a detailed study of the long-wave ordinary cyclotron waves in the direct vicinity of the frequency (8).

It can be assumed that the α_n with $n \geq 2$ are small compared with unity. Then, neglecting them throughout, except in the resonant expressions, we can write the following solution of Eq. (3):

$$-\omega + \Omega(1+\alpha_2) = k^2v^2/10\Omega, \tag{10}$$

which is obtained when the inequalities (9) are satisfied.

The importance of studying the extraordinary waves in the vicinity of $\omega_{1,2}(0)$ is also due to the fact that, at not too large wavelengths, oscillations can arise near the frequency $\omega_{1,4}(0) = \Omega(1+\alpha_4)$, which by virtue of the smallness of α_4 is close to the frequency $\omega_{1,2}(0)$. According to the theory of the natural oscillations of a Fermi liquid [11], there is an infinite number of such frequencies. However, in order to be able to distinguish between them, it is necessary to satisfy the inequality

$$\alpha_n\Omega\tau \gg 1, \tag{11}$$

where τ is the electron free path time. Therefore the determination of the parameters α_n with large numbers n cannot be carried out in samples that are not very pure. Bearing in mind this remark, we can use in the analysis of the experimental data the theoretical formulas obtained under the assumption that a small number of coefficients α_n differs from zero. Thus, in the case of $\alpha_n = 0$ for $n > 2$, we get 2)

$$\varepsilon_{zz} = 1 - \frac{3\omega_p^2}{\omega^2}\frac{M_{zz}(\omega,k)}{D_{zz}(\omega,k)}, \tag{12}$$

where

$$M_{zz}(\omega,k) = b^{0,0}_{1,0;1,0}[(1+\alpha_2 - {}^5/_6\,\alpha_2 b^{-1,1}_{2,1;2,1})(1+\alpha_2-{}^5/_6\,\alpha_2 b^{1,1}_{2,1;2,1})$$
$$-({}^5/_6\,\alpha_2 b^{-1,1}_{2,1;2,1})^2] + {}^5/_6\,\alpha_2(b^{0,-1}_{1,0;2,1})^2(1+\alpha_2-{}^5/_6\,\alpha_2 b^{1,1}_{2,1;2,1})$$
$$+{}^5/_6\,\alpha_2(b^{0,1}_{1,0;2,1})^2(1+\alpha_2-{}^5/_6\,\alpha_2 b^{-1,-1}_{2,1;2,1})$$
$$+{}^{25}/_{18}\,\alpha_2^2 b^{0,1}_{1,0;2,1} b^{1,-1}_{2,1;2,1} b^{-1,0}_{2,1;1,0}, \tag{13}$$

$$D_{zz} = (1-3\alpha_1 a^{0,0}_{1,0;1,0})[(1-{}^5/_6\,\alpha_2 a^{-1,-1}_{2,1;2,1})(1-{}^5/_6\,\alpha_2 a^{1,1}_{2,1;2,1})$$
$$-({}^5/_6\,\alpha_2 a^{1,-1}_{2,1;2,1})^2] - {}^5/_2\,\alpha_1\alpha_2(a^{0,-1}_{1,0;2,1})^2(1-{}^5/_6\,\alpha_2 a^{1,1}_{2,1;2,1})$$
$$-{}^5/_2\,\alpha_1\alpha_2(a^{0,1}_{1,0;2,1})^2(1-{}^5/_6\,\alpha_2 a^{-1,-1}_{2,1;2,1})$$
$$-{}^{25}/_6\,\alpha_1\alpha_2^2 a^{0,-1}_{1,0;2,1} a^{-1,+1}_{2,1;2,1} a^{1,0}_{2,1;1,0}, \tag{14}$$

$$a^{\nu\mu}_{nm;rs} = \frac{1}{2}\sum_{l=-\infty}^{+\infty}\frac{l\Omega}{\omega-l\Omega}\int_0^\pi d\theta\sin\theta J_{l+\nu}\left(\frac{kv}{\Omega}\sin\theta\right)$$
$$\times J_{l+\mu}\left(\frac{kv}{\Omega}\sin\theta\right)P_n^m(\cos\theta)P_r^s(\cos\theta), \tag{15}$$

$$b^{\nu\mu}_{nm;rs} = \frac{1}{2}\sum_{l=-\infty}^{+\infty}\frac{\omega}{\omega-l\Omega}\int_0^\pi d\theta\sin\theta J_{l+\nu}\left(\frac{kv}{\Omega}\sin\theta\right)$$
$$\times J_{l+\mu}\left(\frac{kv}{\Omega}\sin\theta\right)P_r^m(\cos\theta)P_r^s(\cos\theta). \tag{16}$$

Here $J_l(x)$ is the Bessel function and $P_n^m(\cos\theta)$ are the associated Legendre polynomials. We note that the coefficients a and b are connected by the relation

$$a^{\nu\mu}_{nm,rs} = b^{\nu\mu}_{nm,rs} - \delta_{\nu\mu}\,{}^1/_2\int_0^\pi d\theta\sin\theta P_n^m(\cos\theta)P_r^s(\cos\theta). \tag{17}$$

These coefficients determine the dielectric constant of the electron liquid also in the case of an arbitrary number of non-zero α_n. Formula (16) can be written also in the form

$$b^{\nu\mu}_{nm;rs} = (-1)^\nu\frac{\pi x}{2\sin\pi x}\int_0^\pi\sin\theta\,d\theta P_n^m(\cos\theta)P_r^s(\cos\theta)$$
$$\times J_{{}^1/_2\mu-{}^1/_2\nu-x}\left(\frac{kv}{\Omega}\sin\theta\right)J_{{}^1/_2\mu+{}^1/_2\nu+x}\left(\frac{kv}{\Omega}\sin\theta\right), \tag{18}$$

where $x = \omega/\Omega$.

If the inequalities (9) are satisfied, the dispersion equation of the ordinary wave takes the form

$$M_{zz}(\omega,k) = 0. \tag{19}$$

Hence, assuming that α_2 is small compared with unity, we obtain for long waves

$$\frac{\omega}{\Omega} - 1 = \frac{1}{2}\left\{\alpha_2 - \frac{k^2v^2}{10\Omega^2}\right. \tag{20}$$
$$\left.\pm\left[\left(\alpha_2-\frac{k^2v^2}{10\Omega^2}\right)^2 - \frac{\alpha_2}{294}\frac{k^4v^4}{\Omega^4}\left(\alpha_2-\frac{k^2v^2}{5\Omega^2}\right)\right]^{1/2}\right\}.$$

When $\alpha_2 > 0$, the two oscillation branches described by this formula, come closer together or, as is customarily stated, interact strongly in the vicinity of the point $k^2v^2 = 10\alpha_2\Omega^2$. Outside this small vicinity, the oscillation spectra corresponding to (20) can be written in the form (10) and

$$\frac{\omega}{\Omega} - 1 = \frac{\alpha_2}{1176}\frac{k^4v^4}{\Omega^4}\frac{\alpha_2 - k^2v^2/5\Omega^2}{\alpha_2 - k^2v^2/10\Omega^2}. \tag{21}$$

The results show that in the vicinity of cyclotron resonance the number of waves propagating in the Fermi liquid turns out to be larger than in the electron gas of noninteracting particles.

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1) The Larmor frequency of an electron can be determined both by means of an experiment not connected with the propagation of cyclotron waves, and from data on short-wave cyclotron waves.

2) The structure of formula (12) is similar in principle to the expressions for the dielectric constant obtained in [13], which differ qualitatively from the corresponding expressions for the electron gas in that the zeroes of D_{zz} depend on the wave vector.

We note that in accordance with formula (20) the number of waves is conserved also when $\alpha_2 = 0$. This does not mean actually that both waves described by formula (20) are also possible in an electron gas. The point is that in the case of an electron gas D_{zz} has a zero of first order at $\omega = \Omega$, so that the waves of the second branch are forbidden. To the contrary, in the case $\alpha_2 \neq 0$, it can be easily verified that for long waves the zeroes of D_{zz} are incompatible with the zeroes of M_{zz}. It must be emphasized, however, that for the oscillation branch described by formula (20), with a minus sign in front of the square root, in order to be able to speak of such an incompatibility in an electron liquid with collisions, it is necessary to satisfy the inequality $\alpha_2^2 \Omega\tau > 1$ or $\alpha_2 \Omega\tau > 100$. In such strong fields, condition (11) for α_4 may be satisfied, and, strictly speaking, formula (20) may turn out to be incomplete. In this case, however, besides the oscillation branch (10), there will arise also a branch with limiting frequency $\Omega(1 + \alpha_4)$, which apparently will be closer to the Larmor frequency than $\omega_{1,2}(0)$.

Let us dwell briefly on the solution of Eq. (9) in the region investigated experimentally by Walsh and Platzman[8,9]. Namely, in the vicinity of cyclotron resonance at double the Larmor frequency we obtain for the extraordinary wave

$$k^2v^2 / \Omega = 5(2\Omega[1 + \alpha_2] - \omega). \qquad (22)$$

It is assumed here that the coefficients α_n are small. Here, too, in analogy with formula (10), the wave vector vanishes not when $2\Omega = \omega$, as follows from the theory of the electron gas. α_2 can be determined from the shifts, predicted by formula (22), of the limiting frequency of the extraordinary cyclotron waves with spectrum (22).

Finally, let us touch upon plasma waves, which have almost longitudinal polarization of the electric vector. Such waves are possible in the vicinity of $\epsilon_{xx} = \infty$. Since, for example, we have in the approximation $\alpha_n = 0$ for $n \geq 2$.

$$\varepsilon_{xx} = 1 + \frac{3\omega_p^2}{2\omega kv}\frac{M_{xx}}{D_{xx}}, \qquad (23)$$

where

$$M_{xx} = a^{0,1}_{0,0;1,1}\{(1 - \alpha_0 a^{0,0}_{0,0;0,0})(1 - {}^3/_2\alpha_1 a^{-1,-1}_{1,1;1,1}) - {}^3/_2\alpha_0\alpha_1 (a^{0,-1}_{0,0;1,1})^2 + {}^3/_2\alpha_1 [a^{1,-1}_{1,1;1,1}(1 - \alpha_0 a^{0,0}_{0,0;0,0}) + \alpha_0 a^{0,1}_{0,0;1,1} a^{-1,0}_{0,0;1,1}]\}$$
$$+ a^{0,-1}_{0,0;1,1}\{(1 - \alpha_0 a^{0,0}_{0,0;0,0})(1 - {}^3/_2\alpha_1 a^{1,1}_{1,1;1,1}) - {}^3/_2\alpha_0\alpha_1 (a^{1,0}_{1,1;0,0})^2 + {}^3/_2\alpha_1 [a^{-1,1}_{1,1;1,1}(1 - \alpha_0 a^{0,0}_{0,0;0,0}) + \alpha_0 a^{0,1}_{0,0;1,1} a^{-1,0}_{1,1;0,0}]\}, \qquad (24)$$

$$D_{xx} = [(1 - \alpha_0 a^{0,0}_{0,0;0,0})(1 - {}^3/_2\alpha_1 a^{1,1}_{1,1;1,1}) - {}^3/_2\alpha_0\alpha_1 (a^{1,0}_{1,1;0,0})^2]$$
$$\times [(1 - \alpha_0 a^{0,0}_{0,0;0,0})(1 - {}^3/_2\alpha_1 a^{-1,-1}_{1,1;1,1}) - {}^3/_2\alpha_0\alpha_1 (a^{0,-1}_{0,0;1,1})^2]$$
$$- ({}^3/_2\alpha_1)^2 [a^{1,-1}_{1,1;1,1}(1 - \alpha_0 a^{0,0}_{0,0;0,0}) + \alpha_0 a^{0,-1}_{0,0;1,1} a^{1,0}_{1,1;0,0}]^2, \qquad (25)$$

it follows that the condition for the propagation of the plasma waves is given by

$$D_{xx} = 0 \qquad (26)$$

under the condition that M_{xx} does not vanish simultaneously. In the region of long waves, this equation has a solution

$$\omega = \Omega\left[1 + \alpha_1 + \left(\frac{\alpha_0}{6} - \frac{2\alpha_1}{5}\right)\frac{k^2v^2}{\Omega^2}\right]. \qquad (27)$$

The shift of the limiting frequency compared with the Larmor frequency is determined by the value of α_1, which can thus be determined experimentally. We note that formula (27) presupposes that α_1 is small. As in the thoroughly discussed case of ordinary waves near the Larmor frequency, near double the Larmor frequency there can also appear, in principle, other oscillation branches having the limiting frequencies predicted in[11] and given by the formula

$$\omega_{n,m}(0) = m\Omega(1 + \alpha_n), \quad m \leqslant n. \qquad (28)$$

However, in order for such waves to appear it is necessary to have larger fields than for the waves with the spectrum (22), (27). On the other hand, a study of cyclotron waves in metals using strong fields and samples of high purities may make it possible to determine the different α_n, and thus obtain an appreciable amount of information on the properties of the electron liquid of metals.

Note added in proof (12 January 1968). In a recently published paper (P. M. Platzman and W. M. Walsh, Phys. Rev. Lett. **19**, 514 (1967)) an experimental study of plasma waves in potassium yielded values $\alpha_0 = -0.71$ and $\alpha_1 = -0.08$.

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