THE STATISTICAL CHARACTER OF MULTIPLE HADRON PRODUCTION IN THE THEORY OF COMPLEX ANGULAR MOMENTA

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Submitted October 10, 1967

Zh. Eksp. Teor. Fiz. 54, 1001–1009 (March, 1968)

It is shown by means of reggeon diagram techniques that in the high energy region the production of hadrons in a definite kinematic configuration occurs independently for each hadron, if the quantum numbers of the hadrons differ from those of the vacuum. The kinematic region where this happens is the region where all pair energies are large, the transverse momenta of the two fastest particles are finite, and the transverse momenta of all other particles are very small. Under these conditions the production amplitude for n particles turns out to be proportional to the amplitude of a two-particle process, in the same manner as for the emission of soft photons in electrodynamics.

1. INTRODUCTION

GRIBOV [1] developed a reggeon diagram technique for the computation of the asymptotic behavior of the amplitudes of two-particle processes. In the same paper arguments were given indicating that in case the momentum transfer is much smaller than the particle masses the most important contribution to the asymptotic behavior of the "enhanced reggeon diagrams" described in detail in [1].

The reggeon diagram technique can be generalized without difficulty to include the case of the transformation of two particles into three particles [2,3]. In [3] the authors have shown that under certain conditions the amplitude for three-particle production $a + b \to 1 + 2 + 3$ (Fig. 1) is proportional to the amplitude of the two-body reaction $a + b \to 1 + 3$. These conditions are the following.

1. The pair energies $s_{12}^{1/2}$ and $s_{23}^{1/2}$ of the produced particles must be large (the "genuinely inelastic case" [4]), and $s_{12}s_{23} \sim s$ ($s^{1/2}$ is the total energy in the c.m.s.). In this case $\kappa_1^2 = -t_{a1}$ (the square of the momentum transfer from the initial particle a to the produced particle 1) and $\kappa_3^2 = -t_{b3}$ (the square of the momentum transfer from particle b to particle 3) (Fig. 1) remain finite. Only in this case do reggeon diagrams occur at all.

2. Only the enhanced diagrams must be essential, which is possible when the momentum transfers $|\kappa_1|$ and $|\kappa_2|$ are much smaller than the particle masses.

3. The momentum component of particle 2 perpendicular to the direction of motion of the incident particles, i.e., $|\kappa_2|$, must be much smaller than the perpendicular components $|\kappa_1|$ and $|\kappa_3|$ ($\kappa_2^2 \ll \kappa_1^2, \kappa_3^2$). Since $\kappa_2 = \kappa_1 + \kappa_3$ the latter condition signifies that $\kappa_1^2 \approx \kappa_3^2 \equiv -t$[1)].

4. The particle 2 is emitted by the reggeon α, but owing to its quantum numbers it cannot be emitted by

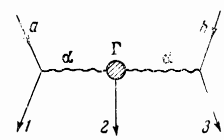

FIG. 1

the vacuum reggeon. (For example, the particle 2 may be a ρ meson and the reggeon α a ρ or K^* reggeon.)

If these conditions are fulfilled the amplitude F_3 of the process $a + b \to 1 + 2 + 3$ turns out to be independent of s_{12} and s_{23} separately and depends only on their product which equals sm_2^2. The amplitude differs from the amplitude $A(s, t)$ of the two-particle process $a + b \to 1 + 3$ only by a constant factor γ:

$$F_3(s_{12}, s_{23}, t) = \gamma A(s, t), \quad s = s_{12}s_{23} / m^2. \tag{1}$$

Equation (1) is a consequence of an identity which relates the Green's function of the α-reggeon, $G_j(t)$, to the "vertex part" $\Gamma_{j_1j_2}(t)$ (cf. Fig. 1) which determines the asymptotic behavior of the production process under the specified conditions. The relation between G and Γ is analogous to the generalized Ward identity in quantum electrodynamics, and its existence is determined by condition 4. The diagrams represented in Fig. 1 contain one α-reggeon line which carries a definite quantum number, "charge" (baryon number, isospin, etc.) and an arbitrary number of vacuum reggeons. Other reggeon diagrams, containing a larger number of "charged" lines, contribute an asymptotically small amount and thus there appears the conservation of some charge, running along the α-line, in a manner equivalent to the conservation of electric charge along an electron line in quantum electrodynamics. Since, according to condition 4, particle 2 can be emitted only from a charged α-line, the situation turns out to be very reminiscent of the emission of photons from an electron line in quantum electrodynamics. The vertex part turns out to be related to the Green's function by a Ward identity.

The present paper is devoted to a generalization of the result (1) to the case of n-particle production. Here we have to assume that conditions analogous to 1–4

1)It is easy to see that the three-particle production amplitude F_3 depends in general on three variables s_{12}, s_{23} and t. The total energy s is expressed in terms of s_{12} and s_{23}: $s = s_{12}s_{23}/m^2$ (m is the mass of the second particle).

must be satisfied. For the case of n-particle production this means that all pair energies (squared) $s_{12}, \ldots, s_{n-1,n}$ must be large (their product $s_{12}s_{23}\ldots s_{n-1,n} \sim s$, cf., e.g.,[5]) and the transverse momenta of the produced particles satisfy $\kappa_1^2, \kappa_n^2 \ll m^2$ (in order that only enhanced diagrams be important). In addition the perpendicular momenta of the "interior" particles, $\kappa_2^2, \ldots, \kappa_{n-1}^2 \ll \kappa_1^2, \kappa_n^2$. Here again $\kappa_1^2 \approx \kappa_n^2 \equiv -t$. If in addition condition 4, as formulated above, holds, it will be shown in the present paper that the amplitude F_n for the production of n identical particles is equal to

$$F_n(s_{12}, \ldots, s_{n-1,n}; t) = \gamma^{n-2}A(s, t), \quad s = s_{12}\ldots s_{n-1,n} / (m^2)^{n-2}. \tag{2}$$

Equation (2) is a consequence of a relation which relates the "vertex part" for many-particle production to the Green's function of the α-reggeon. This relation is a generalization of the indicated relation between the vertex part and the Green's function for three-particle production. The expression (2) for the amplitude F_n means that in the kinematic configuration under discussion the emission of particles occurs independently of one another.

Section 2 contains a derivation of the "Ward identity" for the vertex part corresponding to n-particle production, and also a derivation of Eq. (2). In Sec. 3 an expression is obtained for the cross section for n-particle production, integrated over the accessible (under the imposed restrictions) region of phase space. In the Conclusion the average number of emitted particles is computed for the given configuration. This number increases logarithmically with the energy. Several concrete examples of application of the results are discussed.

2. THE n-PARTICLE PRODUCTION AMPLITUDE

Under the assumption that all pair energies s_{12}, $s_{23}, \ldots, s_{n-1,n}$ are large, the n-particle production amplitude can be represented as a Mellin transform of a function depending on the $(n-1)$ complex angular momenta[3]:

$$F^{(n)}(s_{12}, \ldots, s_{n-1,n}; \varkappa_1, \ldots, \varkappa_n) = \xi_0 \int_{b-i\infty}^{b+i\infty} \frac{dj_1}{2\pi i} \int_{b-i\infty}^{b+i\infty} \frac{dj_2}{2\pi i} \cdots \int_{b-i\infty}^{b+i\infty} \frac{dj_{n-1}}{2\pi i} \times F^{(n)}_{j_1\ldots j_{n-1}}(\varkappa_1, \ldots, \varkappa_n) s_{12}^{j_1} \ldots s_{n-1,n}^{j_{n-1}}. \tag{3}$$

All integration paths are situated to the right of the singularities of $F^{(n)}_{j_1\ldots j_{n-1}}$. The amplitude $F^{(n)}_{j_1\ldots j_{n-1}}$ is determined by the sum of all possible reggeon diagrams as explained in detail in[3] for the case n = 3. Formula (3) contains the constant ξ_0 which is equal to the value of the signature factor ξ_j for $j = \alpha(0)$ ($\alpha(t)$ is the trajectory of the pole of the α-reggeon)[2]. This was done in order to bring into agreement the rules for computing reggeon diagrams for $F^{(n)}_{j_1\ldots j_{n-1}}$

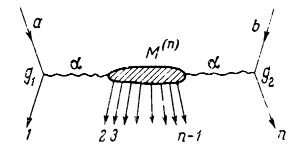

FIG. 2

with the rules formulated in[1] for the diagrams which determine the Green's function for the α-reggeon. If the momentum transfers $\kappa_1 \ll m^2$ and only amplified diagrams are important, the quantity $F^{(n)}_{j_1\ldots j_{n-1}}$ becomes (Fig. 2)

$$F^{(n)}_{j_1\ldots j_{n-1}}(\varkappa_1, \ldots, \varkappa_n) = g_1 g_2 G_{j_1}(\varkappa_1^2) M^{(n)}_{j_1\ldots j_{n-1}}(\varkappa_1, \ldots, \varkappa_n) G_{j_{n-1}}(\varkappa_n^2), \tag{4}$$

where $G_j(\kappa^2)$ is the exact Green's function of the α-reggeon, the block $M^{(n)}_{j_1\ldots j_{n-1}}$ is represented in Fig. 2, and g_1 and g_2 are the coupling constants of a and 1, and b and n to the α-reggeon.

The particles 2, 3, ..., n-1 are emitted from the α-reggeon line passing through the block $M^{(n)}_{j_1\ldots j_{n-1}}$ as explained in the Introduction. We now show that the quantity $F^{(n)}_{j_1\ldots j_{n-1}}$ can be expressed in terms of the Green's function of the α-reggeon, if all perpendicular momenta of the "internal" particles satisfy the inequalities $|\kappa_2|, \ldots, |\kappa_{n-1}| \ll |\kappa_1|, |\kappa_n|$. Since momentum conservation implies $\kappa_1 + \ldots + \kappa_n = 0$, the latter condition also means that $\kappa_1^2 \approx \kappa_n^2 \equiv -t$, so that all quantities in (3) and (4) depend only on one momentum transfer.

The amplitude $M^{(n)}_{j_1\ldots j_{n-1}}(t)$ can be decomposed into a sum of "irreducible" blocks, each of which does no longer contain parts connected by one α-line. For the case n = 5, for instance, this decomposition is illustrated in Fig. 3. The irreducible vertex part with two α-reggeons and $(n-2)$ emitted particles will be denoted by $\Gamma^{(n)}_{j_1\ldots j_{n-1}}(t)$. We now assert that $\Gamma^{(n)}_{j\ldots j_{n-1}}(t)$ satisfies the following "Ward identity"

$$\Gamma^{(n)}_{j_1\ldots j_{n-1}}(t) = -\gamma^{n-2}\Big\{\frac{G_{j_1}^{-1}(t)}{(j_2-j_1)(j_3-j_1)\ldots(j_{n-1}-j_1)} + \frac{G_{j_2}^{-1}(t)}{(j_1-j_2)(j_3-j_2)\ldots(j_{n-1}-j_2)} + \ldots + \frac{G^{-1}_{j_{n-1}}(t)}{(j_1-j_{n-1})(j_2-j_{n-1})\ldots(j_{n-2}-j_{n-1})}\Big\}, \tag{5}$$

where γ is the coupling constant for the transition of the α-reggeon into an α-reggeon plus the emitted particle.

In order to prove (5) it is necessary to consider all reggeon diagrams for the vertex part $\Gamma^{(n)}$. The integrals corresponding to these diagrams contain products of different free Green's functions of the α-reggeon: $G_l^{(0)}(t) = [l - \alpha(t)]^{-1}$, depending on the angular momenta $l = j_i - j'_k$, where j'_k are the integration variables. Such products can be transformed by making use of the identity

$$G_{l_1}^{(0)}(t)\ldots G_{l_N}^{(0)}(t) = \frac{G_{l_1}^{(0)}(t)}{(l_2-l_1)\ldots(l_N-l_1)} + \frac{G_{l_2}^{(0)}(t)}{(l_1-l_2)\ldots(l_N-l_2)} + \ldots + \frac{G_{l_N}^{(0)}(t)}{(l_1-l_N)\ldots(l_{N-1}-l_N)}. \tag{6}$$

[2]In computing the asymptotic behavior with logarithmic accuracy (which is assumed when only amplified diagrams are considered) all factors which have a weak dependence on j, in particular ξ_j, can be taken outside the integral for $j = \alpha(0)$.

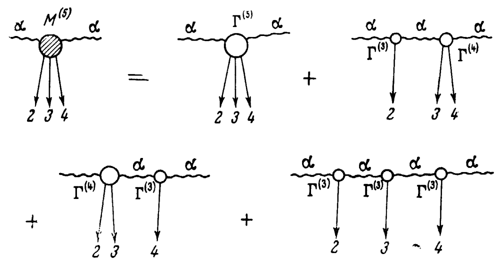

FIG. 3.

The latter relation can be checked easily by observing that both sides can be represented as the integral

$$\int_{b-i\infty}^{b+i\infty} \frac{dl}{2\pi i} G_l^{(0)}(t) \frac{1}{(l_1-l)(l_2-l)\ldots(l_N-l)}, \tag{7}$$

where the integration path passes to the right of the pole $l = \alpha(t)$ of $G_l^{(0)}$ and is situated to the left of the poles: $l = l_1, \ldots, l_N$. Closing the contour in (7) to the left we obtain the left hand side of Eq. (6), and closing it to the right we obtain the right hand side of the same equation.

Utilizing (6) it is easy to establish that in perturbation theory the left hand side and the right hand side of Eq. (5) are equal, since the individual terms in (6) will either cancel mutually, or correspond to perturbation theory diagrams for the self-energy part of the α-reggeon $\Sigma_j(t) = G_j^{(0)-1}(t) - G_j^{-1}(t)$.

Summing sequences of diagrams of the type represented in Fig. 3 one can see with the help of (5) that for the amplitude $F^{(n)}_{j_1\cdots j_{n-1}}(t)$ which determines the asymptotic behavior of the process, there exists the identity:

$$F^{(n)}_{j_1\ldots j_{n-1}}(t) = g_1 g_2 \gamma^{n-2}\left\{\frac{G_{j_1}(t)}{(j_2-j_1)\ldots(j_{n-1}-j_1)}\right.$$
$$\left.+\frac{G_{j_2}(t)}{(j_1-j_2)\ldots(j_{n-1}-j_2)}+\ldots+\frac{G_{j_{n-1}(t)}}{(j_1-j_{n-1})\ldots(j_{n-2}-j_{n-1})}\right\}. \tag{8}$$

The right hand side of this identity can be written as an integral of the type (7), but containing the exact Green's function:

$$F^{(n)}_{j_1\ldots j_{n-1}}(t) = g_1 g_2 \gamma^{n-2}\int_{b-i\infty}^{b+i\infty}\frac{dj}{2\pi i}G_j(t)\frac{1}{(j_1-j)\ldots(j_{n-1}-j)},$$
$$\operatorname{Re} j < \operatorname{Re} j_1, \ldots, \operatorname{Re} j_{n-1}. \tag{9}$$

We substitute the expression (9) into Eq. (3). We integrate over $j_1, j_2, \ldots, j_{n-1}$ by means of closing the contour to the left, since the product $s_{12}^{j_1}\ldots s_{n-1,n}^{j_{n-1}}$ decreases when $\operatorname{Re} j_1, \ldots, \operatorname{Re} j_{n-1} \to -\infty$. Owing to the condition $\operatorname{Re} j < \operatorname{Re} j_1, \ldots, \operatorname{Re} j_{n-1}$ the poles in the j-plane ($j = j_1, \ldots, j_{n-1}$) are inside the integration contour and we obtain:

$$F^{(n)}(s_{12}, s_{23}, \ldots, s_{n-1,n}, t)$$
$$= g_1 g_2 \xi_0 \gamma^{n-2}\int_{b-i\infty}^{b+i\infty}\frac{dj}{2\pi i}G_j(t)[s_{12}s_{23}\ldots s_{n-1,n}]^j. \tag{10}$$

Up to a constant factor, of which we can get rid by means of a redefinition of the constant γ, the product $s_{12}\ldots s_{n-1,n}$ is the total energy s. The integral (10) together with the multipliers $g_1 g_2 \xi_0$ represents the asymptotic form of the amplitude $A(s, t)$ for the two-particle process $a + b \to 1 + n$, and thus

$$F^{(n)}(s_{12}, \ldots, s_{n-1,n}, t) = \gamma^{n-2}A(s, t). \tag{11}$$

3. THE CROSS SECTIONS FOR INELASTIC PROCESSES

Equation (11) allows us to find the cross sections for inelastic processes, integrated over the accessible part of phase space, as restricted by the kinematic constraints described in the introduction. The constraints related to the fact that all momentum transfers must be smaller than the masses do not seem to be very restrictive, since the admissible region includes almost all of the diffraction cone. However, the conditions $|\kappa_2|, \ldots, |\kappa_{n-1}| \ll |\kappa_1| \approx |\kappa_n| \equiv (-t)^{1/2}$ restricts the integration to an insignificant part of the diffraction cone for the momenta $\kappa_2, \ldots, \kappa_{n-1}$. The integrations over the lengths of $\kappa_2, \ldots, \kappa_{n-1}$ can be extended only up to $\kappa_0 \ll (-t)^{1/2}$. Therefore the cross section computed below represents only an insignificant part of the total cross section for the process. On the other hand the restrictions imposed on the pair energies, $m^2 \ll s_{ik} \ll s$ are not very essential. This is related to the fact that for $s_{ik} \sim s$ the momentum transfers become large and the amplitude decreases rapidly, whereas the integration over the region where even one of the pair energies is of the order $s_{ik} \sim m^2$ leads to the loss of one power of ln s in the total cross section, as can be seen from the computation of phase space given below.

The kinematic analysis (cf., e.g.,[5]) shows that there exist $(n-1)$ configurations of momenta of the produced particles in which all pair energies are large and the momentum transfers are smaller than, or of the order of, the masses. In these configurations the particles 1, 2, ..., n divide up into two groups, such that particles 1, 2, ..., c are emitted almost in parallel directions to the momentum p_a of the incident particle, and the particles c + 1, ..., n are emitted in the backward direction $p_b = -p_a$, in such a manner that the parallel components of the momenta in each group are large and are strictly ordered:

$$k_1 \gg k_2 \gg k_3 \gg \ldots \gg k_c \gg m, \quad m \ll k_{c+1} \ll \ldots \ll k_n. \tag{12}$$

The momenta $k_1 \approx k_n \approx \frac{1}{2}(s)^{1/2}$. In all other cases either one of the pair energies turns out to be small ($\sim m^2$), or one of the momentum transfers is large.

The expression for the part of the n-particle production cross section under consideration is

$$\Delta\sigma_n = \frac{1}{2s}\frac{1}{(2\pi)^{3n-4}}\int\ldots\int\prod_{i=1}^{n}\frac{d^3p_i}{2E_i}|F^{(n)}|^2$$
$$\times\delta^{(4)}\Big(\sum_{i=1}^{n}p_i - p_a - p_b\Big). \tag{13}$$

In view of (12) all parallel momentum components can be neglected inside the delta function, with the exception of k_1 and k_n. In addition, since $|\kappa_2|, \ldots, |\kappa_{n-1}| \ll |\kappa_1| \approx |\kappa_n|$ one can neglect also the perpendicular components of the $(n-2)$ "internal" particles. The phase space will decompose into the product of the two-particle phase space of the first and n-th particle, successive independent integrations over $d^2\kappa_2 \ldots d^2\kappa_{n-1}$

and the integrations over $dk_2 \ldots dk_{n-1}$, which have to be done taking into account (12). Substituting (11) into (13) we obtain

$$\Delta\sigma_n = \sigma_2\left[\frac{|\gamma|^2}{2(2\pi)^3}\int d^2\varkappa\right]^{n-2}$$

$$\times \sum_{c=1}^{n-1}\left(\int_{m^2}^{1/2\sqrt{s}}\frac{dk_2}{k_2}\int_{m^2}^{k_2}\frac{dk_3}{k_3}\cdots\int_{m^2}^{k_{c-1}}\frac{dk_c}{k_c}\right)$$

$$\times\left(\int_{m^2}^{1/2\sqrt{s}}\frac{dk_{n-1}}{k_{n-1}}\int_{m^2}^{k_{n-1}}\frac{dk_{n-2}}{k_{n-2}}\quad\int_{m^2}^{k_{c+2}}\frac{dk_{c+1}}{k_{c+1}}\right. \tag{14}$$

It is understood that if $c = 1$ (or $c = n - 1$) the integrations over the k_i in the first (second) parentheses are absent.

A direct computation of the right hand side of (14) yields

$$\Delta\sigma_n(s) = \sigma_2(s)\frac{[\Gamma\ln(s/m^2)]^{n-2}}{(n-2)!}, \qquad \Gamma = \frac{|\gamma|^2}{2(2\pi)^3}\int d^2\varkappa. \tag{15}$$

In view of what was said at the beginning of the present section, the cross section $\sigma_2(s)$ can practically be identified with the total cross section for the two-particle process $a + b \to 1 + n$, whereas $\Delta\sigma_n(s)$ is only part of the total cross section for the process $a + b \to 1 + 2 + \ldots + n$. The latter circumstance is a result of the fact that the integration over $d^2\kappa$ can be extended only up to $|\kappa| = \kappa_0 \ll m$[3].

We remark that the factor $1/(n-2)!$ occurs in (15) due to the part of phase space admitted by the kinematic constraints and is not related to identity of the particles. Another factor of $1/(n-2)!$ which should be there because of identity of the particles, cancels with a factor $(n-2)!$ coming from taking into account the $(n-2)!$ noninterfering reggeon diagrams with large contributions, which differ from one another by permutations of the identical particles. The expression (15) is thus valid for the production of nonidentical particles also, if they are emitted along the α-reggeon line only in a definite sequence. However, in this case $|\gamma|^{2(n-2)}$ should be replaced by the product $|\gamma_2\gamma_3\ldots\gamma_{n-1}|^2$ where the γ_i are the coupling constants for the emission of the appropriate particles.

It is easy to find an example where the factor $1/(n-2)!$ will be absent from the expression (15). Let us imagine that along the α-reggeon line nonidentical particles are emitted which do not change the quantum numbers of the α-line (such "particles" can be, for example, pairs $\pi\pi$, $K\bar{K}$ etc. with fixed total masses). Then $(n-2)!$ identical diagrams differing by permutations of the pairs, will contribute, leading to a cancelation of the factor $1/(n-2)!$ in (15).

4. CONCLUSION

As examples of application of Eq. (15) one can consider processes for which the asymptotic behavior is dominated by a boson pole of the type of a ρ-meson or a K^*-resonance (taking into account Mandelstam branch cuts). We consider, for example the emission of ρ-mesons. (As explained in[3], the application of formulas like (15) to the emission of pions and to processes for which the asymptotic behavior is dominated by a fermion pole, runs into difficulties.)

Consider, for instance,

$$\pi^+ + p \to K^+ + \Sigma^+ + n\rho^0,$$

where a K^* meson plays the role of the α-reggeon. It is easy to compute the part $\Delta\sigma_{tot}$ of the total cross section which includes the production of particles with the appropriate momentum configurations. We have

$$\Delta\sigma_{tot}(s) = \sum_{n=0}^{\infty}\Delta\sigma_n(s) = \sigma_0(s)\left(\frac{s}{m^2}\right)^{\Gamma}. \tag{16}$$

If we had $\sigma_0(s) \to$ const for $s \to \infty$, Eq. (16) would seem to be inconsistent, since $\Delta\sigma_{tot}$ would increase with energy. However, our results are applicable only to the case when the asymptotic behavior of $\sigma_0(s)$ is determined by an α-reggeon which carries quantum numbers differing from those of the vacuum (in our concrete example the α-reggeon is a K^*, so that the cross section $\sigma_0(s) \sim s^{2\alpha(0)-2}$ decreases according to a power law as s increases). It is possible that the inapplicability of (16) to the case when the asymptotic behavior of $\sigma_0(s)$ is determined by the vacuum pole is a sign of self-consistency of the theory.

It is very likely that another variant is possible. The quantity Γ is proportional to κ_0^2, the limit of integration over the momentum transfers admitted by the conditions listed in the Introduction. However, if a logarithmic narrowing of the cone occurs as the energy increases, it is not sufficient for the applicability of our results that the transverse momenta of the "internal" particles, $|\kappa_2|^2, \ldots, |\kappa_{n-1}|^2$ be considerably smaller than $|\kappa_1|^2 \approx |\kappa_n|^2 = -t$. It is also required that $\kappa_i^2 \ll m^2/\ln s$ ($i = 2, \ldots, n-1$), since otherwise the dependence of the amplitude on κ_i^2 becomes essential, and the Ward identities based on the neglect of all κ_i would not be valid. But then the quantity Γ in (16) cannot be considered constant for very large energies, and the factor $(s/m^2)^{\Gamma}$ may not increase.

With the help of (16) it is easy to compute the average number of ρ^0 mesons:

$$\bar{n} = \frac{\sum_{n=0}^{\infty} n\sigma_n}{\Delta\sigma_{tot}} = \Gamma\ln\frac{s}{m^2}, \tag{17}$$

which increases logarithmically if Γ does not depend on the energy.

We now consider the emission of two kinds of neutral particles, e.g. ρ^0 and A_2^0. The cross section for the process of emission of $n_1\rho^0$ mesons and $n_2 A_2^0$ mesons is

$$\sigma_{n_1 n_2}(s) = \sigma_0(s)\frac{(\Gamma_1\ln(s/m^2))^{n_1}(\Gamma_2\ln(s/m^2))^{n_2}}{n_1! n_2!}. \tag{18}$$

The numerical factor in (18) comes about in the following way. There are $(n_1 + n_2)!$ large reggeon diagrams which contribute identically and differ by permutations of arbitrary particles. In addition the phase space of the ρ-mesons must be divided by $n_1!$ and that of the A_2^0 mesons by $n_2!$. We thus have $(n_1 + n_2)!/(n_1 + n_2)!n_1!n_2! = 1/n_1!n_2!$. Equation (17) now determines the average number of ρ^0-mesons for $\Gamma = \Gamma_1$ (A_2^0-mesons for $\Gamma = \Gamma_2$).

[3]We have asserted earlier that $\kappa_0 \ll (-t)^{1/2}$. It is easy to see that after integrating over t in (15) this condition can be replaced by $|\kappa_0| \ll m$.

If we consider the production of charged ρ-mesons together with the production of ρ^0, we have

$$\sigma_{n_0 n_{ch}} = \sigma_0(s)\left(\Gamma_0 \ln\frac{s}{m^2}\right)^{n_0}\left(\Gamma_{ch} \ln\frac{s}{m^2}\right)^{n_{ch}} \Big/ n_0! n_{ch}!, \quad (19)$$

where n_0 is the number of neutral mesons, n_{ch} is the total number of charged ρ mesons, The numerical factor in (19) can be obtained in the following manner. The sequence of ρ^+ and ρ^- mesons along the reggeon line is strictly determined, since the K^{*+} can emit only a ρ^+ and the K^{*-} can emit only a ρ^-. Therefore the total number of large diagrams is now equal to the total number of diagrams with all permutations, minus the permutations among ρ^+ and ρ^- mesons, i.e. $N = (n_0 + n_{ch})![n_{ch}!/n_+!n_-!]^{-1}$ ($n_{ch} = n_+ + n_-$, $n_\pm$ are the numbers of $\rho^\pm$ mesons). The final numerical factor is obtained by multiplying N by $1/(n_0 + n_{ch})!$, coming from the phase space computation, and by $(n_0!\,n_+!\,n_-!)^{-1}$, which takes into account the identity of the ρ^0, ρ^+, and ρ^- mesons.

From (19) it follows that

$$\bar{n}_0 = \Gamma_0 \ln(s/m^2), \quad \bar{n}_{ch} = \Gamma_{ch} \ln(s/m^2). \quad (20)$$

Since isospin invariance implies $\Gamma_{ch} = 2\Gamma_0$, it follows that $\bar{n}_{ch} = 2\bar{n}_0$ or $\bar{n}_+ = \bar{n}_- = \bar{n}_0 \Gamma_0 \ln(s/m^2)$.

In conclusion we note that even if the amplified diagrams really contribute the main part of the amplitude and the theory is correct, its experimental verification seems to be very difficult. The results obtained are valid only under the rather unrealistic condition $\ln(s/m^2) \gg 1$; in addition one must select cases belonging only to an insignificant part of phase space. It may be that it is the most realistic to observe the independence of the cross section for n particle production of the pair energies, as discussed in detail in [3].

We are grateful to V. N. Gribov, B. L. Ioffe, E. M. Levin and K. A. Ter-Martirosyan for very interesting discussions.

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Translated by M. E. Mayer
114