

*ELECTRIC CONDUCTIVITY OF A PLASMA IN A STRONG MAGNETIC FIELD*

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The anomalous resistance effect in a plasma is considered under the assumption that the Larmor frequency of the electron exceeds its Langmuir frequency. It is shown that in this case the ion-acoustic instability does not lead to any appreciable change in the conductivity. However, if accelerated electrons appear in the plasma, they may become isotropic at the expense of instability in the anomalous Doppler effect and thus increase the plasma pressure. Experimentally, this effect can appear as an anomalous resistance.

1. INTRODUCTION

As is well known, because of the long-range character of the forces between charged particles, relaxation processes in a plasma frequently take the form of collective effects associated with the excitation of oscillations. In particular, effects of such a nature can take place when an electric current flows through a completely ionized plasma. Under conditions in which the electron temperature  $T_e$  is much greater than the ion temperature  $T_i$ , a buildup of ion-acoustic oscillations is possible if the current is sufficiently large. The scattering of electrons by these oscillations should lead to a more rapid exchange of momentum between the electron and ion components, i.e., to an anomalous resistance.<sup>[1-3]</sup> In the absence of a magnetic field (more precisely, for  $H > (4\pi m_e c^2 n_0)^{1/2}$ , where  $n_0$  is the electron density) the presence of noise with a level at least one order of magnitude greater than thermal significantly increases the effective frequency of electron-acoustic collisions. This is connected with the fact that the interaction of "unmagnetized" electrons with the slow oscillations has the character of elastic scattering, wherein the directed velocity is rapidly lost. As a result, the conductivity can be much less than the collisional.<sup>[1-3]</sup>

In experiments on the Joule heating of a plasma in the Tokamak, which is a toroidal setup with a strong longitudinal magnetic field, it was discovered<sup>[4]</sup> that the effect of increased resistance is observed even for  $H > (4\pi m_e c^2 n_0)^{1/2}$ , i.e.,  $\omega_H > \omega_0$ , where  $\omega_H = eH/m_e c$  is the cyclotron and  $\omega_0$  the Langmuir frequency. Speaking more precisely, it was shown in these experiments that the conductivity of the plasma  $\sigma$  is much less than the classical conductivity  $\sigma_0 = e^2 n_0 \tau_0 / m_e$  computed from the electron temperature  $T_0$ , which is found from measurements of the diamagnetic signal, i.e., the plasma pressure  $p = n_0 T_0$  (the contribution of the ions to the pressure is small). In this connection, it is of interest to consider the problem of the anomalous resistance of the plasma when  $\omega_H > \omega_0$ .

It must be kept in mind that when  $\omega_H > \omega_0$  the character of the interaction of the electrons with the oscillations changes considerably. In this case, the Larmor radius of the electrons is less than the Debye radius, so that the transverse energy of the thermal electrons is an adiabatic invariant—it remains unchanged in the interaction with the oscillations. Therefore, the re-

sults for a weak magnetic field<sup>[1-3]</sup> cannot be transferred to the case  $\omega_H > \omega_0$ .

In the present work we discuss the physical processes which can take place when current flows through a plasma in a strong magnetic field, and estimate the value of the anomalous resistance.

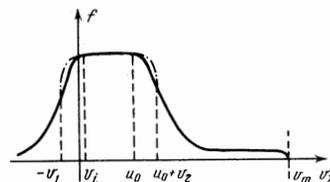
2. QUALITATIVE CONSIDERATION

The study of weakly nonlinear collective processes in a plasma is naturally divided into three stages. First, in the linear approximation, it is necessary to investigate what sort of instabilities can develop in the plasma. Then it is necessary to consider the inverse effect of the action of the oscillations on the velocity distribution function of the particles. For small amplitude, of these oscillations, it suffices to use the quasilinear approximation in this case, for which the development of the oscillations is accompanied by the formation of a "plateau" in the distribution function in the region of resonance interaction of the particles with the waves. Finally, for the complete solution of the problem and the determination of the noise spectrum and macroscopic effects, we must solve the nonlinear kinetic equation for the waves with account of scattering of the waves by particles and processes of transformation of one kind of waves into the other.

Let us consider a problem that is of interest from this viewpoint. We assume that  $T_e \gg T_i$  and that the directed velocity of the electrons  $u = j/en_0$  is much larger than the sound velocity  $c_s = \sqrt{T_e/m_i}$ . We shall take it into account that the magnetic field is not very great, so that

$$\omega_{ci} = (4\pi e^2 n_0 / m_i)^{1/2} \gg \omega_{Hi} = eH / m_i c.$$

Here the ions can be regarded as unmagnetized, and the frequency of the ion-acoustic oscillations with a wavelength greater than the electron Debye wavelength, i.e.,  $k < \omega_0/v_e$ , is equal to  $\omega = kc_s$ . The phase velocity of such waves along the magnetic field is equal to  $v_{ph} = \omega/k_z = kc_s/k_z$ , where  $k_z$  is the longitudinal



component of the wave vector  $k$ . Depending on the inclination of the vector  $k$  with respect to the direction of the magnetic field,  $v_{ph}$  can change over wide limits, beginning with  $c_s$  and above. For  $k > \omega_0/v_e$ , the phase velocity decreases to  $v_{ph} \sim v_i = T_i/m_i$ . Thus, in the range  $v_i < v_z < v_e = \sqrt{T_e/m_e}$ , it is possible for the electrons to enter into resonance interaction with the waves, i.e., to radiate ion-acoustic waves by the Cerenkov mechanism. As a consequence, in the entire range of velocities  $v_i < v_{ph} = v_z < v_p \sim u$ , where, in the absence of oscillations, the derivative of the distribution function  $\partial f/\partial v_z$  would have been positive, and a buildup of ion-acoustic oscillations and the formation of a "plateau" in the electron distribution function would take place (see the drawing).

If the electric field  $E$  is much less than the critical  $E_c = m_e v_e / e \tau_e$ , then there will be no high-frequency instabilities other than the ion-acoustic instability, and the value of the anomalous conductivity is determined by the width and location of the plateau in the distribution function. For  $u \ll v_e$ , the corresponding effect is very small. However, the formation of the plateau does not lead to complete stabilization of the instability—as the result of collisions in the plateau region, a certain positive slope of the distribution function exists,  $\partial f/\partial v_z > 0$ , so that the growth of oscillations continues, although with a much smaller increment. The increase in the amplitude of oscillation leads to the result that nonlinear effects of the type of scattering by electrons and ions come into play. Scattering by electrons is seen to be greater than that by ions. It takes place only from electrons outside the plateau region and is accompanied by a transfer of energy to them. As a result, as will be shown below, the plateau region is somewhat broadened; however, even in this case, the effect of additional resistance is not large.

Thus, for  $\omega_H > \omega_0$  and  $E \ll E_c$  the effect of the anomalous resistance is small. We now discuss qualitatively the case  $E \sim E_c$ .

When the longitudinal electric field  $E$  approaches the critical value  $E_c$ , the number of escaping electrons increases exponentially,<sup>[5]</sup> so that a "tail" appears on the electron distribution function (see the drawing). It is not difficult to see that these electrons also can lead to a buildup of oscillations of the plasma. In fact, in the presence of a magnetic field, in addition to the resonance at  $\omega - k_z v_z = 0$ , there is a resonance at harmonics of the cyclotron frequency; with account of the Doppler effect,  $\omega - n\omega_H - k_z v_z = 0$ , where  $n$  is an arbitrary number, positive or negative.<sup>[6]</sup> For  $n < 0$ , where there is a so-called "anomalous" Doppler effect, for an oscillation frequency  $\omega \ll \omega_H$ , the resonance condition takes the approximate form  $v_z \cong |n| \omega_H / k_z$ , i.e.,  $v_z \cong \omega_H / k_z$ , for  $|n| = 1$ .

In the case of longitudinal oscillations of a rarefied plasma, there exist two branches of oscillations with  $\omega \ll \omega_H$ —ion-acoustic and Langmuir. For Langmuir oscillations,  $k_z$  cannot exceed the inverse of the Debye radius with respect to the electron temperature,  $k_z < \omega_0/v_e$ , i.e., the resonance interaction can take place only for  $v_z > v_m = \omega_H v_e / \omega_0$ . Ion-acoustic waves can be propagated also at somewhat larger  $k_z$ , up to

$$k_z \sim \omega_0/v_i \sim \sqrt{T_e/T_i} \omega_0/v_e,$$

however, as will be shown below, the amplitude of the oscillations is very small in this region because of scattering from the ions. Thus the resonance interaction of the electrons with the anomalous Doppler effect generally begins when  $v_z > v_m$ .

From the quantum-mechanical viewpoint, the resonance condition can be regarded as the law of energy conservation for radiation

$$\hbar\omega = n\hbar\omega_H + \hbar k_z v_z,$$

where  $\hbar\omega$  is the plasmon energy,  $n$  the change (decrease) in the magnetic quantum number of the electron, and

$$\hbar k_z v_z \cong -\{(m_e v_z - \hbar k_z)^2 - m_e^2 v_z^2\}/2m_e$$

the decrease in the longitudinal energy of the electron due to the "recoil" occurring upon radiation of a plasmon with longitudinal momentum  $\hbar k_z$  (in the transition to the classical case, the Planck constant  $\hbar$  must tend to zero). Thus, for the anomalous Doppler effect, the radiation of the plasmon ( $\omega > 0$ ) is accompanied by a decrease in the longitudinal energy of the electron ( $k_z > 0$  and an increase in its transverse energy ( $n < 0$ ), and in the case of the normal Doppler effect part of the transverse energy goes over into the longitudinal one (here  $n > 0$ , and for  $\omega < \omega_H$  we have  $k_z < 0$ , i.e., a backward wave with  $v_{ph} = \omega/k_z < 0$  is radiated). For  $\omega \ll \omega_H$  the energy of the plasmon is much less than the increase of the transverse energy of the electron, i.e., the transition takes place at almost constant energy of the electron. In other words, for such an interaction, the electron distribution function becomes isotropic. As a consequence, the buildup of longitudinal energy by the electrons in the region  $v_z > v_m$  should be somewhat decelerated, since the energy acquired from the electric field is partly transformed into longitudinal energy.

The accelerated electrons undergo virtually no pair collisions. Moreover, there are no sufficiently intense oscillations in the plasma to change the sign of the longitudinal velocity of the runaway electrons. Therefore the runaway electrons should be distributed more or less isotropically in the hemisphere  $v_z > v_m$  of velocity space.

We denote by  $v'$  the mean velocity and by  $n'$  the density of the electrons. Their current density  $j'$  is of the order of  $en'v'$ . Under stationary conditions, the value of  $v'$  is determined by the balance between the acceleration of the particles in the field  $E$  and their dissipation. By  $\tau_E$  we mean the lifetime of the fast particles (in order of magnitude, it is equal to the so-called energy lifetime). Then, approximately,  $v' \cong eE\tau_E/m_e$ . Then we get for the total electric current density  $j$

$$j = \sigma_e E + j' = \sigma_e (1 + \Gamma) E, \quad (1)$$

where

$$\sigma_e = e^2 n_0 \tau_e / m_e, \quad \Gamma \cong n' \tau_E / n_0 \tau_e,$$

$\tau_e$  is the mean collision time of thermal electrons with the ions. The quantity  $\Gamma$  indicates what fraction of the fundamental current consists of the current of runaway electrons. For  $\tau_E \gg \tau_e$  the contribution to the current from the runaway electrons can be significant even if  $n' \ll n_0$ . In the presence of traveling waves with  $v_{\perp}$

$\sim v_{\parallel} \sim v'$  the total plasma pressure consists of the thermal pressure  $nT_e$  (under the assumption  $T_e \gg T_i$ ) and the pressure of runaway electrons  $\sim m_e n' v'^2$ . Introducing the notation  $\alpha = E/E_c$  and  $p = n_0 T_0$ , and taking it into account that  $v' \cong eE\tau_E/m_e$  and  $E_c \cong m_e v_e / e\tau_e$ , we obtain, approximately,

$$T_e = T_e(1 + A\Gamma), \quad (2)$$

where  $A = \alpha^2 \tau_E / \tau_e$ .

Let us consider relations (1) and (2). Since the value of  $\Gamma$  is proportional to  $n'$ , it then depends exponentially on  $E/E_c$ . In accord with [5], the number of electrons escaping per unit time is approximately equal to

$$n \cong \frac{n}{\tau_e} \exp\left(-\frac{E_c}{E}\right), \quad E_c = \frac{L}{8} \frac{e}{d^2} = 1.6 \cdot 10^{-15} \frac{n_0}{T_e}$$

( $L \approx 15$ ,  $T_e$  in eV). From the conditions of balance in the number of fast electrons  $n \cong n'/\tau_E$ , we find

$$\alpha = E/E_c = 1/\ln(\tau_E n_0 / \tau_e n').$$

For  $\tau_E n_0 \gg \tau_e n'$ , the value of  $\alpha$  can be assumed to be constant, with logarithmic accuracy. In other words, over wide limits of change of the number of fast electrons  $n'$ , the value of  $E/E_c$  can be considered to be "frozen" for a value of  $\alpha = \text{const} \ll 1$ .

Upon increase of  $E/E_c$ , the value of  $\Gamma$  increases exponentially, so that the contribution of fast electrons to the current density (1) and the pressure (2) becomes appreciable. Depending on the value of  $A$  the following phenomena must be considered here. If  $A \ll 1$ , then, in accord with (1), the conductivity of the plasma increases sharply, while the temperature can remain virtually constant. Conversely, for  $A \gg 1$ , an increase in the "temperature"  $T_0$  should be observed for the "frozen" conductivity  $\sigma$ , which outwardly appears as an "anomalous" resistance. If we introduce the "classical" conductivity  $\sigma_0$ , computed from the "temperature"  $T_0$ , as is done in the treatment of the experimental data, then, with account of the fact that the classical conductivity is proportional to  $T^{3/2}$ , we obtain from (1)

$$\frac{\sigma_0}{\sigma} = \frac{1}{1 + \Gamma} \left(\frac{T_0}{T_e}\right)^{3/2}. \quad (3)$$

For a comparison with experiment, it is convenient to introduce the critical field  $E_{c0}$  for the "temperature"  $T_0$ . Taking it into account that  $E_{c0} \propto 1/T_0$  while the electric field in the plasma  $E$  is "frozen" at a value  $E = \alpha E_c \propto \alpha/T_e$ , we get from (3)

$$\frac{\sigma_0}{\sigma} = \frac{\alpha^{-3/2}}{1 + \Gamma} \left(\frac{E}{E_{c0}}\right)^{3/2}. \quad (4)$$

Here, for  $A\Gamma \gg 1$ , the value of  $\Gamma$  can be represented as

$$\Gamma = \frac{1}{A} \frac{T_0}{T_e} = \frac{\tau_e}{\tau_E \alpha^3} \frac{E}{E_{c0}}.$$

Thus, with increasing  $E/E_{c0}$ , the ratio  $\sigma_0/\sigma$  which characterizes the effect of "anomalous resistance" should increase at first like  $(E/E_{c0})^{3/2}$  and then, when  $\Gamma$  becomes greater than unity, like  $(E/E_c)^{1/2}$ . The relation (4) corresponds qualitatively to the experimental data obtained on the TM-3 Tokamak.

We also note that inasmuch as the isotropization of the escaping electrons with respect to velocity begins at  $v' > \omega_H v_e / \omega_0$ , the effect of an increase in the

transverse pressure in comparison with the thermal pressure, and consequently, the effect of "anomalous conductivity," can be observed only for

$$T_0/T_e > n' \omega_H^2 / n_0 \omega_0^2.$$

On the other hand, at very high energy of the "hot" electrons in the plasma of a toroidal pinch, the following effect becomes important. As soon as the transverse velocity  $v'$  of the accelerated electrons becomes larger than

$$\sqrt{R/a} v_i' \geq \sqrt{R/a} v_m,$$

where  $R$  is the large and  $a$  the small radius of the plasma tube, they are trapped by magnetic mirrors on the internal circuit of the torus and are transformed from transit electrons to trapped electrons. Because of the reflection from the magnetic mirrors, the trapped electrons reverse their velocity  $v_z$  and then, as the result of interaction with the Langmuir (or ion-acoustic) waves in the normal Doppler effect they can lose their transverse energy and be transformed into transit electrons with  $v_z < 0$ . As the result of this, a complete isotropization of electrons with energy

$$W \sim m_e v'^2 > \frac{R}{a} m_e v_m^2 \sim \frac{R}{a} \frac{\omega_H^2}{\omega_0^2} T_e.$$

should take place. Correspondingly, the acquisition of energy by such electrons from the external field is sharply reduced, so that it is natural to expect that  $T_0/T_e$  will not exceed the value

$$\frac{R}{a} \frac{n' \omega_H^2}{n_0 \omega_0^2} < \frac{R}{a} \frac{\omega_H^2}{\omega_0^2}.$$

As a result, it can be expected that  $T_0/T_e$  will have the order of magnitude of  $\omega_H^2 / \omega_0^2$ , i.e.,

$$\sigma_0/\sigma = c_1 \omega_H^3 / \omega_0^3, \quad (5)$$

where  $c_1$  is some numerical factor of the order of unity.

The relation (5) also agrees qualitatively with the experimental results on the TM-3.

### 3. CASE OF A WEAK ELECTRIC FIELD

We now consider in more detail the case  $E \ll E_c$ , when there are no runaway electrons and only the buildup of ion-acoustic oscillations of a nonisothermal plasma can lead to additional resistance.

For a quantitative description of the weak turbulence that is developed upon passage of the longitudinal current, the set of kinetic equations for waves and particles can be used.<sup>[7]</sup> This system has the form

$$\frac{\partial f}{\partial t} + \frac{e}{m_e} E \frac{\partial f}{\partial v_z} = \text{St}(f) + \text{St}_q(f) + \text{St}_s(f), \quad (6)$$

$$\frac{k^2}{8\pi} \frac{\partial \epsilon'}{\partial t} \left\{ \frac{\partial I_{\mathbf{k}}}{\partial t} + \frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{k}} \nabla I_{\mathbf{k}} - \frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{r}} \frac{\partial I_{\mathbf{k}}}{\partial \mathbf{k}} \right\} = -(\epsilon_i'' + \epsilon_e'') \frac{k^2}{4\pi} I_{\mathbf{k}} + S. \quad (7)$$

Here  $f$  is the electron distribution function,  $\text{St}(f)$  is the Coulomb integral of collisions of electrons with electrons and ions, and  $\text{St}_q(f)$  is a quasilinear term of interaction of electrons with waves. In a strong magnetic field  $\omega_H \gg \omega_0$  it has the form

$$\text{St}_q(f) = \frac{\partial}{\partial v_z} D_q \frac{\partial f}{\partial v_z}, \quad D_q = \frac{\pi e^2}{m_e^2} \int k^2 I_{\mathbf{k}} \delta(\omega_{\mathbf{k}} - k_z v_z) d\mathbf{k}. \quad (8)$$

The term  $\text{St}_s(f)$  takes into account the scattering of electrons by waves,  $\epsilon'$  and  $\epsilon''$  are the real and

imaginary parts of the dielectric constant, while  $\epsilon''$  is connected with the increment  $\gamma_{\mathbf{k}}$  by the relation  $\gamma_{\mathbf{k}} = -\epsilon''(\partial\epsilon'/\partial\omega)^{-1}$ . We recall that for ion-acoustic oscillations, with neglect of ion damping ( $T_i \ll T_e$ ), the frequency and the increment are determined by the relations

$$\omega_{\mathbf{k}}^2 = \frac{k^2 c_s^2}{1 + k^2 d^2}, \quad \gamma_{\mathbf{k}} = \frac{\pi}{2} \frac{\omega_{\mathbf{k}}^3 m_i}{k^2 m_e n_0} \int k_2 \delta(\omega_{\mathbf{k}} - k_2 v_z) \frac{\partial f}{\partial v_z} dv_z, \quad (9)$$

where

$$c_s^2 = T_e / m_i, \quad d^2 = T_e / 4\pi e^2 n_0.$$

In Eq. (7), the integral term  $S$  takes into account the scattering of the waves by electrons and ions.

The electron distribution function should be determined by means of Eqs. (6) and (7). So far as the ions are concerned, one can assume approximately that the Maxwell distribution at temperature  $T_i$  holds for them.

It is seen from Eq. (9) that the oscillations take place only in that region of phase velocities ( $\omega/k_z > 0$ ) where  $\partial f/\partial v_z > 0$ . Precisely in this region the noise intensity  $I_{\mathbf{k}} \neq 0$  (more exactly, it is much greater than the thermal level) and, consequently,  $D_{\mathbf{q}} \neq 0$ .

Under the assumption  $E \ll E_c$ , the energy input to the plasma per unit time is  $eunE \ll n_0 T_e / \tau_e$ , so that the distribution function does not differ appreciably from Maxwellian and Eq. (6) can be averaged over  $v_{\perp}$  under the assumption that  $T_{\perp} = T_{\parallel}$ . Keeping the same designation  $f$  for the integral of the distribution function over the transverse velocities, we get from (6) for the stationary case in the quasilinear approximation

$$\frac{e}{m_e} E \frac{\partial f}{\partial v_z} = \frac{v_e^2}{\tau_e} \frac{\partial}{\partial v_z} \left( \frac{\partial f}{\partial v_z} + \frac{m_e v_z}{T_e} f \right) + \frac{\partial}{\partial v_z} D \frac{\partial f}{\partial v_z}, \quad (10)$$

where the first component on the right approximates the Coulomb part of the collisions.

According to (9) and (10), the oscillations grow rapidly in amplitude in the region where  $\partial f/\partial v_z > 0$  and produce a plateau in the distribution function in this region. After a single integration of (10), we get

$$\frac{\partial f}{\partial v_z} = \frac{1}{\tau_e} f(u_0 - v_z) \left( D_{\mathbf{q}} + \frac{v_e^2}{\tau_e} \right)^{-1}, \quad (11)$$

where  $u_0 = e\tau_e E/m_e$ . It is then seen that the derivative  $\partial f/\partial v_z$  is positive for  $v_z < u_0$ , where  $u_0$  is determined by the same relation as the directed velocity  $u = j/en_0$  in the absence of oscillations. For  $E \ll E_c$  we get  $u_0 \ll v_e$ . Since the formation of a plateau changes the value of the longitudinal current but little (at most, by  $\sim u/v_e$ ), the effect of anomalous resistance in the quasilinear approximation is not large.

Let us assume that the amplitude of the oscillations is so large that the effect of scattering of waves by electrons begins to play an important role. For  $kd \sim 1$ , it suffices to consider in the scattering term only the Thomson scattering by "hot" electrons, neglecting polarization corrections. A corresponding expression can be obtained, for example, from the results of [7], taking it into account that in a strong magnetic field ( $\omega_H \rightarrow \infty$ ) the operators  $\mathbf{k} \cdot \mathbf{g}_{\mathbf{k}\omega}$  are replaced by

$$\frac{e}{m_e} \frac{k_z}{\omega - k_z v_z + i\nu} \frac{\partial}{\partial v_z}.$$

Moreover, it is sufficient to take into account the scattering by electrons only in the region outside the plateau, where  $\partial f/\partial v_z$  is not small. (Account of the scattering by electrons in the plateau region would

surpass the accuracy in the framework of the theory of weak turbulence considered by us.) With account of scattering, Eq. (10) keeps the same form, except that  $D_{\mathbf{q}}$  outside the plateau is replaced by

$$D_s = \frac{\pi e^4}{2m_e} \int \frac{(k_z - k'_z)^2 \delta(\omega - \omega' - k_z v_z + k'_z v'_z) I_{\mathbf{k}} I_{\mathbf{k}'}}{(v_z - \omega/k_z)^2 (v_z - \omega'/k'_z)^2} dk dk', \quad (12)$$

while the corresponding contribution to the term of scattering of the waves has the form

$$S = \frac{\pi e^4}{m_e^3} \int \frac{(k_z - k'_z) \delta(\omega - \omega' - k_z v_z + k'_z v'_z) I_{\mathbf{k}} I_{\mathbf{k}'}}{(v_z - \omega/k_z)^2 (v_z - \omega'/k'_z)^2} \frac{\partial f}{\partial v_z} dv_z dk'. \quad (13)$$

It is seen that the scattering of phonons by electrons leads to an additional diffusion of the electrons in velocity space. Since  $D_s$  falls off very rapidly with  $v_z$ , this diffusion simply broadens the region of the plateau: wherever  $D_s \gg v_e^2/\tau_e$ , the derivative  $\partial f/\partial v_z$  is near zero, while outside this region, the function  $f$  is close to a shifted Maxwellian (cf. (11)). The value of the anomalous resistance depend on the extent to which the plateau region broadens.

To estimate the width of this region, one can use the relations of energy and momentum balance of the ion-acoustic waves. We multiply (7) by  $\omega$  and  $k_z$  successively and integrate over  $\mathbf{k}$ . On the left side, we obtain the loss of energy and momentum due to transfer of the waves to the periphery of the pinch, i.e., in order of magnitude  $c_s \mathcal{E}/a$  and  $c_s P_z/a$ , respectively, where  $\mathcal{E}$  is the energy and  $P_z$  the momentum of the waves,  $a$  the radius of the plasma pinch.

So far as the right hand side is concerned, neglecting damping and scattering by the ions, it is expressed in terms of the integral of  $\epsilon_e'' k^2 I_{\mathbf{k}}/4\pi$  and  $S_e$ . The value of  $\epsilon_e''$  is expressed in terms of the derivative  $\partial f/\partial v_z$ , which is given by Eq. (11). In this relation  $v_e^2/\tau_e$  can be neglected in comparison with  $D_{\mathbf{q}}$  in the plateau region. It is not difficult to show that the numerator of the integral of  $\epsilon_e'' k^2 I_{\mathbf{k}}/4\pi$  contains an integral proportional to  $D_{\mathbf{q}}$ , so that  $D_{\mathbf{q}}$  can be cancelled out. In exactly the same way, in the integration of  $S_e$  with subsequent symmetrization over  $\mathbf{k}$  and  $\mathbf{k}'$ , the integral in the numerator cancels  $D_s$  in the denominator. As a result, the equations of balance of energy and momentum of the waves take the form

$$\frac{c_s}{a} \mathcal{E} = \int \frac{m_e v_z (u_0 - v_z)}{\tau_e} f dv_z, \quad (14)$$

$$\frac{c_s}{a} P_z = \int \frac{m_e (u_0 - v_z)}{\tau_e} f dv_z, \quad (15)$$

where the integration is carried out over the plateau region:  $-v_i < v_z < u_0 + v_e$ . Here the integral over the region of the quasilinear plateau  $0 < v_z < u_0$  is positive, in correspondence with the fact that the electrons in this region generate waves. The electrons outside the region of the quasilinear plateau absorb energy in the scattering and therefore the complete plateau cannot be significantly wider than the quasilinear one; otherwise the integral in (14) becomes negative and the amplitude of the noise begins to decrease.

Thus, in the absence of runaway electrons, the development of ion-acoustic instability of a nonisothermal plasma with  $\omega_H > \omega_0$  leads only to a small distortion of the electronic distribution function in the form of the formation of a plateau having a width on the order of  $u = e\tau_e E/m_e$ . The corresponding effect of the decrease

in the conductivity at the expense of a buildup of the oscillations is shown to be small, of the order of  $\delta\sigma \sim -\sigma u/v_e$ .

### CONCLUSION

It has been shown in the work that in the case of a strong magnetic field, when the cyclotron frequency of the electrons exceeds the Langmuir frequency, in the case of passage of a longitudinal current with directed velocity  $u = j/en_0$  less than the thermal velocity  $v_e = T_e/m_e$ , the effect of anomalous resistance in the true meaning of this word is small; the conductivity decreases only by an amount of the order of  $\delta\sigma \sim -\sigma u/v_e$ . However, in the presence of sufficiently long-lived accelerated electrons, an effect can take place which outwardly appears as an anomalous resistance. Namely, because of the interaction with the oscillations in the anomalous Doppler effect, the runaway electrons are made isotropic to a significant degree with respect to their velocities and can give an appreciable contribution to the plasma pressure. Here the "temperature"  $T_0 = p/n_0$ , which is determined from the pressure  $p$ , can be much greater than the temperature  $T_e$  of the main electrons. Correspondingly, the classical conductivity computed from  $T_0$  will be greater than the actual value. It is not excluded that just such an effect is ob-

served in the Tokamak toroidal systems.<sup>[4]</sup> A direct answer to this question could be given by x-ray measurements, from which one could judge the presence or absence of runaway electrons.

<sup>1</sup>E. C. Field and B. D. Fried, *Phys. Fluids* **7**, 1937 (1964).

<sup>2</sup>L. V. Korablev and L. I. Rudakov, *Zh. Eksp. Teor. Fiz.* **50**, 220 (1966) [*Soviet Phys. JETP* **23**, 145 (1966)].

<sup>3</sup>L. M. Kovrizhnykh, *Zh. Eksp. Teor. Fiz.* **51**, 1795 (1966) [*Soviet Phys. JETP* **24**, 1210 (1967)].

<sup>4</sup>L. A. Artimovich, G. A. Bobrovskii, S. V. Mirnov, K. A. Razumova and V. S. Strelkov, *Atomnaya énergiya* **22**, No. 4 (1967).

<sup>5</sup>H. Dreicer, *Phys. Rev.* **115**, 238 (1959); **117**, 329 (1960).

<sup>6</sup>V. D. Shafranov, in the collection *Voprosy teorii plazmy* (Problems of Plasma Theory) (No. 3, Atomizdat, 1963), p. 81.

<sup>7</sup>B. B. Kadomtsev, in the collection *Voprosy teorii plazmy* (Problems of Plasma Theory) (No. 4, Atomizdat, 1964), pp. 254-264.

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