THE GYROMAGNETIC EFFECT IN AN ANTIFERROMAGNET AT LOW TEMPERATURES

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The angular momentum of a spin system in an antiferromagnet at temperatures low compared with the Neel point is calculated. It is shown that in terms of the sublattice magnetization vector field the angular momentum is a sum of the "orbital" and "spin" field angular momenta. Owing to spin-spin dipole interaction the dependences of the angular momentum on the temperature and field intensity differ from those of the magnetic moment. In particular, the momentum does not vanish in zero field when the magnetic moment is zero.

THE properties of the gyromagnetic effect in a ferromagnet at low temperatures have been considered by one of the authors in[1]. It was shown that when the temperature is comparable with the energy of the spin-spin dipole interaction, then the temperature contributions to the magnetic moment and to the angular momentum of the spin system turn out not to be proportional to one another: their ratio is temperature dependent. At higher temperatures the gyromagnetic ratio coincides with the atomic g-factor.

In this paper, we consider the analogous problem for an antiferromagnet consisting of two magnetic sublattices. Since the magnetic moment and angular momentum of an antiferromagnet in the ground state (at zero temperature) vanish, the temperature effects are in this case the main effects, and not small contributions as in the ferromagnet.

In the absence of an external magnetic field an antiferromagnet has zero magnetic moment, whereas for small fields the magnetic moment depends linearly on the field. The field dependence of the angular momentum is at sufficiently low temperatures, as will be shown, quite different. In particular, for zero field the angular momentum does not vanish, as a result of which an antiferromagnet should have a spontaneous angular momentum of the spin system for zero magnetic moment.

1. The energy density of a uniaxial antiferromagnet with a positive anisotropy constant with a constant applied field H_0 is[2]:

$$\mathscr{H}=\frac{\alpha}{2}\left\{\left(\frac{\partial \mathbf{M}_1}{\partial x_i}\right)^2+\left(\frac{\partial \mathbf{M}_2}{\partial x_i}\right)^2\right\}+\alpha_{12}\frac{\partial \mathbf{M}_1}{\partial x_i}\frac{\partial \mathbf{M}_2}{\partial x_i}+\delta\mathbf{M}_1\mathbf{M}_2$$
$$-\frac{\beta}{2}\{(\mathbf{M}_1\mathbf{n})^2+(\mathbf{M}_2\mathbf{n})^2\}-(\mathbf{M}_1+\mathbf{M}_2,\mathbf{H}_0)-(\mathbf{M}_1+\mathbf{M}_2,\mathbf{h})-\frac{h^2}{8\pi}. \quad (1)$$

Here $\mathbf{M}_1$ and $\mathbf{M}_2$ are the sublattice magnetizations ($|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$); δ is the uniform and α and α_{12} the nonuniform exchange-interaction constants; β is the anisotropy constant, n—the unit vector along the axis of easiest magnetization, and h—the magnetic field describing the dipole magnetic interaction and satisfying the equations of magnetostatics.

In the ground state the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ do not depend on the coordinates and are antiparallel to one another: $\mathbf{M}_1 = -\mathbf{M}_2 = \mathbf{M}_0$. We shall denote by $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ the components of the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ perpendicular to the easy axis. Then

$$\mathbf{M}_1=\boldsymbol{\mu}_1+\mathbf{n}\sqrt{M_0^2-\mu_1^2},$$
$$\mathbf{M}_2=\boldsymbol{\mu}_2-\mathbf{n}\sqrt{M_0^2-\mu_2^2}.$$

Near the ground state the contributions $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, as well as the field h, are small compared with M_0. Confining ourselves to terms quadratic in the small contributions, we obtain

$$\mathscr{H}=\mathscr{H}_0+\frac{\alpha}{2}\left\{\left(\frac{\partial \boldsymbol{\mu}_1}{\partial x_i}\right)^2+\left(\frac{\partial \boldsymbol{\mu}_2}{\partial x_i}\right)^2\right\}+\alpha_{12}\frac{\partial \boldsymbol{\mu}_1}{\partial x_i}\frac{\partial \boldsymbol{\mu}_2}{\partial x_i}+\frac{\delta}{2}(\boldsymbol{\mu}_1+\boldsymbol{\mu}_2)^2$$
$$+\frac{\beta}{2}(\mu_1^2+\mu_2^2)+\frac{H_0}{2M_0}(\mu_1^2-\mu_2^2)-(\boldsymbol{\mu}_1+\boldsymbol{\mu}_2)\mathbf{h}-\frac{h^2}{8\pi}, \quad (2)$$

where $\mathscr{H}_0$ is the energy density of the ground state which does not depend on the constant field $\mathbf{H}_0$.

As in[1], we set up the density of the Lagrangian

$$L=\frac{1}{2gM_0}([\dot{\boldsymbol{\mu}}_1\mathbf{n}]\boldsymbol{\mu}_1-[\dot{\boldsymbol{\mu}}_2\mathbf{n}]\boldsymbol{\mu}_2)-\mathscr{H}, \quad (3)^*$$

from which one obtains as the Lagrange equations in approximation linear in $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_2$, and h the equations of motion for the sublattice magnetizations[3]

$$\frac{\partial \mathbf{M}_j}{\partial t}=-g[\mathbf{M}_j\mathbf{H}_{j\ \mathrm{eff}}],\quad \mathbf{H}_{j\ \mathrm{eff}}=-\frac{\delta\mathscr{H}}{\delta\mathbf{M}_j};\quad j=1,2, \quad (4)$$

and the magnetostatics equation for the field h

$$\operatorname{div}\mathbf{h}=-4\pi\operatorname{div}(\boldsymbol{\mu}_1+\boldsymbol{\mu}_2). \quad (5)$$

Here one must choose as the generalized coordinates the components of the vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, and the scalar potential φ of the field $\mathbf{h}$ ($\mathbf{h} = \nabla\varphi$).

2. With the aid of the density of the Lagrangian (3) one can obtain an expression for all the dynamic characteristics of the spin system of an antiferromagnet, forming the energy-momentum tensor[4]:

$$T_{\alpha\beta}=L\delta_{\alpha\beta}-\frac{\partial\boldsymbol{\mu}_1}{\partial x_\alpha}\frac{\partial L}{\partial(\partial\boldsymbol{\mu}_1/\partial x_\beta)}-\frac{\partial\boldsymbol{\mu}_2}{\partial x_\alpha}\frac{\partial L}{\partial(\partial\boldsymbol{\mu}_2/\partial x_\beta)}-\frac{\partial\varphi}{\partial x_\alpha}\frac{\partial L}{\partial(\partial\varphi/\partial x_\beta)};$$
$$\alpha,\beta=1,2,3,4,\quad x_{1,2,3}=x,y,z,\quad x_4=t.$$

Then, in particular, the momentum density is

$$p_i=T_{i4}=\frac{1}{2gM_0}\left([\boldsymbol{\mu}_1\mathbf{n}]\frac{\partial\boldsymbol{\mu}_1}{\partial x_i}-[\boldsymbol{\mu}_2\mathbf{n}]\frac{\partial\boldsymbol{\mu}_2}{\partial x_i}\right),\quad i=1,2,3. \quad (6)$$

Hence we find the "orbital" angular momentum of the spin system

*$[\boldsymbol{\mu}_i\mathbf{n}] \equiv \boldsymbol{\mu}_i \times \mathbf{n}$.

$$J_{orb} = \int [\mathbf{rp}]\, dV, \tag{7}$$

the integration being over the volume of the antiferromagnet.

The dipole interaction leads to nonconservation of the momentum J_{orb}. As follows from the linearized Eqs. (4), for the sublattice magnetization

$$\frac{d}{dt}(\mathbf{J}_{orb})_z = \int [\mathbf{h}, \mu_1 + \mu_2]_z dV \tag{8}$$

(the z axis coincides with the easy magnetization axis).

If the field H_0 is directed along the easy axis, then it follows from the same Eqs. (4) that

$$\int [\mathbf{h}, \mu_1 + \mu_2]_z dV = -\frac{1}{2gM_0}\frac{d}{dt}\int (\mu_1{}^2 - \mu_2{}^2)\, dV. \tag{9}$$

The quantity

$$(\mathbf{J}_{sp})_z = \frac{1}{2gM_0}\int (\mu_1{}^2 - \mu_2{}^2)\, dV \tag{10}$$

can be considered as the "spin" angular momentum of the field of the magnetizations μ_1 and μ_2. With an accuracy within a factor of $-1/g$, this quantity coincides with the z component of the magnetic moment of the system.

It follows from (8) and (9) that the angular momentum

$$J_z = (\mathbf{J}_{orb})_z + (\mathbf{J}_{sp})_z \tag{11}$$

is conserved, and constitutes the total angular momentum of the spin system of an antiferromagnet. In the case when H_0 is directed at an angle to the easy axis the axial symmetry is destroyed, and the momentum J_Z ceases to be an integral of the motion. However, in this case too the quantity J_Z has the meaning of the projection of the total angular momentum on the z axis.

3. In order to calculate the observed average value of the momentum in the region of low temperatures, one must carry out the quantization of the fields μ_1 and μ_2, i.e., consider the sublattice magnetizations as operators. In the approximation when the Hamiltonian is quadratic and Eqs. (4) are linear in μ_1 and μ_2, the transverse components of the magnetizations obey the Bose commutation relations. If one introduces the operators a_1 and a_2 by means of the equalities[1)]

$$\mu_1^- \equiv \mu_1^x - i\mu_1^y = \sqrt{2g\hbar M_0}\, a_1, \quad \mu_2^+ \equiv \mu_2^x + i\mu_2^y = \sqrt{2g\hbar M_0}\, a_2, \tag{12}$$

then it is readily verified that the following commutation rules hold:

$$[a_j(\mathbf{r}) a_l(\mathbf{r}')] = 0, \quad [a_j(\mathbf{r}) a_l^+(\mathbf{r}')] = \delta_{jl}\delta(\mathbf{r} - \mathbf{r}'); \quad j, l = 1, 2 \tag{13}$$

(it is assumed that $M_1^Z = -M_2^Z = M_0$).

The equations for the operators a_1 and a_2 follow from the linearized Eqs. (4) and are of the form

$$i\dot{a}_1 + gM_0\left\{\alpha\Delta a_1 + \alpha_{12}\Delta a_2^+ - \left(\delta + \beta + \frac{H_0}{M_0}\right)a_1 - \delta a_2^+ + \frac{h^-}{\sqrt{2g\hbar M_0}}\right\} = 0$$

$$i\dot{a}_2 + gM_0\left\{\alpha\Delta a_2 + \alpha_{12}\Delta a_1^+ - \left(\delta + \beta - \frac{H_0}{M_0}\right)a_2 - \delta a_1^+ + \frac{h^+}{\sqrt{2g\hbar M_0}}\right\} = 0, \tag{14}$$

where $h^{\pm} = h^x \pm ih^y$. The remaining two equations are obtained from (14) by Hermitian conjugation.

In order to diagonalize the Hamiltonian (2) and the momentum (11), we go over first of all to the Fourier components of the operators a_1 and a_2:

$$a_j(\mathbf{r}) = (2\pi)^{-3/2}\int a_{\mathbf{k}j}\, e^{i\mathbf{kr}}\, d^3k, \quad j = 1, 2. \tag{15}$$

The Fourier component of the field h is obtained from Eq. (5), and is

$$\mathbf{h}_\mathbf{k} = -2\pi k^{-2}\sqrt{2g\hbar M_0}\{k^+ a_{\mathbf{k}1} + k^- a_{-\mathbf{k}1}^+ + k^- a_{\mathbf{k}2} + k^+ a_{-\mathbf{k}2}^+\}\,\mathbf{k}. \tag{16}$$
$$k^{\pm} = k^x \pm ik^y.$$

As a result one obtains from (14) a system of four equations relating $a_{\mathbf{k}1}$, $a_{-\mathbf{k}1}^+$, $a_{\mathbf{k}2}$, and $a_{-\mathbf{k}2}^+$. Therefore the final diagonalization is carried out by means of linear transformations into new Bose operators $b_{\mathbf{k}1}$ and $b_{\mathbf{k}2}$ whose subscripts enumerate the branches of the normal oscillations:

$$a_{\mathbf{k}1} = u_{11} b_{\mathbf{k}1} + u_{12}^* b_{-\mathbf{k}2}^+ + v_{11}^* b_{-\mathbf{k}1}^+ + v_{12} b_{\mathbf{k}2},$$
$$a_{-\mathbf{k}2}^+ = u_{21} b_{\mathbf{k}1} + u_{22}^* b_{-\mathbf{k}2}^+ + v_{21}^* b_{-\mathbf{k}1}^+ + v_{22} b_{\mathbf{k}2}. \tag{17}$$

We introduce the matrix notation:

$$a_\mathbf{k} = \begin{pmatrix} a_{\mathbf{k}1} \\ a_{-\mathbf{k}2}^+ \end{pmatrix}, \quad b_\mathbf{k} = \begin{pmatrix} b_{\mathbf{k}1} \\ b_{-\mathbf{k}2}^+ \end{pmatrix}, \quad u = \begin{pmatrix} u_{11} u_{12}^* \\ u_{21} u_{22}^* \end{pmatrix}, \quad v = \begin{pmatrix} v_{11} v_{12}^* \\ v_{21} v_{22}^* \end{pmatrix}. \tag{18}$$

The linear transformation is then written in the form:

$$a_\mathbf{k} = u b_\mathbf{k} + v^* b_{-\mathbf{k}}^+ \tag{19}$$

(the asterisk denotes the complex conjugate, and the cross refers only to second-quantization operators).

The requirement that transformation (17) preserve the Bose commutation relations reduces, as is readily verified, to two matrix equations

$$\bar{u}u - \bar{v}v = \mathrm{I}, \quad \bar{u}v^* - \bar{v}u^* = 0, \tag{20}$$

where $\bar{u} = \sigma\tilde{u}^*\sigma$, and σ is the Pauli matrix

$$\sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{21}$$

Equations (14) can also be represented in a matrix form if one goes over to the Fourier components (15) and takes into account (16):

$$i\hbar\dot{a}_\mathbf{k} = \hat{G} a_\mathbf{k} + \hat{F}^* a_{-\mathbf{k}}^+, \tag{22}$$

where

$$\hat{G} = \begin{pmatrix} A_\mathbf{k} + g\hbar H_0 & B_\mathbf{k} \\ -B_\mathbf{k} & -A_\mathbf{k} + g\hbar H_0 \end{pmatrix}, \quad \hat{F} = \begin{pmatrix} C_\mathbf{k} & C_\mathbf{k} \\ -C_\mathbf{k} & -C_\mathbf{k} \end{pmatrix}, \tag{23}$$

and the elements $A_\mathbf{k}$, $B_\mathbf{k}$, and $C_\mathbf{k}$ are defined by the equations

$$A_\mathbf{k} = g\hbar M_0\left(\alpha k^2 + \delta + \beta + \frac{2\pi}{k^2}k^+k^-\right),$$
$$B_\mathbf{k} = g\hbar M_0\left(\alpha_{12}k^2 + \delta + \frac{2\pi}{k^2}k^+k^-\right),$$
$$C_\mathbf{k} = \frac{2\pi g\hbar M_0}{k^2}(k^+)^2. \tag{24}$$

Substituting (19) in (22) and comparing the coefficients of $b_\mathbf{k}$ and $b_\mathbf{k}^+$, we obtain a system of matrix equations

$$\hat{G}u + \hat{F}^* v = u\hat{\varepsilon}, \quad \hat{F}u + \hat{G}v = -v\hat{\varepsilon}, \tag{25}$$

where

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{\mathbf{k}1} & 0 \\ 0 & -\varepsilon_{\mathbf{k}_2} \end{pmatrix}. \tag{26}$$

[1)]The different definition of the operators a_1 and a_2 is due to the fact that the magnetic moments of the sublattices in the ground state are antiparallel ($\mathbf{M}_1 = \mathbf{M}_0$, $\mathbf{M}_2 = -\mathbf{M}_0$).

(Use has been made here of the diagonality of the Hamiltonian expressed in terms of the operators b_k and b_k^+.)

As is seen from (23), the matrix $\hat{F}$ is singular, which does not enable one to express v in terms of u (or vice versa) directly from Eqs. (25). Multiplying the first of Eqs. (25) on the right by $\hat{\epsilon}$ and making use of the second equation, and also of the equality $\hat{F}^*\hat{F} = 0$, we obtain

$$v = [\hat{G}, \hat{F}^*]^{-1}(u\hat{\varepsilon}^2 - \hat{G}^2 u), \quad [\hat{G}, \hat{F}^*] \equiv \hat{G}\hat{F}^* - \hat{F}^*\hat{G}. \tag{27}$$

Analogously

$$v\hat{\varepsilon}^2 = \hat{G}^2 v + [\hat{G}, \hat{F}]u.$$

Substituting here the value of v from (27), we obtain the equation

$$u\hat{\varepsilon}^4 - \{\hat{G}^2 + [\hat{G}, \hat{F}^*]\hat{G}^2[\hat{G}, \hat{F}^*]^{-1}\}u\hat{\varepsilon}^2 + \{[\hat{G}, \hat{F}^*]\hat{G}^2[\hat{G}, \hat{F}^*]^{-1}\hat{G}^2 - [\hat{G}, \hat{F}^*][\hat{G}, \hat{F}]\}u = 0.$$

One can verify by direct calculation that the expressions in the brackets are proportional to the unitary matrix. Since the matrices $\hat{\epsilon}^2$ and $\hat{\epsilon}^4$ are diagonal, we obtain the dispersion equation

$$\varepsilon^4 - 2(A_k^2 - B_k^2 + g^2\hbar^2 H_0^2)\varepsilon^2 + (A_k^2 - B_k^2 - g^2\hbar^2 H_0^2)^2 - 4|C_k|^2(A_k - B_k)^2 = 0,$$

whence we find two branches of the spectrum:

$$\varepsilon_{k1,2}^2 = A_k^2 - B_k^2 + g^2\hbar^2 H_0^2 \pm 2[g^2\hbar^2 H_0^2(A_k^2 - B_k^2) + |C_k|^2(A_k - B_k)^2]^{1/2}. \tag{28}$$

Substituting expression (27) in the first of Eqs. (25), we obtain an equation for the matrix u. In expanded form this equation is a system of four equations with real coefficients. The matrix of this system is, as one can verify, of second rank. Two matrix elements, for instance u_{11} and u_{12} can, therefore, be given arbitrarily and the others can be expressed in terms of these. The system of equations (25) allows one, therefore, to express the remaining six matrix elements in terms of two. These two elements can be found with accuracy up to a phase factor from conditions (20). We note that the second of conditions (20) is a consequence of the system (25) from which it follows that the matrix $\bar{u}v^* - \bar{v}u^*$ anticommutes with the diagonal matrix $\hat{\epsilon}$. Since $\epsilon_{k_1} \neq \epsilon_{k_2}$, this is only possible under the condition $\bar{u}v^* - \bar{v}u^* = 0$. As regards the matrix $\bar{u}u - \bar{v}v$, it follows from (25) that it commutes with the matrix $\hat{\epsilon}$. This results in the vanishing of the nondiagonal matrix elements. Therefore conditions (20) reduce to the requirement that the diagonal elements of the matrix $\bar{u}u - \bar{v}v$ be equal to unity:

$$|u_{11}|^2 - |u_{21}|^2 - |v_{11}|^2 + |v_{21}|^2 = 1,$$
$$|u_{22}|^2 - |u_{12}|^2 - |v_{22}|^2 + |v_{12}|^2 = 1. \tag{29}$$

Conditions (29) determine with an accuracy up to a phase factor two matrix elements in terms of which all the remaining ones are expressed. If these two elements are chosen real, then the matrix u will also be real and the matrix v will be proportional to $(k^+)^2 \sim e^{2i\varphi_k}$ (φ_k is the azimuthal angle of the wave vector **k**).

4. Let us now express the angular momentum (11) in terms of the Bose operators b_k and b_k^+. From (6), (7), and (10) we find with the aid of (12)

$$J_z = \frac{\hbar}{2}\int (a_1^+\hat{l}_z a_1 - a_1\hat{l}_z a_1^+ + a_2^+\hat{l}_z a_2 - a_2\hat{l}_z a_2^+ + a_1^+a_1 + a_1a_1^+ - a_2^+a_2 - a_2a_2^+)\,dV,$$

where $\hat{l}_z = -i(x\partial/\partial y - y\partial/\partial x)$. After going over to the Fourier components (15), we have

$$J_z = \frac{\hbar}{2}\int \{a_{k1}^+\hat{l}_k^z a_{k1} - a_{-k2}\hat{l}_k^z a_{-k2}^+ - a_{-k1}\hat{l}_k^z a_{-k1}^+ + a_{k2}^+\hat{l}_k^z a_{k2} + a_{k1}^+a_{k1} - a_{-k2}a_{-k2}^+ + a_{-k1}a_{-k1}^+ - a_{k2}^+a_{k2}\}\,d^3k,$$

$$\hat{l}_k^z = -i\left(k_x\frac{\partial}{\partial k_y} - k_y\frac{\partial}{\partial k_x}\right).$$

The latter equality can be written in matrix form:

$$J_z = \frac{\hbar}{2}\int \{a_k^+\sigma\hat{l}_k^z a_k - a_{-k}\sigma\hat{l}_k^z a_{-k}^+ + a_k^+\sigma a_k + a_{-k}\sigma a_{-k}^+\}\,d^3k. \tag{30}$$

Here σ is the Pauli matrix (21); the operators a and a^+ which stand on the right of σ are determined by the column of (18), and the operators on the left of σ by the corresponding row.

After substituting transformation (19) in expression (30), we find that in going over to the operators b and b^+ in which the Hamiltonian is diagonal, the momentum J_z retains the form (30):

$$J_z = \frac{\hbar}{2}\int \{b_k^+\sigma\hat{l}_k^z b_k - b_{-k}\sigma\hat{l}_k^z b_{-k}^+ + b_k^+\sigma b_k + b_{-k}\sigma b_{-k}^+\}\,d^3k. \tag{31}$$

Use has been made here of conditions (20), as well as of the relation $\hat{l}_k^z v = 2v$ and of the fact that the matrix u is real.

The operator of the momentum J_z is diagonalized together with the Hamiltonian in the representation of $k_\perp$, k_z, and m where $k_\perp = (k_x^2 + k_y^2)^{1/2}$, and $m = 0, \pm 1, \ldots$ is the eigenvalue of the single-particle operator $\hat{l}_k^z$:

$$b_k \equiv b_{k_\perp, k_z, \varphi_k} = \frac{1}{\sqrt{2\pi}}\sum_m b_{k_\perp, k_z, m}\, e^{im\varphi_k}. \tag{32}$$

If, in addition, one goes over to discrete quantum numbers, taking into account the finite volume of the antiferromagnet and the cylindrical symmetry, we obtain for the Hamiltonian and for the momentum the following expressions:

$$\mathcal{H} = \mathcal{H}_0 + \sum_{m,n,k_z}(\varepsilon_{1mnk_z} b_{1mnk_z}^+ b_{1mnk_z} + \varepsilon_{2mnk_z} b_{2mnk_z}^+ b_{2mnk_z}), \tag{33}$$

$$J_z = \hbar\sum_{m,n,k_z}\{(m+1)b_{1mnk_z}^+ b_{1mnk_z} + (m-1)b_{2mnk_z}^+ b_{2mnk_z}\}, \tag{34}$$

where the values of k_z are determined by the condition of periodicity and the number n enumerates the roots of the Bessel function $J_m(k_\perp R)$ (R is the radius of the cylinder), so that $k_\perp = k_{\perp mn}$.

We note that formulas (33) and (34) correspond to choosing as ϵ_{k_1} the branch of the spectrum with a plus sign of the root in (28). In the opposite case one must interchange the subscripts 1 and 2 in (34). This can be followed most simply by formally assuming $C_k = 0$ [see (24)]. Then, as follows from (25), in the first case the matrix $v = 0$, and in the second case $u = 0$. This corresponds to the indicated interchange of subscripts. In addition, in accordance with a choice of the sign before the root in ϵ_{k_1}, formulas (33) and (34) are appropriate when in the first place the matrix u and in the second the matrix v is real. (Otherwise the values of the index m in the energy of the spin wave ϵ and in the operators b will differ.)

5. Averaging (34) with an equilibrium Gibbs distribution, we find

$$\bar{J}_z = \hbar \sum_{m,n,k_z} (\bar{n}_{1mnk_z} - \bar{n}_{2mnk_z}),$$

where $\bar{n}_j = \langle b_j^+ b_j \rangle$ $(j = 1, 2)$ is the equilibrium Bose function with $\bar{n}_{j\,m,n,k_z} = n_{j\,-m,n,k_z}$. Returning to the momentum representation and going over from the sums to integrals, we obtain

$$\bar{J}_z = \frac{V\hbar}{(2\pi)^2} \int_0^\infty k^2 dk \int_0^\pi \sin\vartheta\, d\vartheta\, \{(e^{\varepsilon_{k1}/T} - 1)^{-1} - (e^{\varepsilon_{k2}/T} - 1)^{-1}\}, \quad (35)$$

where V is the volume of the antiferromagnet.

Since for the indicated choice of the spectrum branches $\epsilon_{k_1} > \epsilon_{k_2}$, then $\bar{J}_z < 0$. This means that the angular momentum of the spin system is directed in a direction opposite to that of the magnetic field (the positive direction of the z axis coincides with the direction of the field). Owing to the dipole-dipole interaction, the momentum (35) does not vanish for $H_0 = 0$ and $T \neq 0$.

Thus the relativistic magnetic interaction in an antiferromagnet produces at nonzero temperature in the spin system of the antiferromagnet a spontaneous angular momentum, just like the exchange interaction leads to a spontaneous momentum in a ferromagnet. As regards the direction of the spontaneous momentum, it is determined, as also in the case of a ferromagnet, by the external field which then tends to zero (as $V \to \infty$).

A calculation of the spontaneous momentum at a temperature $T \gg \beta g\hbar M_0 \approx 1^\circ K$ leads to the following result:

$$\bar{J}_z \approx -\frac{V\hbar}{a^3}\frac{g\hbar H_0}{T}\left(\frac{T}{\Theta_N}\right)^3. \quad (36)$$

where a is the lattice constant, and $\Theta_N = g\hbar M_0 a^{-1}[2\delta(\alpha - \alpha_{12})]^{1/2}$ coincides in order of magnitude with the Neel temperature. For comparison we present an expression for the momentum of an antiferromagnet located in an external magnetic field H_0 $(T \gg g\hbar H_0)$ when the dipole interaction is ignored[2]:

$$\bar{J}_z \approx -\frac{V\hbar}{\delta a^3}\left(\frac{T}{\Theta_N}\right)^3, \quad (37)$$

If, for example, $H_0 = 10^3$ Oe, $T = 10^\circ$, and $\delta = 10^2$, then the values obtained from (36) and (37) coincide in order of magnitude.

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