ROLE OF THE NARROW LAYER OF OPEN TRAJECTORIES IN THE THEORY OF GALVANOMAGNETIC PHENOMENA

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It is shown that the existence of a narrow layer of open trajectories leads, on the one hand, to a peculiar dependence of the resistance on the magnetic field H, and on the other to different temperature dependences of the resistance at various values of H within the same temperature range (in the case of electron-phonon interaction). A "temperature breakdown" phenomenon is predicted, which results in the dependence of the resistance on temperature even when the main cause of the resistance is electron scattering by impurities.

1. INTRODUCTION

THE kinetic properties of a metal in a magnetic field depend significantly on the dynamics of the conduction electrons, which in turn is determined by the topology of the Fermi surface.[1, 2] In the case of a closed Fermi surface, a sufficiently strong magnetic field ($r_H/l = \gamma \ll 1$, r_H—Larmor radius, l—mean free path) leads to the occurrence of a localized state (the electron moves on a closed orbit). Therefore a nonzero current ($j \neq 0$) along an electric field perpendicular to the magnetic field can occur only as result of electron scattering. If the Fermi surface is open, then a nonlocalized state is produced at certain directions of the magnetic field H (the electron moves on an open trajectory). Therefore a finite value of the current ($j \neq \infty$) along the electric field can be ensured only by some dissipation mechanism.

For a qualitative estimate of the electric conductivity it is possible to use the diffusion terminology. The conductivity is determined by the well known formula $\sigma = ne^2u$, where n is the number of carriers, u the mobility, which is obtained from the Einstein equation modified for the case of degenerate electron gas, $u = D/\varepsilon_F$ (ε_F—Fermi energy, D—diffusion coefficient). As is well known, the mean-square displacement $\overline{r^2}$ of a diffusing particle in a time t is determined by the expression $\overline{r^2} = Dt$. In the case of a closed Fermi surface, the electron is displaced as a result of collisions through a distance r_H within the relaxation time τ, and therefore $r_H^2 = D\tau$, that is, $D = r_H^2/\tau$. In the case of an open Fermi surface, the electron is displaced during the relaxation time τ a distance equal to the mean free path l, i.e., $l^2 = D\tau$ or $D = lv_F$ (v_F—Fermi velocity). Thus, for closed trajectories we have in order of magnitude $\sigma \approx ne^2r_H^2/\varepsilon_F l \approx \sigma_0\gamma^2$ (σ_0—conductivity of the metal without the magnetic field), and for open trajectories $\sigma = ne^2lv_F/\varepsilon_F \approx \sigma_0$.

The electron mean free path is determined by collisions with the impurities and the phonons. For rough estimates we can use the Matthiessen rule $l^{-1} = l_{imp}^{-1} + l_{ph}^{-1}$ (l_{imp}—electron-impurity mean free path, l_{ph}—electron-phonon mean free path). From the expression for the transverse conductivity in the magnetic field it follows that when $l_{ph} \ll l_{imp}$ (ultrapure samples) the conductivity for closed trajectories is proportional to $(T/\Theta)^5$ (Θ—Debye temperature), and for open ones to $(\Theta/T)^5$ (the phonon mean free path in the magnetic field without allowance for quantization effects, as usual, is proportional to $(\Theta/T)^5$ (see, for example [3]).

If the Fermi surface is open, some of the trajectories are closed and some are open. Under ordinary conditions the closed trajectories make an utterly insignificant contribution to the transverse conductivity. However, if the layer of the open trajectory is sufficiently narrow (the thickness of the "bridge" is $\delta p \ll p_F$, where p_F is the characteristic momentum, of the order of the Fermi momentum), then allowance for the closed trajectories is essential and the transverse conductivity takes the form[2]

$$\sigma_{yy} \approx \frac{\delta p}{p_F} ne^2 \frac{v_F l}{\varepsilon_F} + \gamma^2\sigma_0 \qquad (1)$$

(p_x is the direction of the openness, $\mathbf{H} \parallel \mathbf{z}$; in $\mathbf{r}$-space infinite motion takes place along the y axis). The mean free path which enters in the first term of (1) is in fact the distance which the electron has time to cover before either one of two events takes place: either it loses its momentum on the order of p_F, or else it "jumps out" of the layer of the open trajectories. If $\delta p \approx p_F$, then, naturally, this is the ordinary mean free path l. But if $\delta p \ll p_F$, then the "jumping out" will take place in a length l_{eff} much earlier than the momentum loss p_F ($l_{eff} \lesssim l$).

In estimating the effective mean free path l_{eff} it is also necessary to take into account the relation between δp and the characteristic change of the momentum Δp in one collision. If the principal role is played by collisions with impurities, then $\Delta p \approx p_F$ and $l_{eff} = l$. But if the main mechanism of the dissipation is scattering by phonons, then $\Delta p \approx q_0 = T/s$, where T is the temperature and s is the speed of sound ($p_F s \approx \Theta$). When $\delta p \gg q_0$ then, as can be readily seen, $l_{eff} = (\delta p/p_F)l$, and l as usual, is proportional to $(\Theta/T)^5$. Let us recall (using diffusion terminology) the origin of the fifth power of the temperature. The mean free path is $l = v_F\tau$, and τ is the time of diffusion of the electron in $\mathbf{p}$-space over a "distance" p_F, that is, $D_p\tau = p_F^2$,

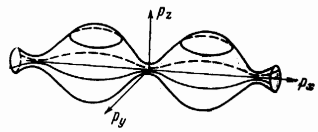

FIG. 1.

where $D_p = \overline{(\Delta p)^2}\,\nu_{ph}$. The collision frequency per unit time ν_{ph} is proportional to the number of phonons, that is, $\nu_{ph} \propto (T/\Theta)^3$, and $\overline{(\Delta p)^2} = q_0^2 \propto (T/s)^2$. Hence $l = l(\Theta)(\Theta/T)^5$.

But if $\delta p \ll q_0 = T/s$, then each collision knocks the electron t out from the layer of infinite motion, and this again leads to a sharp decrease in the effective mean free path. In this case the temperature dependence of l_{eff} changes. It is proportional to $(\Theta/T)^3$, and not to $(\Theta/T)^5$ as usual, since the frequency of collisions per unit time, as already mentioned, is proportional to T^3.

A small group of infinite-motion electrons can occur in the following cases.

A. The Fermi energy ε_F is close to the critical energy ε_c at which the open equal-energy surfaces become closed. In this case there are near the Fermi surface (which can be either closed or open) open equal-energy surfaces, representing a set of almost closed cavities joined by narrow "bridges" (see Fig. 1). The thickness of the "bridge" depends on the energy ε and its order of magnitude is $p_F\sqrt{|\delta\varepsilon|/\varepsilon_F}$, where $\delta\varepsilon = \varepsilon - \varepsilon_c$.

It follows from the foregoing estimate that the width of the layer of open trajectories δp can depend on the temperature. For open Fermi surfaces we have

$$\delta p \approx p_F[(|\delta\varepsilon_F| + kT)/\varepsilon_F]^{1/2}, \quad |\delta\varepsilon_F| = |\varepsilon_F - \varepsilon_{\kappa}| \ll \varepsilon_F.$$

On the other hand, if the Fermi surface is closed, then the smallness of the quantity $|\delta\varepsilon_F|$ can lead to a unique phenomenon—to "temperature breakdown," which consists in the following. When $k_T \ll |\delta\varepsilon_F|$, the main contribution to the current is made by the electrons on the closed equal-energy surfaces, so that transverse components of the electric conductivity $\sigma_{xx} \approx \sigma_{yy} \approx \gamma^2\sigma_0$. With increasing T, the value of kT becomes $\gtrsim |\delta\varepsilon_F|$. At these temperatures, the relative number of electrons on the open trajectories is of the order of $\sqrt{kT/\varepsilon_F}$, which corresponds to a value $\delta p \approx p_F\sqrt{kT/\varepsilon_F}$. According to the estimates made above, the contribution of this layer of open trajectories to the transverse current can become predominant, and this changes appreciably the dependence of σ_{yy} on H.

B. A narrow layer of open trajectories can occur also as a result of magnetic breakdown.[4] To this end it is necessary that there exist in p-space a line of points of intersection of two energy bands (line of conical points), passing through the Fermi surface. (On such a surface there is a unique open self-intersecting trajectory passing through the conical point (see Fig. 2; 1 and 2 are the numbers of the energy bands).) It can be shown that in this case the effective width of the layer of open trajectories is $\delta p \approx p_F\sqrt{\hbar\Omega/\varepsilon_F}$ (Ω—Larmor frequency).

In the next section we shall investigate the kinetic equation for the electrons in a magnetic field, in the case when the metal contains a small group of electrons executing infinite motion. In Sec. 3 we shall estimate

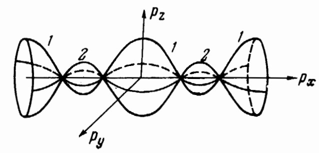

FIG. 2.

with the aid of the obtained formulas the different electron scattering mechanisms, and will consider in detail the influence of the magnetic breakdown and "temperature breakdown" on the transverse resistance of metals.

2. INVESTIGATION OF THE KINETIC EQUATION

The Boltzmann equation linearized over the electric field has, for electrons in constant magnetic and electric fields, the form

$$\partial f_1/\partial t - \partial f_1/\partial t_{col} = -e\mathbf{E}\mathbf{v}\,\partial f_0/\partial\varepsilon, \qquad (2)$$

where f_0 is the Fermi electron distribution function, t the time of motion of the electron on the trajectory $\varepsilon(\mathbf{p}) = \text{const}$, $p_z = \text{const}$, and $\partial f_1/t_{col}$—collision integral.

If we introduce the vector function ψ_i with the aid of the relation

$$f_1 = -eT\frac{\partial f_0}{\partial\varepsilon}\psi_i E_i,$$

then Eq. (2) is transformed into

$$\partial\psi_i/\partial t + \hat{W}_p\{\psi_i\} = v_i/T, \qquad (3)$$

where

$$\hat{W}_p\{\psi_i\} = -\left(T\frac{\partial f_0}{\partial\varepsilon}\right)^{-1}\frac{\partial}{\partial t_{cT}}\left(T\frac{\partial f_0}{\partial\varepsilon}\psi_i\right)$$
$$= \left(T\frac{\partial f_0}{\partial\varepsilon}\right)^{-1}\int K_{\mathbf{p},\mathbf{p}'-\mathbf{p}}\{\psi_i(\mathbf{p}') - \psi_i(\mathbf{p})\}\,d\mathbf{p}'; \qquad (3a)$$

$K_{\mathbf{p},\mathbf{p}'-\mathbf{p}}$ determines the number of transitions from the state $\mathbf{p}$ into the state $\mathbf{p}'$ in a unit time. The quantity $K_{\mathbf{p},\mathbf{p}'-\mathbf{p}}$ as a function of the difference between momenta of the initial and final states differs significantly from zero in the interval Δp (Δp is the characteristic momentum transferred to the electron during the scattering time).

The boundary condition for (3) is obtained by averaging this equation over the period of motion of the electron in the magnetic field:[1, 2]

$$\overline{\hat{W}_p\{\psi_i\}} = \bar{v}_i/T. \qquad (4)$$

In the system of coordinates chosen above $\bar{v}_x = 0$ in the entire interval of variation of p_z, while $\bar{v}_y \approx v_F$ on the interval $(-\delta p, +\delta p)$ and $\bar{v}_y = 0$ outside this interval. It follows therefore that the presence of a narrow layer of infinite motion does not affect the x-component of the distribution function ψ_x. The dependence of ψ_x on the field and on the temperature is exactly the same as in the case of a closed Fermi surface. The other component of the distribution function, ψ_y, experiences appreciable changes, owing to the existence of the narrow layer of open trajectories. To determine the zeroth term of the expansion of ψ_y in the reciprocal magnetic field, it is necessary to solve the following integral equation:

$$\left(\frac{\partial f_0}{\partial \varepsilon}\right)^{-1}\int \overline{K}_{\varepsilon,p_z;\varepsilon',p_z'}\{\psi_y^{(0)}(\varepsilon', p_z') - \psi_y^{(0)}(\varepsilon, p_z)\}d\varepsilon' dp_z' = \bar{v}_y(\varepsilon, p_z). \quad (5)$$

It is easy to see that the characteristic interval of variation of $\psi_y^{(0)}(\varepsilon_F, p_z)$ with respect to p_z is δp. This integral equation makes it possible to obtain in general form the temperature dependence of $\psi_y^{(0)}$ in two limiting cases: $\delta p \ll \Delta p$ and $\delta p \gg p$.

In the case $\delta p \ll \Delta p$, the velocity $\bar{v}_y$, and consequently also $\psi_y^{(0)}(\varepsilon, p_z)$ has a sharp maximum in the interval $(-\delta p, +\delta p)$, which is much smaller than the interval of integration in (5). Consequently, inside the indicated interval, the main contribution to $\psi_y^{(0)}(\varepsilon, p_z)$ is made by the integral terms of (5). It is therefore perfectly natural to represent the general expression for $\psi_y^{(0)}(\varepsilon, p_z)$ in the form

$$\psi_y = -\frac{\partial f_0}{\partial \varepsilon}\frac{\bar{v}_y}{\bar{v}_0} + \frac{\delta p}{\Delta p}\chi_y(\varepsilon, p_z), \quad (6)$$

where

$$\bar{v}_0 = \int \overline{K}_{\varepsilon,p_z;\varepsilon',p_z'}d\varepsilon' dp_z',$$

and χ_y is defined by the equation

$$\int \overline{K}_{\varepsilon,p_z;\varepsilon',p_z'}\chi_y(\varepsilon', p_z')d\varepsilon' dp_z' = \bar{v}_0(\varepsilon, p_z)\chi_y(\varepsilon, p_z) + \frac{\Delta p}{\delta p}\int \overline{K}_{\varepsilon,p_z;\varepsilon',p_z'}\frac{\partial f_0}{\partial \varepsilon'}\frac{1}{\bar{v}_0(\varepsilon', p_z')}\bar{v}_y(\varepsilon', p_z')d\varepsilon' dp_z'. \quad (7)$$

$\nu_0(p)$ is the number of electron collisions per unit time. For example, in the case of electron-phonon interaction $\nu_0 \propto T^3$, that is, it is proportional to the number of phonons at the given temperature.

Usually the frequency of the collision between the electron and different lattice defects or phonons depends little on the region of the Fermi surface in which the electron is situated during the time of the collision. Therefore we can assume for estimates that $\nu_0(p)$ is a continuous function of its argument in the interval $(-\delta p, +\delta p)$. It is then easy to see that

$$\int \overline{K}_{\varepsilon,p_z;\varepsilon',p_z'}\frac{\partial f_0}{\partial \varepsilon'}\bar{v}_0^{-1}(\varepsilon', p_z')\bar{v}_y(\varepsilon', p_z')d\varepsilon' dp_z' \sim v_F\int_{-\delta p}^{\delta p}\overline{K}_{\varepsilon,p_z;\varepsilon',p_z'}d\varepsilon' dp_z' \Big/ \int_{-\Delta p}^{\Delta p}K_{\varepsilon,p_z;\varepsilon',p_z'}d\varepsilon' dp_z' < v_F\frac{\delta p}{\Delta p}$$

and consequently Eq. (7) contains neither small nor large parameters. Therefore the function χ_y, which is a solution of (7), is of the same order of magnitude as the first term in (6). Thus, in the zeroth approximation in the conductivity γ is determined only by the electrons that execute infinite motion.

In order to take into account the contribution made to the conductivity by the electrons on the closed cross sections, it is necessary to calculate the higher terms of the expansion (up to second-order terms, since the terms of the diagonal part of the matrix σ that are odd in the field vanish.[2]). Finally, in the zeroth approximation with respect to the parameter $\delta p/\Delta p \ll 1$ we obtain

$$\psi_x \approx \frac{c}{eH}\frac{1}{T}(C_x^{(1)} - p_y) + \left(\frac{c}{eH}\right)^2\frac{m}{T}\left(\int_0^{\varphi}\hat{W}_p\{p_y\}d\varphi' - \int_0^{\varphi}\hat{W}_p\{C_x^{(1)}\}d\varphi'\right),$$

$$\psi_y \approx -\bar{v}_y\bar{v}_0^{-1}\frac{\partial f_0}{\partial \varepsilon} + \frac{c}{eH}\frac{1}{T}(C_y^{(1)} + p_x) + \left(\frac{c}{eH}\right)^2\frac{m}{T}\left(\int_0^{\varphi}\hat{W}_p\{p_x\}d\varphi' - \int_0^{\varphi}\hat{W}_p\{C_y^{(1)}\}d\varphi'\right). \quad (8)$$

Here $\varphi = t(eH/mc)^{-1}$, and the functions $C_x^{(1)}$ and $C_y^{(1)}$ which enter into these expressions are determined from the integral equations (accurate to $\delta p/\Delta p$)

$$\overline{W}_p\{C_x^{(1)}\} = \overline{\hat{W}_p\{p_y\}}, \qquad \overline{W}_p\{C_y^{(1)}\} = -\overline{\hat{W}_p\{p_x\}}.$$

These equations have the same structure as in the case of a closed Fermi surface. This is explained by the fact that in the first approximation in γ the main contribution to the conductivity is made by the electrons on the closed trajectories (the ratio of the number of electrons on the open trajectories to the number of electrons on the closed trajectories is proportional to $\delta p/p_F \ll 1$). Consequently, the entire system of electrons can be broken up into two weakly interacting subsystems: the electrons of the open trajectories and the electrons on the closed ones. As seen from (8), each of these subsystems makes an additive contribution to the conductivity. Accordingly, the conductivity tensor can be represented in the form

$$\sigma_{yy} = \sigma_{yy}^{\text{open}} + \sigma_{yy}^{\text{closed}}, \quad (9)$$

where $\sigma_{yy}^{\text{open}}$ is determined by the first term of the expression for ψ_y in (8). Owing to the average electron velocity which is contained in this term, it differs from zero only in the interval $(-\delta p, +\delta p)$, that is, $\sigma_{yy}^{\text{open}} \sim \delta p/p_F$. We call attention to the fact that the quantity ν_0 which enters in $\sigma_{yy}^{\text{open}}$ is the sum of the collision frequencies.

The inequality $p_F \gg \Delta p$ can be satisfied when the electron scattering is due mainly to collisions with the phonons. Owing to the condition $\delta p \gg \Delta p = q_0$, the velocity $\bar{v}_y$, and consequently also the function ψ_y, varies little over the interval q_0, making it possible to reduce (5) to a second-order differential equation with respect to the variable p_z (leaving it integral with respect to ε). Since the function ψ_y changes appreciably over the interval $(-\delta p, +\delta p)$, we get $\psi_y \sim (\delta p/p_F)^2(\Theta/T)^5$. The conductivity σ_{yy} is proportional to the quantity

$$T\int v_y\psi_y\frac{\partial f_0}{\partial \varepsilon}d\mathbf{p},$$

hence

$$\sigma_{yy} \approx (\delta p/p_F)^3(\Theta/T)^5\sigma_0(\Theta). \quad (10)$$

Here $\sigma_0(\Theta)$ is the conductivity of the metal in the absence of a magnetic field at $T = \Theta$.

3. INVESTIGATION OF THE GENERAL FORMULAS

The electric conductivity of a metal is determined usually by the electron-phonon and electron-impurity collisions, that is, $\nu_0 = \nu_{ph} + \nu_{imp}$, where ν_{ph} is the electron-phonon collision frequency and ν_{imp} is the electron-impurity collision frequency. At sufficiently high temperatures (in pure samples) $\nu_{ph} \gg \nu_{imp}$, that is, the presence of the impurities can be neglected. Using (8) and (9), as well as the results of [2, 3] for $\sigma_{ik}^{\text{closed}}$, we can readily write the dependence of the components of the electric conductivity tensor on the magnetic field and on the temperature ($T \ll \Theta$, the number of electrons n_1 is not equal to the number of holes n_2):

$$\sigma_{yy} = \sigma_{yy}^{\text{open}} + \sigma_{yy}^{\text{closed}} \approx \frac{\delta p}{p_F}\left(\frac{\Theta}{T}\right)^3 a_{yy}' + \left(\frac{T}{\Theta}\right)^5 \frac{a_{yy}''}{(\Omega\tau_\Theta)^2},$$
$$\sigma_{xx} = \sigma_{xx}^{\text{замкн}} \approx \left(\frac{T}{\Theta}\right)^5 \frac{a_{xx}}{(\Omega\tau_\Theta)^2}, \quad \sigma_{xy} = -\sigma_{yx} = \frac{ce(n_1 - n_2)}{H}. \quad (11)$$

Here τ_Θ is the relaxation time at $T = \Theta$ and a_{ik} are quantities which do not depend on the magnetic field due to coincide in order of magnitude with the conductivity without the magnetic field at $T = \Theta$.

The magnetoresistance tensor ρ_{ik} is obtained by inverting the conductivity matrix $\hat{\sigma}$. The most interesting is the dependence of the component ρ_{xx} of the resistance tensor on the magnetic field and on the temperature. The dependence of this component on the magnetic field is shown in Fig. 3.

If the magnetic field satisfies the condition $(\Omega\tau_\Theta)^{-2} \gg (\delta p/p_F)(\Theta/T)^8$, then the presence of a narrow bridge of the Fermi surface does not affect the resistance, and therefore ρ_{xx} tends to saturate with respect to the field and tends to a temperature dependence $(T/\Theta)^5$. If $(\Omega\tau_\Theta)^{-2} \ll (\delta p/p_F)(\Theta/T)^8$, then the electrons on the open trajectories begin to contribute, and ρ_{xx} increases quadratically with increasing H, and in this case

$$\rho_{xx} \approx (\Omega\tau_\Theta)^2 \frac{\delta p}{p_F}\left(\frac{\Theta}{T}\right)^3 \frac{1}{\sigma_0(\Theta)}. \quad (12)$$

With decreasing temperature, the electron-phonon frequency decreases and becomes comparable with, and subsequently even much lower than the electron-impurity frequency. In the latter case the resistance (as well as the mobility) tends to a constant value with respect to temperature. In the intermediate region, in which the inequality

$$\frac{1}{l_\Theta}\left(\frac{T}{\Theta}\right)^5 \ll \frac{1}{l_{\text{imp}}} \ll \frac{1}{l_\Theta}\left(\frac{T}{\Theta}\right)^3$$

is satisfied, the electrons on the closed cross sections collide effectively only with impurities ($l \gg l_{\text{imp}}$), whereas for the electrons on the open trajectories the effective collisions remain as before those with the phonons ($l_{\text{eff}} \approx (\Theta/T)^3 l_\Theta \ll l_{\text{imp}}$). Therefore $\sigma_{ik}^{\text{closed}}$ does not depend on the temperature, and $\sigma_{yy}^{\text{open}} \sim (\Theta/T)^3(\delta p/p_F)$. At these temperatures in magnetic fields satisfying the condition $(\Omega\tau_{\text{imp}})^{-2} \gg (\delta p/p_F)(\Theta/T)^3$ (τ_{imp}—electron-impurity free-path time) the presence of electrons moving in infinite fashion does not have any effect, and therefore ρ_{xx} does not depend on T. With increasing field ($(\Omega\tau_{\text{imp}})^{-2} \lesssim (\delta p/p_F)(\Theta/T)^3$), a narrow layer appears, and ρ_{xx} begins to increase. When $(\Omega\tau_{\text{imp}})^{-2} \ll (\delta p/p_F)(\Theta/T)^3$ the resistance component coincides with (12).

The component ρ_{yy} in the entire interval of the magnetic fields is determined by the electrons on the closed

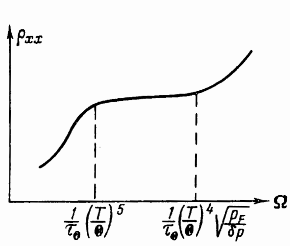

FIG. 3.

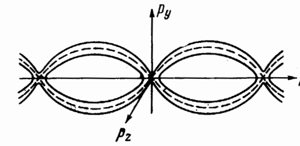

FIG. 4.

cross sections and therefore does not depend on the temperature.

The "temperature breakdown" (see Sec. 1) leads to a dependence of the resistance on the temperature even in the case when the phonons have already completely "frozen out" and the electrons are scattered electrically by the impurities. The cross section of the equal-energy surfaces that correspond to three different energies $\varepsilon < \varepsilon_c$, $\varepsilon = \varepsilon_c$, and $\varepsilon > \varepsilon_c$ are shown in Fig. 4. We recall that ε_c is that energy at which the closed equal-energy surfaces change into open ones. The fact that the temperature spreading of the Fermi step gives rise to a small group of infinitely moving electrons makes it possible to use formulas (8) to estimate the components of the conductivity tensor:

$$\sigma_{xx} \approx \frac{\sigma_0}{(\Omega\tau_{\text{imp}})^2}, \quad \sigma_{xy} = -\sigma_{yx} = \frac{ce(n_1 - n_2)}{H},$$
$$\sigma_{yy} \approx \sqrt{\frac{T}{\varepsilon_F}} \exp\left\{-\frac{|\varepsilon_c - \varepsilon_F|}{T}\right\}\sigma_0 + \frac{\sigma_0}{(\Omega\tau_{\text{imp}})^2} \quad (13)$$

From this we readily get for the resitance component ρ_{xx} the expression

$$\rho_{xx} \approx \frac{1}{\sigma_0}\left[1 + (\Omega\tau_{\text{imp}})^2 \sqrt{\frac{T}{\varepsilon_F}} \exp\left\{-\frac{|\varepsilon_c - \varepsilon_F|}{T}\right\}\right]. \quad (14)$$

In fields satisfying the condition

$$\frac{1}{(\Omega\tau_{\text{imp}})^2} \gg \sqrt{\frac{T}{\varepsilon_F}} \exp\left\{-\frac{|\varepsilon_c - \varepsilon_F|}{T}\right\},$$

the resistance, as usual, does not depend on the temperature and tends to saturate with respect to the field. If

$$\frac{1}{(\Omega\tau_{\text{imp}})^2} \lesssim \sqrt{\frac{T}{\varepsilon_F}} \exp\left\{-\frac{|\varepsilon_c - \varepsilon_F|}{T}\right\},$$

then ρ_{xx} begins to depend on T and increases with the increasing magnetic field:

$$\rho_{xx} \approx (\Omega\tau_{\text{imp}})^2 \sqrt{\frac{T}{\varepsilon_F}} \exp\left\{-\frac{|\varepsilon_c - \varepsilon_F|}{T}\right\}\frac{1}{\sigma_0},$$
$$\frac{1}{(\Omega\tau_{\text{imp}})^2} \ll \sqrt{\frac{T}{\varepsilon_F}} \exp\left\{-\frac{|\varepsilon_c - \varepsilon_F|}{T}\right\}.$$

If the Fermi surface is made up of periodically repeating cavities belonging to two different energy bands and interconnected by conical points (Fig. 2), then the narrow layer of open trajectories is the result of magnetic breakdown. To this end it is necessary that the line of conical points (see Sec. 1) be straight and that the magnetic field be perpendicular to it. The thickness of the layer δp now depends on the magnitude of the magnetic field:[4] $(\delta p/p_F) \approx \sqrt{\hbar\Omega/\varepsilon_F}$. This leads, on the one hand, to a somewhat different than usual dependence of the magnetoresistance on the magnetic field,[4] and on the other hand to a unique temperature variation of the magnetoresistance as a function of H. If the magnetic field is such that the existence of the electrons moving in infinite fashion does not affect the magnetoresistance, then the transverse component $\rho_{xx} \propto (T/\Theta)^5$ (we neglect the scattering by the impurities). In the same temperature interval, but in fields satisfying the condition $(T/\Theta) \gg \sqrt{\hbar\Omega/\varepsilon_F} \gg (T/\Theta)^8 (\Omega\tau_\Theta)^{-2}$, the transverse magnetoresistance is

$$\rho_{xx} \approx (\Theta / T)^3 \sqrt{\hbar\Omega / \varepsilon_F} (\Omega\tau_\Theta)^2 \sigma_0^{-1}(\Theta).$$

This is connected with the fact that the narrow layer of open trajectories is already significant in such fields, but the thickness of the layer is equal to $p_F\sqrt{\hbar\Omega/\varepsilon_F} \ll q_0 = p_F(T/\Theta)$, and therefore the formulas (8) and (11) are valid.

In even stronger fields $\sqrt{\hbar\Omega/\varepsilon_F} \gg T/\Theta)$ the thickness of the layer becomes much larger than the characteristic phonon momentum q_0 and

$$\rho_{xx} \approx (\Theta / T)^5 (\hbar\Omega / \varepsilon_F)^{3/2} (\Omega\tau_\Theta)^2 \sigma_0^{-1}(\Theta).$$

In the derivation of the latter estimate we used formula (10).

One of the symptoms whereby a narrow layer of open trajectory can be observed experimentally is the increase of the resistance with increasing H after the $\rho_{xx}(H)$ dependence saturates (Fig. 3). Such a behavior of the resistance was observed by Borovik and Volotskaya,[5] who measured the $\rho_{xx}(H)$ dependence of pure aluminum at helium and hydrogen temperatures. The deviation from Kohler rule observed in [5] agrees with the results given above. However, for a complete comparison of theory with experiment it is necessary to measure the temperature dependence of $\rho_{xx}(T)$ at different values of the magnetic field.

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