SOVIET PHYSICS JETP VOLUME 26, NUMBER 3 MARCH, 1968

STARK EFFECT IN HIGH FREQUENCY STOCHASTIC FIELDS IN A PLASMA

E. V. LIFSHITZ

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted February 15, 1967

Zh. Eksp. Teor. Fiz. 53, 943–952 (September, 1967)

The Stark effect in high-frequency stochastic fields in a plasma is analyzed theoretically. Relations are between the spectral line width and the electric field strength for the cases of stochastic oscillations and a superposition of regular and stochastic oscillations. It is shown that the Stark effect in alternating stochastic and regular high-frequency or low frequency fields can be employed effectively to determine the main parameters of an unstable turbulent plasma.

As is well known, strong high-frequency electric fields can become excited when instabilities develop in a plasma. A peculiarity of these fields is that they very rarely represent oscillations with fixed phase and frequency, and are for the most part stochastic. Such fields can be excited in the plasma also by external sources, for example in the stochastic method of heating and accelerating electrons or plasma ions. Measurement of the intensity of such fields is of considerable interest. Besides being of interest in themselves, these measurements are also important because they make it possible to determine directly a plasma parameter $n_p kT_e$, which is important for many applications and which is connected with the field intensity by several relations. By determining the field intensity from the half-width of the Stark contour, it is possible to determine $n_p kP_e$ directly. In addition, knowing n_p, we can also determine from these measurements T_e.

An advantage of this method for measuring the main parameters of the plasma is that it does not lead to plasma perturbation. Its efficiency increases with increasing electric field intensity, and, starting with field intensity values $\sim 2-3$ kV/cm, it yields reliable results. To use this method it is necessary to find a relation between the width of the spectral line and the intensity of the stochastic high-frequency fields. It is also of interest to determine in detail the contour of the spectral line. In the presence of high frequency oscillations of frequency ω, satellites spaced $n\omega$ apart appear in the Stark contour. Thus, from the form of the contour it is possible to detect the existence of high-frequency oscillations and to measure their frequency. Since the form of the contour depends significantly on the degree of stochasticity of the oscillations, it also becomes possible to determine the character of the exciting oscillations.

Whereas the Stark effect in external regular alternating field has been investigated quite thoroughly by Schrödinger,[1] the particular case of monochromatic high frequency oscillations was investigated in detail by Blokhintsev[2] and Mitsuk,[3] the Stark effect in stochastic high frequency fields, insofar as we know, has never been considered before. The purpose of the present investigation was to determine the shape of the spectral line due to the high frequency stochastic fields, and to derive relations between its half-width and the intensity of the electric field. It is assumed here (an assumption which is satisfied in a larger number of experiments) that the Stark effect in high frequency fields is much stronger than the Stark effect due to internal Coulomb fields. As is well known, the theory of the latter has been well investigated by Holtsmark,[4] by Mandel'shtam and Sobolev,[5] and by others. The main investigations of the Stark broadening of lines in a high-temperature plasma were made by Kogan[6] and by Kolb, Griem, and Schen,[7] while the nonstationary processes of pair collisions were taken into account by Vaĭnshteĭn and Sobel'man.[8]

It must be noted that the recent experimental investigations have demonstrated the possibility of measuring the intensity of high-frequency electric fields excited by external sources in a plasma, by means of a method based on the use of the Stark effect in regular alternating fields.[3, 9, 10] Preliminary measurements of the electric field intensity and of the value $n_p kT_e$ by means of the Stark effect in irregular alternating fields, produced as a result of collective intereactions between an electron and a plasma, give results which are in good agreement with measurement made with the aid of other methods.[11] We point out that the problem of determining the shape of the spectral line in the case of the Stark effect in alternating stochastic fields is similar to a considerable degree with the well-known radiophysics problem of determining the frequency spectrum of a signal which is modulated in phase or in frequency by noise.[12]

The simplest method of determining the contour of the spectral line is one based on the use of correlation functions. As is well known, the spectral density of emission is determined in accordance with Khinchin-Wiener by expansion of the correlation function in a Fourier series or integral (for the discrete or continuous spectrum, respectively).[13] In the case of a discrete spectrum, it is determined by the relation

$$I(\omega_n) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} K(\tau) e^{-i\omega_n \tau} d\tau. \quad (1)$$

The correlation function $K(\tau)$ is here equal to[17]

$$K(\tau) = \overline{\exp i[\psi(t+\tau) - \psi(t)]}, \quad \psi(t) = \alpha \int_0^t E(t) dt, \quad (2)$$

α is the constant of the linear Stark effect.

We now consider the question of the Stark broadening due to oscillations excited when an initially-monoenergetic beam of electrons passes through a plasma.[14] As is well known, collective interaction causes in this case first excitation of oscillations at a frequency close to the Langmuir frequency, with a very narrow spectrum with respect to the wave numbers k ($k \approx \omega/v_{beam}$), and then, as a result of the reaction of the excited oscillations, the beam decelerates and its temperature increases. This starts the buildup of the neighboring part of the spectrum (with respect to k), and the total oscillation energy increases. In the steady-state turbulence state reached as a result of the reaction of the growing oscillations on the beam distribution function, broad packets of waves with random phases are excited in the plasma, and the oscillation frequency remains as before close to the Langmuir frequency. Thus, the electric fields acting on the atoms and on the ions are a superposition of waves with specified amplitudes E_k and with random phases.

An import circumstance for what follows is that in strong magnetic field $\omega_H^2 \gg \omega_p^2$ (ω_H is the Larmor frequency of the electrons, ω_p is the Langmuir frequency), when the transverse motion of the electrons is forbidden, the most intensely excited waves propagate in the direction of beam motion; they are quasi-one-dimensional. This can be seen also from the fact that the oscillation growth increment $\gamma \sim \cos\theta$, where θ is the angle between the direction of motion of the beam and the direction of wave propagation (such a relation takes place, for example in the case when the elementary excitation mechanism is the Cerenkov effect). It is precisely such a quasihomogeneous character which is possessed by waves observed in a large number of experiments on collective interactions between beams and plasma. The one dimensional character of the excited waves simplifies the solution of the problem of the Stark broadening of the lines, for in this case it is legitimate to use the scalar law of addition of the perturbations causing the Stark broadening of the lines. In the case when the transverse components of the field E_r are sufficiently large and the waves are not one-dimensional, it is necessary to use the "vector law" of perturbation addition. This more complicated case, which is of undisputed interest, will not be considered here.

Since the frequencies of the high-frequency oscillations are lower than the frequencies of the light radiation, the perturbation due to these fields can be regarded as adiabatic, and consequently it does not cause any transitions between different states. Thus, in the case of the Stark effect due to the high frequency one-dimensional waves, the function $\psi(t)$ is equal to

$$\psi(t) = \alpha \sum_{k=1}^{N} \frac{E_k}{\omega_k}[\sin(\omega_k t + \varphi_k) - \sin\varphi_k], \tag{3}$$

where $\varphi_k = kx_0 + \varphi_{0k}$, φ_{0k} is the random phase, and ω is connected with k by the dispersion relation

$$\omega^2 = \omega_p^2(1 + 3k^2a^2), \tag{4}$$

ω_p is the Langmuir frequency of the electrons, k the wave number, and a the Debye radius.

Since $k^2a^2 \ll 1$, we have in first approximation

$$\psi(t) \approx \frac{\alpha}{\omega_p} \sum_{k=1}^{N} E_k[\sin(\omega_p t + \varphi_k) - \sin\varphi_k]. \tag{5}$$

During the saturation stage, the intensity of the high frequency oscillations is maximal and is determined by a relation derived by Shapiro,[15]

$$|E_k|^2 = 4\pi^2 n_1 m \frac{\omega_p^2}{k^3} \frac{(\omega_p/k) - v_1}{v_2 - v_1} \approx 4\pi^2 n_1 m \frac{\omega_p^3}{k^4 v_0}, \tag{6}$$

where v_0 and n_1 are respectively the velocity and the density of the beam electrons, and $v_2 - v_1$ is the width of the "plateau" produced on the beam-electron distribution function as a result of the reaction of the excited high-frequency oscillations on the beam electrons

$$v_2 = v_0\left[1 + \left(\frac{n_1}{n_p}\right)^{1/3}\right], \quad v_1 \approx v_{Te} \ln\left[\frac{n_p}{n_1}\frac{v_0}{v_{Te}}\right].$$

The wave number k varies between the limits

$$k_1 \leqslant k \leqslant k_2, \quad k_1 = \frac{\omega_p}{v_0}, \quad k_2 = \sqrt{2}\Big/ \ln\left[\frac{n_p}{n_1}\frac{v_0}{v_{Te}}\right] a;$$

v_{Te} and n_p are the thermal velocity and the density of the plasma electrons.

In the case when the number of terms in the sum $N \to \infty$, the function $\psi(t)$ is described by a Gaussian distribution and we can use for the calculation of the correlation functions the corresponding relations obtained in the theory of phase or frequency modulation of the signal by noise. The correlation function for a finite number N, without assuming a Gaussian distribution, can be obtained by direct calculation.

It is necessary to obtain

$$K(\tau) = \overline{\exp i\Phi} = \int_{-\infty}^{\infty} e^{i\Phi} W(\Phi)\, d\Phi,$$

$$\Phi = \alpha \sum_{k=1}^{N} \psi_k = \alpha \sum_{k=1}^{N} \frac{E_k}{\omega_k}[\sin(\omega_k t + \varphi_k) - \sin\varphi_k],$$

where W is the probability of Φ.

According to the Markov's method[6, 16]

$$K(\tau) = \prod_{k=1}^{N} A_k(1), \quad A_k(\rho) = \int e^{i\rho\psi_k} W_1(\psi_k)\, d\psi_k.$$

Here $W_1(\psi_k)$ is the probability of ψ_k. Assuming that all the values of the phase φ_k are equally probable, we have for $A_k(\rho)$ the following formula:

$$A_k(\rho) = \frac{1}{2\pi}\int_0^{2\pi} \exp\left\{i\rho\alpha\frac{E_k}{\omega_k}[\sin(\omega_k t + \varphi_k) - \sin\varphi_k]\right\} d\varphi_k$$
$$= J_0\left[2\rho\alpha\frac{E_k}{\omega_k}\sin\omega_k t\right], \tag{7}$$

where J_0 is a Bessel function. Therefore

$$K(\tau) = \prod_{k=1}^{N} J_0\left[2\alpha\frac{E_k}{\omega_k}\sin\omega_k\tau\right]. \tag{8}$$

If N is very large, then the amplitude of the field of each individual harmonic is small and $\alpha E_k/\omega_k \ll 1$. Expanding J_0 in a series and confining ourselves to the first terms, we obtain

$$K(\tau) = \exp\left[-\frac{\alpha^2}{2}\sum_{k=1}^{N}\frac{E_k^2}{\omega_k^2}(1 - \cos\omega_k\tau)\right], \tag{9}$$

or, in first approximation over the dispersion equation $k^2a^2 \ll 1$,

$$K(\tau) = \exp\left[-\frac{\alpha^2}{2\omega_p^2}\sum_{k=1}^{N}E_k^2(1 - \cos\omega_p\tau)\right]. \tag{10}$$

The same result can be obtained simply by noting that in the case when $\alpha E_k/\omega_k \ll 1$ we can confine ourselves in the expansion of the exponential in the integrand to the first three terms. Since $\overline{\sin \varphi_k} = 0$, confining ourselves to quadratic terms and multiplying the obtained results for different $\alpha E_k/\omega_k$, we get formula (9).

In the case when $N \to \infty$ and $\psi(t)$ is described by a Gaussian distribution law, relation (10) can also be obtained by using the rather simple method of calculating the mean values of $\overline{\exp i\varkappa[x(t) - x(t')]}$, proposed by Podgoretskiĭ and Stepanov[17, 18] and used by them to determine the Doppler width of the line of an atom executing Brownian motion in a dense gaseous medium. Noting that in the case of a Gaussian distribution for the quantities $x(t) - x(t') = \Delta x$ the odd powers in the expansion of the exponential vanish, and for even $\overline{(\Delta x)^{2n}} = (2n-1)!!\overline{(\Delta x)^2}$ the authors find that

$$\overline{\exp i\varkappa[x(t) - x(t')]} = \exp\left\{-\frac{\varkappa^2}{2}\overline{[x(t) - x(t')]^2}\right\}. \tag{11}$$

In our case

$$x(t) = \sum_{k=1}^{N} \frac{E_k}{\omega_k} \sin(\omega_k t + \varphi_k), \qquad \varkappa \equiv \alpha.$$

When $N \to \infty$ the function $x(t)$ obeys a Gaussian distribution. Substituting this relation in (11) and performing the corresponding calculations, we get (10).

If the field amplitudes are the same $E_k \equiv E$, then the correlation function is equal to[1)]

$$\exp\left[-\frac{2\pi\alpha^2}{\omega_p^2} n_p k(T_e - T_{e0})(1 - \cos\omega_p\tau)\right]. \tag{12}$$

When $\omega_k \equiv \omega_p$ the function $K(\tau)$ is a periodic function of τ; to obtain the contour of the spectral line it is then necessary to expand $K(\tau)$ in a Fourier series. Performing the required calculations, we get

$$K(\tau) = e^{-\delta} I_0(\delta) + \sum_{n=1}^{\infty} e^{-\delta} I_n(\delta) \cos n\omega_p\tau, \tag{13}$$

where

$$\delta = \frac{1}{2}\left(\frac{\Delta\omega}{\omega_p}\right)^2, \quad \Delta\omega = \alpha\left(\sum_{k=1}^{N} E_k^2\right)^{1/2}.$$

Assuming that $\Delta\omega/\omega_p \gg 1$ and using the asymptotic form of the Bessel function for $\delta \gg 1$, $n \gg 1$, and $n \lesssim \delta$, we get

$$K(\tau) = \frac{1}{\sqrt{\pi\delta}} \sum_n e^{-n^2/4\delta} \cos n\omega_p\tau. \tag{14}$$

Thus, the spectral line in the case of a linear Stark effect, due to stochastic high frequency fields, splits into a whole series of satellites, spaced ω_p apart. The maximum of the intensity decreases away from the center of the line like

$$I(n\omega_p) \approx \frac{1}{\sqrt{\pi\delta}} e^{-n^2/4\delta}. \tag{15}$$

The half-width of the envelope, determined from the relation (14), is equal in this case to

$$n\omega_p \approx \Delta\omega = \alpha\left(\sum_k E_k^2\right)^{1/2} \approx \alpha[4\pi n_p k(T_e - T_{e0})]^{1/2}. \tag{16}$$

It is interesting to compare the obtained line contour with the line contour due to the Stark effect in an alternating field in the case when the electric field is a monochromatic oscillation with fixed phase $E = E_0 \cos\omega\tau$.[1, 2] We recall that the intensity of the satellite $n\omega$ is in this case proportional to $J_n^2(x)$ (J_n — Bessel function); $x = \Delta\omega_0/\omega$. Here $J_{n\approx x}^2(x)/J_n^2 \ll x(x) \approx 0.65x^{1/3}$ and the intensity of the satellite is minimal for small n. It increases with n to values $n_{max} \approx x$, and then decreases exponentially when $n > x$. At very large values of x, the unshifted frequency ω_0 vanishes and lines symmetrical about ω_0 appear, shifted by amounts $\pm\Delta\omega_0$, that is, as expected, the shape of the contour coincides in this case with the shape of the contour in the static Stark effect.[2)]

With decreasing x, the intensity of the satellites located between these lines increases, so that when $x \approx 10$ the contour of the spectral line is a rectangle of width $2\Delta\omega_0$; the intensity of the satellites is constant within these limits. In our case of stochastic high-frequency oscillations, the maximum intensity occurs at the unshifted line (ω_0, $n = 0$); with increasing n it drops like $A_n \sim \exp[-n^2/4\delta]$, which differs appreciably from the preceding case up to values $n \le x$.

The shapes of the spectral line differ appreciably from each other in both cases. This enables us to determine from the shape of the spectral line the degree of stochasticity of the high frequency oscillations. We have obtained the form of the contour of the spectral line under the assumption that $\omega_k \equiv \omega_p$. Actually, as follows from the dispersion equation (5), the ω_k are different, although the width of the spectrum with respect to ω is small. Let us find now the form of the line contour with allowance for the dependence of ω on k, in accord with (4).

We need to calculate

$$P(t) = -\frac{\alpha^2}{2}\sum_{k=1}^{N} \frac{E_k^2}{\omega_k^2}(1 - \cos\omega_k t).$$

Replacing the summation by integration with respect to k and then changing over to the variables ω_k, and also recognizing that in accordance with (6)

$$|E_{\omega_k}|^2 = 4\pi^2 3^{3/2} n_1 m \frac{\omega_p^4}{(\omega_k^2 - \omega_p^2)^{5/2}} \frac{v_{Te}}{v_0}$$

we obtain

$$P(t) = -2\pi^{3/2}\alpha^2 n_1 m v_0^2 \frac{v_{Te}}{v_0} \int_{\omega_1}^{\omega_2} (1 - \cos\omega_k t) \cdot \frac{[1 - (\omega_k^2 - \omega_p^2)/\omega_p^2]}{(\omega_k^2 - \omega_p^2)^{5/2}} d\omega_k,$$

$$\omega_1 = \omega_p\left(1 + 3\frac{v_{Te}^2}{v_0^2}\right)^{1/2}, \quad \omega_2 = \omega_p(1 + 3L^2)^{1/2},$$

$$L = \sqrt{2}/\ln\left[\frac{n_{pl}}{n_1}\frac{v_0}{v_{Te}}\right]. \tag{17}$$

---

1) We have used here the fact that in our case $\sum_k E_k^2/4\pi \approx n_p k(T_e - T_{e0})$.

2) We have considered the change of the Stark contour due to high frequency stochastic fields, for the case of only two components of a static Stark structure of the lines. Actually, the spectrum is characterized by a larger number of components. Therefore, to construct the real line contour it is necessary to take into account the contribution made to the intensity by all these components.

Calculating this integral under the assumption that $\alpha^2 n_1 m v_0^2/\omega_p^2 \gg 1$, we obtain in the first approximation in this parameter $P(t) = -\frac{2}{3}\pi\alpha^2 n_1 m v_0^2 t^2$.

To determine the line shape due to the Stark effect in alternating stochastic electric fields of high-frequency oscillations excited by an electron beam passing through the plasma at the saturation stage, it is now sufficient to substitute (17) in (9) and to find with the aid of the so-obtained correlation function $K(\tau)$ the spectral radiation density $I(\omega)$, which is connected with $K(\tau)$ in the case of a continuous spectrum by the relation:[13, 18]

$$I(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(\tau) e^{-i\omega\tau} d\tau.$$

As a result of the calculations we get

$$I(\omega) \approx \exp\{-\omega^2/\Delta\omega^2\}. \tag{18}$$

Therefore the width of the contour of the spectral line is in this case $\Delta\omega = 2\alpha[\frac{2}{3}\pi n_1 m v_0^2]^{1/2}$.

The ion-acoustic oscillations which are excited when large currents flow through the plasma also have an appreciable intensity. The intensity of the ion-acoustic oscillations and the spectral distribution under turbulent-plasma conditions were obtained by Kadomtsev and Petviashvili[19] and are given by the relation

$$E_k^2 \approx k^2 a^2 \varepsilon_k \approx \frac{v_0}{v_{Te}} \frac{T_e}{T_i} \frac{T_e^2}{4\pi^2 k \cdot 7\theta_0^2} \ln\frac{1}{ka}, \tag{19}$$

where ε_k is the spectral energy density, $\varepsilon_k = M_i n_p v_k^2 \approx M_i n_p v_s^2 k^{-3}$, v_0 and n_p are the directed velocity and the density of the plasma electrons, T_e and T_i are respectively the temperatures of the electrons and ions, v_{Te} and v_s are the thermal velocity of the electrons and the speed of sound in a two-temperature plasma, and M_i is the ion mass. It is assumed that the waves build up in a relatively narrow cone of the directions $\mathbf{k}$, where θ_0 is the vertex angle of this cone. Since the quantity $\sum_k E_k^2$ may turn out to be quite appreciable, we shall also consider the Stark effect due to ion-acoustic waves.

To estimate the Stark broadening due to the electric fields of ion-acoustic oscillations, it is sufficient to use relation (19), the dispersion equation for these oscillations, and to calculate the sum

$$P(t) = -\frac{\alpha^2}{2}\sum_k \frac{E_k^2}{\omega_k^2}(1-\cos\omega_k t).$$

As a result of the calculations we get

$$P(t) = -\frac{\alpha^2}{2} n_p k T_e \frac{T_e}{T_i} \frac{v_0}{v_{Te}} \frac{\pi}{7} a^2 k_{max}^2 t^2, \tag{20}$$

where $k_{max} \sim 1/a$. At sufficiently high plasma electron temperature and plasma density, $P(t)$ may turn out to be quite large. The line width determined by the ion-acoustic waves is equal to

$$\Delta\omega_0 = \alpha\left[n_p k T_e \frac{T_e}{T_i} \frac{v_0}{v_{Te}} \frac{2\pi}{7} a^2 k_{max}^2\right]^{1/2} \tag{21}$$

Very frequently the alternating electric fields which cause Stark broadening in a non-equilibrium plasma are superpositions of regular and random high frequency oscillations. It is therefore necessary to determine the form of the Stark contour also in this case. If it is assumed that the electric field in the plasma $E(t)$ and the function $\psi(t)$ are of the form

$$E(t) = E_0\cos\omega_1 t + \sum_{k=1}^{N} E_k \cos(\omega_k t + \varphi_k),$$

$$\psi(t) = \psi_p(t) + \psi_N(t) = \frac{\alpha E_0}{\omega_1}\sin\omega_1 t + \alpha\sum_{k=1}^{N}\frac{E_k}{\omega_k}[\sin(\omega_k t + \varphi_k) - \sin\varphi_k], \tag{22}$$

then the determination of the form of the Stark line in this case reduces to a solution of the problem of determining the spectral density of radiation of a signal whose carrier frequency is modulated by a regular signal and by noise. The mean value $\overline{\exp i[\psi(t+\tau) - \psi(t)]}$, which we need to calculate the line-contour form, will be obtained by using the fact that $\psi_p(t)$ and $\psi_N(t)$ are independent of each other. Then $\overline{\exp i[\psi(t+\tau) - \psi(t)]} = \overline{\exp i[\psi_p(t+\tau) - \psi_p(t)]} \times \overline{\exp i[\psi_N(t+\tau) - \psi_N(t)]}$, and, by using (7) and replacing ω_k by ω_1 in one of the factors, we get

$$K(\tau) = J_0\left[2\frac{\Delta\omega_1}{\omega_1}\sin\frac{\omega_1\tau}{2}\right]\exp\left[-\frac{\alpha^2}{2}\sum_{k=1}^{N}(1-\cos\omega_k\tau)\right]. \tag{23}$$

The same relation can be obtained from the expression for the characteristic function in the case of a superposition of regular signals and noise. According to [12], it is equal to

$$\overline{\exp i[v\psi(t+\tau) + u\psi(t)]} = J_0\left[\frac{\Delta\omega_1}{\omega_1}\sqrt{u^2+v^2+2uv\cos\omega_1\tau}\right]$$

$$\cdot\exp[-\Phi(0)(u^2+v^2) - \Phi_\tau uv], \quad \Phi(0) = \overline{\psi^2(t)}, \quad \Phi_\tau = \overline{\psi(t+\tau)\psi(t)}. \tag{24}$$

Assuming for our case $u = -1$ and $v = 1$, we get (23).

For the case of the linear Stark effect in alternating fields that constitute a superposition of regular and stochastic oscillations, it is necessary in order to determine the form of the spectral line, to substitute (23) in (2). $K(\tau)$ is a periodic function of τ in the case when $\omega_k \equiv \omega$. Expanding this function in a series, assuming that $\Delta\omega/\omega \gg 1$ and $\Delta\omega_1/\omega_1 \gg 1$, we get

$$K(\tau) = \sum I(n\omega)\cos n\omega\tau,$$

where

$$I(n\omega) = \int_{-\infty}^{\infty}\exp\{in\omega t - \tfrac{1}{4}(\Delta\omega)^2 t^2\} J_0(\Delta\omega_1 t)\,dt. \tag{25}$$

In (25) we used the fact that the integrand decreases rapidly, and we have therefore extended the limits of integration to infinity. After a number of calculations we obtain

$$I(n\omega) = \frac{1}{2}\sqrt{\frac{1}{\pi\delta}}e^{-n^2/4\delta}\int_{-\pi}^{\pi}\exp\left\{-\frac{n\gamma}{2\delta}\sin\theta - \frac{\gamma^2}{4\delta}\sin^2\theta\right\}d\theta, \tag{26}$$
$$\delta = \tfrac{1}{4}(\Delta\omega/\omega)^2, \quad \gamma = \Delta\omega_1/\omega_1.$$

In the general case the calculation of the integral (26), which we need to determine the form of the spectral-line contour, is possible only by numerical means. If $n \sim d^{1/2}$ and $\gamma \gg \delta^{1/2}$, that is, the intensity of the regular oscillations greatly exceeds the intensity of the stochastic oscillations, then, as expected, $I(n\omega) \approx 1/\gamma = \text{const}$, that is, the line shape is a rectangle of width $\Delta\omega_1$.

Equation (26) solves the problem of determining the contour of the spectral line for the case of a linear

Stark effect in alternating regular and stochastic fields, when the frequencies of the regular and stochastic oscillations are close to each other. This is precisely the case which takes place, for example, in the excitation of high frequency oscillations in a plasma by an initially-monoenergetic electron beam. In the more general case, to determine the form of the spectral line it is necessary to substitute (23) in (2) and to calculate the corresponding integrals.

In considering the problem of the form of the spectral line due to the Stark effect in alternating stochastic fields, we did not take into account damping effects due to the finite lifetime of the atom or the ion in the excited state, and also pair collisions of the atoms and the ions with the electrons and with each other. Yet allowance for all these effects is essential in order to ascertain whether the individual satellites can be observed. At large values of the pair-collision frequencies or at small values of the lifetime, these effects can greatly change also the form of the envelope. If we denote by $\tau = 1/\nu$ the average lifetime of the atom or ion in the excited state or the corresponding collision frequency then, unlike (15), the form of the spectral line is determined by the relation

$$I(\Omega) = \frac{1}{\pi}\sum_n A_n\left\{\frac{\nu}{(n\omega+\Omega)^2+\nu^2}+\frac{\nu}{(n\omega-\Omega)^2+\nu^2}\right\}, \qquad (27)$$

where $A_n \approx \exp[-n^2/4\delta]$. To be able to resolve the individual satellites $n\omega$, it is necessary that the lifetime be sufficiently long and the collision frequency small, $\omega \gg \nu$. Here, naturally, it is assumed that the experimental line width is also smaller than ω. In order for the envelope, that is, the line contour, not to be too strongly distorted by collisions, it is necessary to have $\Delta\omega \gg \nu$.

We have considered the problem of the Stark effect due to alternating electric fields of stochastic oscillations excited by currents or by electron beams passing through the plasma. The Stark effect leads in this case to a rather appreciable broadening of the lines, and therefore can be simply observed and, by virtue of many of its specific features, it can be separated from the Stark effect due to the Coulomb fields, and from other processes that lead to line broadening. When the stochastic high frequency oscillations are excited by an electron beam, the line width in the nonlinear stage of saturation of these oscillations is, in accordance with (17),

$$\Delta\omega_1 = 2\alpha\left[\frac{2\pi}{3}n_1 m v_0^2\right]^{1/2}.$$

In most experiments, the density of the electrons in the beam ranges from 10^8 to 10^{10} cm^{-3}, and their energy ranges from hundreds of eV to 15–20 keV. Under these conditions, the line broadening $\Delta\omega_1$ changes from tenths of an Angstrom to several Angstroms, and consequently, it can be observed relatively simply.

Thus, the use of the Stark effect in alternating stochastic high frequency and low frequency fields is an effective means of determining the main parameters of an unstable turbulent plasma.

It is my pleasant duty to thank V. D. Shapiro and V. I. Kogan for valuable advice and discussions.

[1] R. von Traubenberg, R. Gebauer, and E. Schrödinger, Z. Physik **78**, 809 (1932).

[2] O. Blochinzew, Phys. Z. Sowjet. **4**, 601 (1933).

[3] V. E. Mitsuk, Zh. Tekh. Fiz. **28**, 1316 (1958) [Sov. Phys.-Tech. Phys. **3**, 1223 (1959)]; Voprosy radioélektroniki, ser. XII, obshchetekhnicheskaya (Problems of Radio Electronics, ser. XII, Gen. Tech.) **17**, 50 (1961).

[4] J. Holtsmark, Ann. Physik **58**, 577 (1919).

[5] S. L. Mandel'shtam and N. N. Sobolev, Zh. Eksp. Teor. Fiz. **20**, 323 (1950).

[6] V. I. Kogan, in: Fizika plazmy i problemy upravlyaemykh termoyadernykh reaktsii (Plasma Physics and Problems of Controlled Thermonuclear Reactions) **4**, AN SSSR, 1958, p. 258.

[7] A. C. Kolb, H. R. Griem, and K. J. Schen, Phys. Rev. **116**, 4 (1959).

[8] L. A. Vaĭnshteĭn and I. I. Sobel'man, Dokl. Akad. Nauk SSSR **90**, 757 (1963); Opt. Spektrosk **6**, 440 (1959).

[9] E. V. Lifshitz, A. I. Egorov, and O. G. Zagorodnov, Ukr. fiz. zh. **6**, 793 (1961).

[10] E. V. Lifshitz, S. S. Krivulya, and V. S. Us, Ukr. fiz. zh. **8**, 1328 (1963).

[11] E. V. Lifshitz, A. K. Berezin, and Yu. M. Lyapkalo, Zh. Teckh. Fiz. **36**, 1087 (1966) [Sov. Phys.-Tech. Phys. **11**, 798 (1966)].

[12] S. O. Rice, Bell System Techn. J. **23**, 282 (1944); O. Middleton, Phil. Mag. **42**, 684 (1951).

[13] S. M. Rytov, Vvedenie v statisticheskuyu radiofiziku (Introduction to Statistical Radiophysics), Nauka, 1966.

[14] I. F. Kharchenko, Ya. B. Faĭnberg, E. A. Kornilov, et al. Paper 2211, Suppl. I, Internat. Conf. on Peaceful Uses of Atomic Energy, Geneva, 1958; Zh. Eksp. Teor. Fiz. **38**, 685 (1960) [Sov. Phys.-JETP **11**, 493 (1960)]; Zh. Tekh. Fiz. **31**, 761 (1961) [Sov. Phys.-Tech. Phys. **6**, 551 (1962)]; A. K. Berezin, Ya. B. Faĭnberg, et al. Atomnaya énergiya **11**, 493 (1961); L. D. Smullin and W. D. Getty, Phys. Rev. Lett. **9**, 3 (1962).

[15] V. D. Shapiro, Zh. Tech. Fiz. **31**, 522 (1961) [Sov. Phys.-Tech. Phys. **6**, 376 (1961)]; Zh. Eksp. Teor. Fiz. **44**, 613 (1963) [Sov. Phys.-JETP **17**, 416 (1963)].

[16] S. Chandrasekhar, Stochastic Problems in Physics and Astronomy (Revs. Modern Phys. **15**, 1–89 (1943).

[17] M. I. Podgoretskiĭ and A. V. Stepanov, Zh. Eksp. Teor. Fiz. **40**, 561 (1961) [Sov. Phys.-JETP **13**, 393 (1961)].

[18] I. I. Sobel'man, Vvedenie v teoriyu atomnykh spektrov (Introduction to the Theory of Atomic Spectra), Fizmatgiz, 1963.

[19] B. B. Kadomtsev and V. I. Petviashvili, Zh. Eksp. Teor. Fiz. **43**, 2234 (1962) [Sov. Phys.-JETP **16**, 1578 (1963)].

Translated by J. G. Adashko
108