PLASMA OSCILLATIONS IN THE PRESENCE OF A PERIODIC PLASMA WAVE

V. I. PETVIASHVILI

Submitted February 1, 1967

Zh. Eksp. Teor. Fiz. 53, 917–925 (September, 1967)

The oscillatory properties of a plasma in the presence of a periodic plasma wave of finite amplitude are investigated. A dispersion equation for the frequency of long-wave oscillations in such a plasma is derived by means of the asymptotic method of averaging. If the amplitude of the background plasma wave is high enough, one finds the excitation of ion oscillations (drift waves, ion acoustic waves and others) which propagate in the opposite direction to the propagation direction of the background plasma wave.

1. The properties of a plasma in which there exists a wave of high amplitude have not been investigated to any great extent in plasma theory to date. Many workers have investigated the propagation of small perturbations ("second sound") against the background of a weakly turbulent plasma. For example, in[1] the authors have investigated small perturbations in the plasma in the presence of random plasma oscillations of low amplitude characterized by a stationary uniform distribution. The results of this work apply only to low-amplitude perturbations against a low-amplitude background. Some of these results are discussed in Sec. 6 of the present work.

It is well known that the plasma equations allow a particular solution in the form of a one-dimensional periodic wave of arbitrary amplitude.[2] In the absence of absorption a reference system exists in which the profile of the periodic wave is fixed. In this system the equations for the periodic wave in the plasma reduce to the equation for a one-dimensional nonlinear oscillator. It is evident that the behavior of small perturbations in a plasma environment in which the periodic wave propagates will differ from the behavior in a uniform plasma.

In the present work we investigate small perturbations in a plasma in the presence of a periodic plasma wave of finite amplitude. The wavelength of the perturbation is assumed to be much greater than the wavelength of the plasma wave so that one can apply the method of averaging to the plasma equations; this is a well-known asymptotic method in the theory of nonlinear oscillations. Expressions are found for the electron velocity and density in the periodic plasma wave in terms of a specified periodic function $f(x + u_1 t + x_0, a)$ of the time t and coordinate x (u_1 is the phase velocity, a is the square of the nondimensional amplitude of the plasma wave, and x_0 is the phase constant).

If such a plasma supports the propagation of a small long-wave perturbation, the electron velocity and density are not periodic functions but can be expressed as functions of f in which the parameters x_0 and a are taken to be slowly varying functions of the coordinate and time (van der Pol method).[3] The problem contains a small parameter, this parameter being the ratio of the wavelength of the background plasma wave to the wavelength of the perturbation. An expansion is made in powers of this small parameter and in the zeroth approximation we obtain the dispersion relation for low-frequency plasma oscillations with the periodic plasma wave from which it is possible to determine ω, the oscillation frequency. If the amplitude $\sqrt{a}$ is sufficiently large, the root ω becomes complex, corresponding to the excitation of a hydrodynamic instability (excitation of low-frequency oscillations by the plasma oscillations).

2. We consider an infinite plasma in a strong uniform magnetic field directed along the x axis. It is assumed that the electrons oscillate along the magnetic field and that they satisfy the hydrodynamic equations

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + v_T{}^2\frac{\partial}{\partial x}\ln n_e = \frac{e}{m_e}\frac{\partial \varphi}{\partial x}, \tag{1}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial n_e v}{\partial x} = 0, \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi e(n_e - n_i). \tag{3}$$

Here, v is the hydrodynamic velocity, v_T is the thermal velocity, n_e is the electron density, n_i is the ion density and φ is the electric potential. We now transform to the new variables

$$x_1 = x + u_1 t, \quad x_2 = x + u_2 t, \tag{4}$$

where u_1 and u_2 are fixed velocities. (It will be found below that u_1 is the phase velocity of the periodic plasma wave while u_2 is the phase velocity of the long-wave perturbation.) Then, substituting $\partial\varphi/\partial x$ from (1) in (3) we have

$$\left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}\right)\left\{\frac{\partial (v+u_1)^2}{\partial x_1} + \frac{\partial (v+u_2)^2}{\partial x_2} + 2v_T{}^2\left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}\right)\ln n_e\right\} = \frac{8\pi e^2}{m_e}(n_e - n_i). \tag{5}$$

In the new variables (2) assumes the form

$$\frac{\partial n_e (v+u_1)}{\partial x_1} + \frac{\partial n_e (v+u_2)}{\partial x_2} = 0. \tag{6}$$

3. We first consider the solutions of (5) and (6) having in view the periodic plasma wave. It is assumed that all quantities depend only on x_1 and that n_i = const = n_0. Under these conditions (6) is replaced by

$$\frac{\partial n_e (v+u_1)}{\partial x_1} = 0, \tag{7}$$

whence

$$n_e(v+u_1) = \text{const} = n_0 u_1. \tag{8}$$

According to (8) the flux density of electrons in the reference system that moves with respect to the ions with velocity u_1 is constant and equal to n_0u_1, the flux density of the ions in this same coordinate system. Assuming that $v_T \ll u_1$, in (5) we neglect the pressure, thus obtaining

$$f'' \equiv \frac{\partial^2 f}{\partial x_1^2} = 2k_0^2\left(\frac{1}{\sqrt{f}} - 1\right), \quad k_0^2 = \frac{4\pi e^2 n_0}{m_e u_1^2}. \tag{9}$$

Here, we have introduced the notation

$$v + u_1 = u_1\sqrt{f} \tag{10}$$

and have substituted n_e from (8).

Multiplying (9) by f', where the primes denote differentiation with respect to x_1, and integrating, we have

$$\tfrac{1}{2}f'^2 = 2k_0^2(2\sqrt{f} - f - 1 + a) \equiv F(f, a), \tag{11}$$

where the constant of integration is written in the form $-1 + a$, in which $u_1\sqrt{a}$ is evidently the amplitude of the oscillations of the velocity v. The relations (9) and (11) are the equations for a one-dimensional oscillator with a potential energy $F(f)$ in which the role of the time is played by the variable x_1. When $0 < a < 1$, (11) exchibits a periodic solution of the form

$$x_1 + x_0 = \pm \int \frac{df}{\sqrt{2F(f)}} = \frac{1}{k_0}\arcsin\frac{\sqrt{f}-1}{\sqrt{a}} \pm \frac{1}{k_0}[a - (\sqrt{f}-1)^2]^{1/2}, \tag{12}$$

where x_0 is a constant of integration. It is evident from (12) that f executes oscillations at a fundamental frequency k_0u_1 equal to the plasma frequency $\omega_0 = \sqrt{4\pi e^2 n_0/m_e}$ and at higher frequencies which are harmonics. Assuming that the amplitude is small, from (12) we have

$$\sqrt{f} = u_1^{-1}(v + u_1) \approx 1 + \sqrt{a}\sin k_0(x_1 + x_0) + a\cos^2 k_0(x_1 + x_0) + \ldots$$

In the general case, for an arbitrary amplitude (12) gives f as a function of a and as a periodic function of $x_1 + x_0 = x + u_1t + x_0$:

$$f = f(x_1 + x_0, a). \tag{13}$$

Thus we have obtained a particular solution of (5) and (6) in the form of a periodic plasma wave that propagates with velocity u_1. The length of this wave is determined from (12):

$$\lambda = \int_0^\lambda dx_1 = 2\int_{(1-\sqrt{a})^2}^{(1+\sqrt{a})^2} \frac{df}{\sqrt{2F(f)}} = \frac{2\pi}{k_0}. \tag{14}$$

Averaging (9) with respect to x_1 we have

$$\langle 1/\sqrt{f}\rangle = 1, \tag{15}$$

where the angle brackets denote the average value. Multiplying (9) by $\sqrt{f}$ and averaging we find

$$2k_0^2(1 - \langle\sqrt{f}\rangle) = -\tfrac{1}{2}\langle f'^2/\sqrt{f}\rangle. \tag{16}$$

Now, substituting f' from (11) and taking account of (15) we have

$$\langle\sqrt{f}\rangle = 1 + a/2. \tag{17}$$

In similar fashion we obtain the relation

$$\langle f\rangle = 1 + \tfrac{3}{2}a. \tag{18}$$

We now consider the average of the electron flux density n_ev and the flux density of electron kinetic energy $n_em_cv^3/2$ in the rest frame of the plasma. Substituting (8) and (10) and taking account of (17) and (18) we find that both of these quantities vanish. This means that in the absence of thermal motion the group velocity of the plasma wave, which is equal to the transport velocity of the energy of the electron oscillations, also vanishes.

4. So far, we have only considered the electron plasma oscillations under the assumption that the ion density is constant because of the large ion mass. Now let us assume that the plasma containing the periodic plasma wave also allows the propagation of a weak ion wave and that the ion density is a function of time and the coordinate in the form $n_i(x + u_2t) = n_i(x_2)$ with a wavelength much larger than the wavelength of the electron plasma wave. Under these conditions, n_e and v, in addition to depending on x_1, will also depend on x_2 and will thus satisfy the general nonstationary equations (5) and (6) which differ from (7) and (9) by small terms. We seek a solution of these equations in the form

$$v = q\sqrt{f} - u_1 \tag{19}$$

as a function (13) of x_1 and the parameters q, a, and x_0, assuming the latter to be functions of x_1 and x_2. The solution n_e is written in the form

$$n_e = n_i(x_2)f^{-1/2}(x_1 + x_0(x_1, x_2),\ a(x_1, x_2)). \tag{19a}$$

Now, (5) and (6) are replaced by

$$\left(\frac{d}{dx_1} + \frac{d}{dx_2}\right)\left\{\frac{dq^2f}{dx_1} + \frac{d(q\sqrt{f}-u)^2}{dx_2}\right\} = \frac{8\pi e^2 n_i}{m_e}\left(\frac{1}{\sqrt{f}} - 1\right), \tag{20}$$

$$\frac{dn_iq}{dx_1} + \frac{d}{dx_2}\left(n_iq - \frac{un_i}{\sqrt{f}}\right) = 0, \quad u \equiv u_1 - u_2. \tag{21}$$

In (20) we have again neglected the electron pressure, assuming for simplicity that $v_g \ll u_2$, where $v_g = v_T^2/u_1$ is the group velocity of the plasma wave; the quantities d/dx_1 and d/dx_2 represent total derivatives taking account of the dependence of x_0 and a in f on x_1 and x_2 respectively.

Thus, using (13) and (19) and (5) and (6) we replace the two unknown functions v and n_e by three new unknown functions q, a, and x_0. As a result we obtain (20) and (21). In order that this substitution be unique we must impose on the new functions additional conditions similar to those that follow in the theory of nonlinear oscillations.[3] These conditions must be expressed in a form such that the second-order system (5) and (6) with respect to the functions v and n_e reduces to a system of three first-order equations with respect to the functions q, a, and x_0. It will be seen below that this requirement is met if

$$\frac{dq^2f}{dx_1} + \frac{d(q\sqrt{f}-u)^2}{dx_2} = q^2f', \tag{22}$$

where the primes denote the derivative $\partial/\partial x_1$ only for the explicitly contained variable x_1.

When (22) is taken into account, (20) assumes the form

$$\left(\frac{d}{dx_1}+\frac{d}{dx_2}\right)q^2f'=\frac{8\pi e^2n_i}{m_e}\left(\frac{1}{\sqrt{f}}-1\right)=\frac{n_i}{n_0}u_1^2f''. \quad (23)$$

Here, we have used the definition of f'' from (9). Expanding the derivative d/dx_1, we write (22) and (23) in the form

$$q^2f_aa'+q^2f'x_0'+2q(q'+\dot{q})f+q^2\dot{f}-2u\frac{d}{dx_2}(q\sqrt{f})=0, \quad (24)$$

$$q^2f_a'a'+q^2f''x_0'+2q(q'+\dot{q})f'+q^2\dot{f}'=\left(\frac{n_i}{n_0}u_1^2-q^2\right)f''. \quad (25)$$

The subscript on f_a denotes the derivative with respect to a and the dot above a symbol denotes d/dx_2, the total derivative with respect to x_2.

We now solve (24) and (25) with respect to a' and x_0'. The determinant of this system is [cf. (9), (11)]

$$f'f_a'-f''f_a=\frac{1}{2}\frac{d}{da}f'^2-f_a\frac{\partial F}{\partial f}=\frac{dF}{da}-\frac{\partial F}{\partial f}f_a=2k_0^2. \quad (26)$$

Using (24), (25) and (26) we have

$$2k_0^2q^2a'=2q(q'+\dot{q})(ff''-f'^2)+q^2(\dot{f}f''-f'\dot{f}')$$
$$-2uf''\left(\dot{q}\sqrt{f}+q\frac{d\sqrt{f}}{dx_2}\right)+\left(\frac{n_i}{n_0}u_1^2-q^2\right)f'f'', \quad (27)$$
$$2k_0^2q^2x_0'=2q(q'+\dot{q})(f'f_a-ff_a')+q^2(\dot{f}'f_a-\dot{f}f_a')$$
$$+2uf_a'\left(\dot{q}\sqrt{f}+q\frac{d\sqrt{f}}{dx_2}\right)-2k_0^2\left(\frac{n_i}{n_0}u_1^2-q^2\right)\left(\frac{1}{\sqrt{f}}-1\right)f_a. \quad (28)$$

Assuming that $n_i' = 0$, we can write (21) in the form

$$q'+\dot{q}=\frac{\dot{n}_i}{n_i}\left(\frac{u}{\sqrt{f}}-q\right)+u\frac{d}{dx_2}\frac{1}{\sqrt{f}}. \quad (29)$$

Now, we substitute q' from (29) and f'' and F from (9) and (11) in (27). In this case we have

$$a'+\dot{a}=\frac{2}{q}\frac{\dot{n}_i}{n_i}\left(\frac{u}{\sqrt{f}}-q\right)(f-3\sqrt{f}+2-2a)-\frac{2u\dot{q}}{q^2}(1-\sqrt{f})$$
$$+\frac{2u}{q}\frac{\dot{f}}{f}-\frac{2u}{q}(1-a)\frac{d}{dx_2}\frac{1}{\sqrt{f}}+\left(\frac{n_iu_1^2}{n_0q^2}-1\right)(2\sqrt{f}-f-1)'. \quad (30)$$

We recall that f is a function given by (12).

Thus, we have obtained a system of first-order equations (28)—(30) which describe the oscillations of the parameters q, a, and x_0 about their fixed values in the unperturbed periodic wave (q oscillates about u_1). These oscillations describe an ion wave, that is, to say, the dependence of n_i on x_2. It is evident from these equations that because of the explicit dependence of f on x_1 the oscillations of the parameters depend on x_1, in which the period of the dependence of the parameters on x_1 is of order λ, the period of the dependence of f and x_1. We have assumed that the wavelength of the perturbation (period of dependence of n_1 on x_2) is much larger than λ so that the dependence of q, a, x_0, and f on x_1 is a rapidly oscillating function as compared with the dependence on x_2.

We can now make estimates of each term in (28)—(30) by means of (12), (13) and (18). For example, the last term on the right side of (30) is of order

$$\frac{q'}{q}a+\left(\frac{n_i}{n_0}-1\right)(a'+k_0x_0').$$

It is not difficult to show that the derivatives of each of the parameters with respect to x_1 and x_2 are of the order of

$$q'\sim\dot{q},\quad a'\sim\dot{a},\quad x_0'\sim\dot{x}_0.$$

Hence, the ratio of the amplitude of the dependence on x_1 to the amplitude of the dependence on x_2 for any parameter is of the order of the ratio of λ to the period of the dependence on x_2, that is to say, a very small quantity. Thus, in the zeroth approximation, to which we limit ourselves here, in which case there is no differentiation with respect to x_1, in (28)—(30) we can neglect the oscillations of q, a, and x_0, assuming then to be functions of x_2 alone.[3]

We now average (29) with respect to x_1 for fixed x_2, taking account only of the explicit dependence of f on x_1. The average of q' vanishes and we have

$$\dot{q}=\frac{\dot{n}_i}{n_i}\left(\left\langle\frac{u}{\sqrt{f}}\right\rangle-q\right)+u\frac{d}{dx_2}\left\langle\frac{1}{\sqrt{f}}\right\rangle. \quad (31)$$

In averaging (30) with respect to x_1 it should be remembered that the last term on the right side, after differentiation by parts, is found to be proportional to q' so that we cannot neglect the dependence of q on x_1 in this term. After averaging we have

$$\dot{a}=\frac{2\dot{n}_i}{qn_i}\left\langle\left(\frac{u}{\sqrt{f}}-q\right)(f-3\sqrt{f}+2-2a)\right\rangle-\frac{2u\dot{q}}{q^2}(1-\langle\sqrt{f}\rangle)$$
$$+\frac{2u}{q}\left\langle\frac{\dot{f}}{f}\right\rangle-\frac{2u}{q}(1-a)\frac{d}{dx_2}\left\langle\frac{1}{\sqrt{f}}\right\rangle+2\left\langle\frac{q'}{q}(2\sqrt{f}-f-1)\right\rangle. \quad (32)$$

In the last term of (32) we have neglected terms that are not linear in the amplitude of the longwave oscillations. In (31) we can substitute the mean value (15) in the unperturbed periodic wave. Thus we find

$$\dot{q}=\frac{\dot{n}_i}{n_i}(u-q)\approx\frac{\dot{n}_i}{n_0}(u-u_1)=-\frac{\dot{n}_i}{n_0}u_2. \quad (33)$$

In (33) we have carried out a linearization process, that is to say, we have neglected terms that are not linear in the amplitude of the oscillations of the parameters and the quantity n_1.

In (32) we now substitute q' from (29) and the average values obtained from (15), (17), and (18) thus obtaining

$$\dot{a}\left(1-\frac{u}{q}\right)=\frac{a\dot{n}_i}{qn_i}(5q-4u)-\frac{a\dot{q}}{q}\left(\frac{u}{q}-1\right). \quad (34)$$

If the electron pressure is considered in going from (5) to (20) the calculations proceed in the same fashion. After a somewhat more complicated calculation (which makes use of an expansion in powers of v_T^2) and after substituting $\dot{q}$ from (33), we find that (34) is replaced by

$$\dot{a}(u_2-v_g)=au_1\frac{\dot{n}_i}{n_0}\left(1+\frac{4u_2}{u_1}-\frac{u_2^2}{u_1^2}\right), \quad (35)$$

where $v_g = v_T^2/u_1$ is the group velocity of the plasma wave. In this equation we have neglected terms containing higher powers of av_T^2. We have also neglected terms that are not linear in the amplitude of the oscillations of the parameters and the quantity n_i.

Thus, we have obtained an equation that relates the infinitesimal longwave oscillations of the ion density to the amplitude of the periodic plasma wave. This equation holds for an arbitrary amplitude of the periodic plasma wave. Equation (35) shows that when the ions are fixed ($\dot{n}_i = 0$) the plasma can support the propagation of free oscillations of the amplitude of the plasma wave with a phase velocity $u_2 = v_g$. A wave packet of plasma waves propagates with the same velocity.

Averaging (28) over x_1 we obtain an equation which gives $\dot{x}_0$ as a function of $q - u_1$.

5. The long-wave oscillations of n_i are accompanied by long-wave oscillations of the potential φ along with the plasma oscillations. We express the oscillations of the parameters of the plasma wave in terms of the longwave oscillations of the potential. For this purpose we average equations (1) written in the variables (4), (19), and (19a), this averaging being carried out over x_1. The average of the total derivatives with respect to x_1 is zero and we find

$$\frac{d}{dx_2}\left\langle \frac{1}{2} q^2 f - uq\sqrt{f} + v_T^2 \ln \frac{n_i}{\sqrt{f}} \right\rangle = \frac{e}{m_e}\frac{d}{dx_2}\langle \varphi \rangle. \tag{36}$$

We neglect the oscillations of q, a and x_0 and make use of the average value as obtained from (17) and (18) in (36). In this way we find

$$\frac{d}{dx_2}\left[\frac{(u-q)^2}{2} + \frac{3}{4}q^2 a - \frac{uqa}{2} + v_T^2 \ln n_i - \frac{1}{2} v_T^2 \langle \ln f \rangle\right] = \frac{e}{m_e}\langle \varphi \rangle. \tag{37}$$

In (35) and (37) we neglect small terms defined by the inequality $u_2 \ll v_T \ll u_1$. Then, eliminating $\dot{a}$ and $\dot{q}$ from this system by means of (33) we have

$$\frac{\dot{n}_i}{n_0}\left(\frac{u_1^3}{4}\frac{a}{u_2 - v_g} + v_T^2\right) = \frac{e}{m_e}\langle \dot{\varphi} \rangle. \tag{38}$$

Here we note that $\langle n_e \rangle = n_i$; $\langle \varphi \rangle$ is the potential which acts on the ions. In addition to depending on x_2, the quantities n_i and $\langle \varphi \rangle$ can depend on the coordinates y and z. Because of the strong magnetic field the electron motion remains one-dimensional and the electron equations, in particular (38), remain unchanged. Linearizing the ion equations of motion in the magnetic field we obtain the relation

$$\dot{n}_i = L\langle \dot{\varphi} \rangle, \tag{39}$$

where L is a linear operator that acts on $\langle \varphi \rangle$. For the ion-acoustic wave, and drift wave, and other familiar forms, the quantity L is well known.

Equations (38) and (39) form a closed linear system that describes the interaction of the periodic plasma wave with phase velocity u_1 and the sinusoidal ion wave with phase velocity (along the magnetic field) u_2. Since these equations are linear the condition that all quantities depend on x and t only through the combination $x_2 = x + u_2 t$ is not limiting. If $n_i = ik_x n_k \exp\{i(\mathbf{k}\cdot\mathbf{r} - \omega t)\}$ where $\mathbf{k}$ is the wave vector, ω is the frequency and n_k is the amplitude, we have $\omega = -k_x u_2$.

We now consider the simplest possible case of a sinusoidal ion-acoustic wave that propagates along the magnetic field. In this case (39) assumes the form

$$\frac{\dot{n}_i}{n_0} = \frac{e}{m_i u_2^2}\langle \dot{\varphi} \rangle. \tag{40}$$

From (38) and (40) we obtain the dispersion equation

$$1 - \frac{\alpha}{u_2 - v_g + \alpha} - \frac{c_s^2}{u_2^2} = 0, \tag{41}$$

where $\alpha = au_1^3/4v_T^2$ and $c_s = \sqrt{m_e v_T^2/m_i}$ is the velocity of propagation for ion sound. On the left side of (41) we have the dielectric constant of the plasma with accuracy to a positive constant. If v_g is small and $\alpha > 0.4\, c_s$, Equation (40) has one real root and two complex roots for u_2. When α is somewhat greater than this critical value the complex roots are given by

$$u_2 \approx -\frac{c_s}{\sqrt{3}} \pm 4ic_s\sqrt{\frac{\alpha}{c_s} - 0{,}4}. \tag{42}$$

Thus, we have an ion-acoustic instability which propagates in the reverse direction to the plasma wave. This instability is due to the fact that the energy of the ion acoustic wave is negative in the region of phase velocities given in (42) (the energy is reduced as the wave amplitude increases [4]). It is difficult to estimate the amplitude of the ion-acoustic wave that results from this instability. It is probable that a high-amplitude wave is established.

Assuming that $c_s \sim v_g \ll u_2 \ll v_T$ and keeping the principal terms, we obtain in similar fashion the dispersion equation for drift waves [5] taking account of the effect of the plasma waves:

$$1 - \frac{\alpha}{u_2 - v_g + \alpha} - \frac{v_d}{u_2} = 0, \tag{43}$$

where v_d is the drift frequency [5] divided by k_x (the magnetic field is along x). When $v_d < 0$, and $\alpha > -(v_g - v_d)^2/v_d$, (43) has unstable solutions for the excitation of the drift wave. In similar fashion we can obtain dispersion equations for other ion oscillations.

6. We now wish to compare the results obtained here with results of other workers on weakly turbulent plasma. Oraevskiĭ and Sagdeev[6] have indicated the existence of an instability of a plasma wave of low amplitude against decay to a plasma wave with the same wavelength propagating in the opposite direction together with a shortwave ion-acoustic wave. It is difficult to investigate this instability in the highly nonlinear case. We can only indicate the possibility of a substantial modification of the results of the analysis of a low-amplitude wave when one considers the highly nonlinear case since the dispersion equation for a plasma with a periodic plasma wave does not allow longwave plasma oscillations.

Vedenov and Rudakov [1] have investigated the interaction of plasmons (quanta associated with the plasma waves) with ion sound, neglecting harmonics of the plasma wave. They found that the derivative of the plasmon distribution function with respect to the wavenumber, like the derivative of the electron distribution function with respect to velocity, can lead to the excitation or damping of ion sound.

In order to make a comparison with the results of the present work we reduce the width of the plasmon distribution function in [1] to zero and find that in this limit the plasma wave interacts with the ions only by virtue of the thermal motion of the electrons. It is evident from (41) and (42) that the thermal motion of the electrons attenuates this interaction. We conclude that for a small width of the plasmon distribution function one cannot neglect the higher harmonics [that is to say, the second term on the right side of (12)] since the instability occurs when the amplitude of the fundamental becomes finite. Furthermore in [1] no division is made of the slow oscillations of the phase of the plasma wave and the oscillations in amplitude, which also effects the results.

Sturrock reports [7] that the group velocity of the plasma wave must be supplemented, in the nonlinear case, by an additional term proportional to the square of the oscillation amplitude. This assertion is based on the result that the mean velocity of the electrons is nonvanishing in the nonlinear case. As we have shown in Sec. 3 of the present work, in the laboratory system in the absence of a current the mean velocity is nonvanishing; however, the mean electron flux and the mean flux of electron kinetic energy do, in fact, vanish if the thermal motion is neglected. This result means that the plasma wave does not transport energy when $v_T = 0$.

The work considered here is also closely related to a paper by Ostrovskiĭ [8] who has considered the propagation of a low-amplitude wave in a nonlinear medium by a semiclassical method. Using the method developed in the present work it should be possible to consider the propagation of low-amplitude long-wave free oscillations in a nonlinear medium in the presence of a periodic wave of arbitrary amplitude (superposition of low-amplitude longwave oscillations and a periodic wave).

The author wishes to thank Academician M. A. Leontovich and B. B. Kadomtsev for valuable comments and discussion.

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Translated by H. Lashinsky
105