ELECTROMAGNETIC FORM FACTOR OF A MEDIUM IN THE SCATTERING, BREMSSTRAHLUNG, AND PAIR-PRODUCTION PROCESSES

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The multiparticle approach to the scattering of charged particles is extended to bremsstrahlung and pair production. The probabilities of all three electromagnetic processes can be expressed in terms of the same electromagnetic form factor of the medium, and it is this fact which underlies the parallelism of the three processes. The properties of the form factor of the medium are considered and the relation of the factor to the dielectric constant of the substance is elucidated. It is noted that there can exist bremsstrahlung and pair production processes in which the excess energy and momentum are transferred to various collective excitations to the medium; the usual case, when the excess energy and momentum are transferred to separate atoms of the medium or to the lattice as a whole, is one of the possibilities. Corrections to the bremsstrahlung and positron spectra, which take into account multiparticle effects, are obtained; the magnitude of the corrections may reach 10% for small Z. Multiparticle effects may be very pronounced if the region of small values of transferred momenta, which is important for them, is separated out by a coincidence technique. It is pointed out that bremsstrahlung and pair production may exist in a narrow channel in matter, in which the usual transfer mechanism is not operative.

1. INTRODUCTION

We shall consider three processes whose realization requires the transfer of energy ω and momentum $\mathbf{q}$ from charge particles to the medium: scattering, bremsstrahlung, and pair production ($\hbar = c = 1$). Since the interaction H' of charged particles with a substance is electromagnetic in all three processes, and the energy and momentum receive the same excitations of the medium, the probabilities of these processes are expressed in the lowest Born approximation in H' (Sec. 2) in terms of the same electromagnetic form factor of the medium $T_{\mu\nu}(\mathbf{q}, \omega)$, defined by formula (7) and representing a generalization of the dynamic form factor introduced in [1, 2] (see also [3]). The symmetry properties of $T_{\mu\nu}(\mathbf{q}, \omega)$ and its connection with the retarded Green's function of the long-wave field in the medium and with the dielectric constant of the medium (formula (18)) are considered in the limiting case of small $\mathbf{q}$ in Sec. 3. This limiting expression in terms of the dielectric constant was obtained earlier in [4] (for bremsstrahlung).

It must be emphasized that any excitation that receives energy ω and momentum $\mathbf{q}$ as a result of scattering of charged particles (the term "scattering" is understood here in the broad sense, and includes, for example, the transfer of energy and momentum from the passing particle to the plasmons or soft transverse photons in Cerenkov radiation), takes part in the bremsstrahlung or pair-production processes, if these values of ω and $\mathbf{q}$ are kinematically admissible (see Sec. 4). Besides the classical well known Bethe-Heitler bremsstrahlung and pair production in nuclei and atoms[5, 6] and bremsstrahlung and pair production in crystals[7, 8] there arises an entire group of multiparticle variants of these processes, in which the energy and momentum are received by different excitations of the medium (this was apparently first noted by Ryazanov[9]). We note, in particular, recently investigated theoretical processes of the bremsstrahlung type, namely hard Cerenkov radiation[10, 4] (transfer of energy and momentum upon radiation of a hard photon to a soft quantum, whose refractive index is larger than unity) and an analogous process in which a plasmon takes part.[9, 11]

A complete theoretical analysis of the probabilities of the three processes in the presence of a medium entails considerable difficulties (of the same character as the difficulties arising near the theoretical calculation of the dielectric constant). Therefore great importance is attached to the possibility of comparing the probabilities of the process when the form factor $T_{\mu\nu}(\mathbf{q}, \omega)$ is not known theoretically. If, for example, the transfer proceeds primarily via a longitudinal field, so that only the component $T_{44} \equiv T_l(\mathbf{q}, \omega)$ of the form factor is significant, then the probability of the processes of scattering, bremsstrahlung, and pair production with the same transfer of $\mathbf{q}$ and ω will be proportional to each other, and the coefficients of proportionality will not depend on the concrete properties of the medium (formula (24) and Sec. 5). It is also clear from the foregoing that the analysis of screening in bremsstrahlung and pair-production processes in condensed media at small momentum transverse $q < q_{at} = 1/a_0 = me^2$ (m—electron mass) should be multiparticle, and not monotonic, as in a gas—that is, it is necessary to take into account the spatial distribution and the dynamics of the entire aggregate of charges of the medium (see Sec. 5).

The difference between the distribution of the charge of collectivized valence electrons in a solid and their distribution in isolated atoms can apparently result in a correction to the soft part of the bremsstrahlung spectrum and the positron spectrum, reaching 10% in substances with small Z. A correction of the same order can result from the influence of the polarization of the medium on the electromagnetic transfer, which hitherto has been taken into account only in the scattering phenomenon, where it leads to an appreciable change in the ionization losses at high energies.[12-14] The corrections under consideration can be approximately larger by one order of magnitude than the vacuum radiative corrections, and they must be taken into account in the bremsstrahlung and pair production processes in substances with small Z.

Multiparticle effects appear more pronouncedly in bremsstrahlung and pair production if one observes only transfers with small q. This can be attained either by choosing the angles and energies for both particles, in the final state, such that the momentum transfer is sufficiently small, or else by passing the charged particles through a narrow channel of diameter D in the medium (in which case only transfers with $q \lesssim D^{-1}$ will take part). In both cases, multiparticle effects in the transfer come to the forefront (see Sec. 5).

The polarization of the medium exerts a strong influence also on the bremsstrahlung quantum itself. The classical theory, which takes this influence into account, was presented by Ter Mikaelyan,[15] while the quantum theory can be obtained by direct generalization of formula (9) of this paper.

2. EXPRESSIONS FOR THE PROBABILITIES OF SCATTERING, BREMSSTRAHLUNG, AND PAIR PRODUCTION IN TERMS OF THE ELECTROMAGNETIC FORM FACTOR OF THE MEDIUM

The system under consideration consists of the nuclei and electrons of the substance, which are coupled by Coulomb forces (Hamiltonian H_s), the transverse electromagnetic field, and fast charged particles which we shall assume to be relativistic and to have a spin $\frac{1}{2}$. The field operators satisfy the vacuum Maxwell's equations. We shall use a Coulomb gauge, whereby the most important—longitudinal—field is described by a potential

$$\varphi(\mathbf{x}) = \sum_\alpha e_\alpha / |\mathbf{x} - \mathbf{x}_\alpha|, \tag{1}$$

which is determined by the instantaneous disposition of the charges of the medium and is not an independent dynamic system. The part $\mathbf{A}^m(\mathbf{x})$ of the transverse field with $|\mathbf{k}| < k_0 \sim 10^6\ \mathrm{cm}^{-1}$ and with Hamiltonian H_{mv} (k_0—upper limit of the ultraviolet region) strongly interacts with the substance (Hamiltonian H_{sv}). This is manifest in the fact that the transverse dielectric constant $\varepsilon_t(\mathbf{k}, \omega)$ differs from unity. On the other hand, for arbitrary $k > k_0$, that is, starting with the x-ray region, we have $\varepsilon_t(\mathbf{k}, \omega) \approx 1$, and for simplicity we shall disregard in the lowest order of perturbation theory the interaction of such hard photons with the medium, neglecting their difference from the vacuum photons.[1)]

Let us unify for simplicity in writing down the formulas the transverse field ($\varepsilon_t \neq 1$), which is strongly coupled with the substance, and the scalar potential (1), into a 4-potential of a field $A_\mu^m = (\mathbf{A}^m, i\varphi)$ which is coupled with the substance. We shall define the "medium" as a system made up of the substance and of the transverse field which is strongly coupled to it (Hamiltonian $H_{med} = H_s + H_{mv} + H_{sv}$ with eigenvalues E_i). A fast particle experiences electromagnetic interaction with the substance and with the transverse field

$$H' = -\int A_\mu{}^m(\mathbf{x}) j_\mu(\mathbf{x})\, d\mathbf{x} - \int A_\mu{}^h(\mathbf{x}) j_\mu(\mathbf{x})\, d\mathbf{x}, \tag{2}$$

where the current $j_\mu = ieN(\bar{\psi}\gamma_\mu\psi)$, and ψ is the Dirac operator. The unperturbed system consists of the medium, the fast particle, and the hard part A_μ^h of the electromagnetic field; this part does not interact with the substance.

The probabilities of our three electromagnetic processes will be considered in the lowest approximation of perturbation theory in the interaction (2). We consider scattering first. The matrix element of the S matrix, corresponding to the transition of the medium from the state i into the state f with energies E_i and E_f and with momenta $\mathbf{P}_i$ and $\mathbf{P}_f$, and to the scattering of the incoming particle from the state $\mathbf{p}_0$, r_0, E_0, into the state $\mathbf{p}$, r, E is equal to

$$S^{sc}_{f\mathbf{p}r,\, i\mathbf{p}_0 r_0} = -2\pi e(\bar{u}_{\mathbf{p}r}, \gamma_\mu, u_{\mathbf{p}_0 r_0})\,(A_\mu{}^M(-\mathbf{q}))_{fi}\,\delta(E_i + \omega - E_f), \tag{3}$$

where $\mathbf{q} = \mathbf{p}_0 - \mathbf{p}$ and $\omega = E_0 - E$ are the momentum and the energy given up by the incoming particle to the medium, and

$$(A_\mu{}^m(-\mathbf{q}))_{fi} = \frac{1}{\Omega}\int d\mathbf{x} e^{i\mathbf{q}\mathbf{x}} (A_\mu{}^m(\mathbf{x}))_{fi}. \tag{4}$$

The matrix element $(A_\mu^m(\mathbf{q}))$ cannot be written out in the general case in explicit form.

The scattering probability $dw_\mathbf{q}$ of a test particle per unit path, with transfer of a momentum $\mathbf{q}$ and energy ω to the medium, summed over the final states f of the medium and the polarization states r and averaged over r_0 and the initial states i of a medium with density matrix $\rho_i = \exp[(F - E_i)/T]$ (T—temperature) is

$$dw_\mathbf{q}{}^{sc} = e^2 \frac{N_\omega + 1}{v_0} S^{sc}_{\mu\nu}(\mathbf{p}_0, \mathbf{q}, \omega)\, T_{\mu\nu}(\mathbf{q}, \omega) \frac{\Omega(d\mathbf{q})}{(2\pi)^3}, \tag{5}$$

where $T_{\mu\nu}(\mathbf{q}, \omega)$ is the electromagnetic form factor of the medium, v_0 the initial velocity of the incoming particle, $N_\omega = (e^{\omega/T} - 1)^{-1}$ in the case of an equilibrium medium,[2)] and the spin sum is

$$S^{sc}_{\mu\nu} = g_\mu [v_\mu v_{0\nu} + v_{0\mu} v_\nu + (1 - \mathbf{v}_0\mathbf{v} - m^2/E_0 E)\,\delta_{\mu\nu}], \tag{6}$$

where $g_\mu = +1$ when $\mu = 1, 2, 3$ and $g_\mu = -1$ when $\mu = 4$, $v_\mu = (\mathbf{v}, i)$, and $\mathbf{v} = \mathbf{p}/E$. This system depends exclusively on the quantities that pertain to the test particle, and is symmetrical with respect to μ and ν. All the necessary information concerning the medium is contained in the electromagnetic form factor

$$T_{\mu\nu}(\mathbf{q}, \omega) = -\frac{\pi g_\mu}{N_\omega + 1} \sum_{i,f} \rho_i (A_\mu{}^m(-\mathbf{q}))_{fi}^* (A_\nu{}^m(-\mathbf{q}))_{fi}\, \delta(E_i + \omega - E_f). \tag{7}$$

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1)In the ultrarelativistic region, allowance for the difference between the dielectric constant $\Sigma_t(\mathbf{k}, \omega_k) = 1 - \omega_p^2/\omega_k^2$ ($\omega_p = (4\pi n e^2/m)^{1/2}$ plasma frequency) of these quanta from unity leads to an appreciable change in the soft part of the bremsstrahlung spectrum [15].

2)In the case of a nonequilibrium medium made up of a substance pierced by a flux of photons of frequency ω (say from a laser), formulas (5), (8), and (11) retain their form, but N_α in them will be the number of quanta per quantum state, which is specified and is independent of T, and formulas (8) and (11) will give in this case the probabilities of not only the spontaneous but also the stimulated bremsstrahlung-quantum emission and pair-production processes.

We shall see later that the structure of the probabilities of the bremsstrahlung and of pair production is the same as in the case of scattering, with the same form factor (7), and only the form of the spin sum changes. We call attention also to the fact that formula (5) includes as particular cases the probabilities of the different processes (see Sec. 4), such as elastic and inelastic scattering by gas atoms, diffraction scattering in a crystal, ionization and polarization losses, and Cerenkov radiation.

We now calculate the probability $dw_{\mathbf{kq}}$ of the bremsstrahlung of a hard quantum with energy $k > k_0$, momentum $\mathbf{k}$, and arbitrary polarization. The incident particle in this case changes momentum from $\mathbf{p}_0$ to $\mathbf{p}$ (we sum over the spin states), the medium goes over from state i to state f, acquiring an energy $\omega = E_0 - E - k$ and a momentum $\mathbf{q} = \mathbf{p}_0 - \mathbf{p} - \mathbf{k}$. The matrix element of the S-matrix will now be

$$S^{\mathrm{br}}_{f\mathbf{p}r\mathbf{k}\lambda,\, i\mathbf{p}_0 r_0} = \sqrt{2\pi\Omega/k}\; 2\pi e^2 \{(\bar{u}_{\mathbf{p}r}\, \hat{e}_\lambda S^c(p_0 - q)\, \gamma_\mu u_{\mathbf{p}_0 r_0}) + (\bar{u}_{\mathbf{p}r}\, \gamma_\mu S^c(p_0 - k)\, \hat{e}_\lambda u_{\mathbf{p}_0 r_0})\}\, (A^{\mathrm{M}}_\mu(-\mathbf{q}))_{fi}\, \delta(E_i + \omega - E_f),$$

where $S^c(p) = -(i/\Omega(i\hat{p} + m))^{-1}$. Acting further in analogy to the scattering case, we obtain

$$dw^{\mathrm{br}}_{\mathbf{kq}} = e^4 \frac{N_\omega + 1}{v_0} S^{\mathrm{br}}_{\mu\nu}\; T_{\mu\nu}(\mathbf{q}, \omega) \frac{\Omega d\mathbf{q} d\mathbf{k}}{(2\pi)^5 m^2 k}, \tag{8}$$

where the electromagnetic form factor $T_{\mu\nu}(\mathbf{q}, \omega)$ is the same as in the case of scattering, and is determined by the same formula (7), but the spin sum is different:

$$\begin{aligned} S^{\mathrm{br}}_{\mu\nu} = {} & [2(1 - \xi - \alpha v)\varkappa_1^2\varkappa_2^2]^{-1}\{-8v_{0\mu}v_{0\nu}[(\varkappa_1 + \varkappa_2)^2 + \varkappa_1\varkappa_2\eta^2] \\ & - 4\varkappa_1(2\varkappa_1 + \varkappa_2\eta^2)\xi_\mu\xi_\nu + 4\varkappa_1[2(\varkappa_1 + \varkappa_2) + \eta^2\varkappa_2](v_{0\mu}\xi_\nu + v_{0\nu}\xi_\mu) \\ & + 2\alpha[2(\varkappa_1 + \varkappa_2)^2 + \varkappa_1\varkappa_2(\varkappa_2 - \varkappa_1 + \eta^2)](v_{0\mu}\eta_\nu + v_{0\nu}\eta_\mu) \\ & - 2\alpha\varkappa_1(\varkappa_1 + \varkappa_2)(2 + \varkappa_2)(\xi_\mu\eta_\nu + \xi_\nu\eta_\mu) \\ & - 4\alpha^2\varkappa_1\varkappa_2^2\eta_\mu\eta_\nu + \alpha^2[-\varkappa_1\varkappa_2(\varkappa_1^2 + \varkappa_2^2) \\ & + 2\eta^2(\varkappa_1 + \varkappa_2)(\varkappa_1\varkappa_2 - \varkappa_1 - \varkappa_2) - 2\varkappa_1\varkappa_2\eta^4]\delta_{\mu\nu}\}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} & \alpha = m/E_0, \quad \boldsymbol{\eta} = \mathbf{q}/m, \quad \nu = \omega/m, \quad \xi = k/E_0, \\ & \mathbf{v}_0 = \mathbf{p}_0/E_0, \quad v_{0\mu} = (\mathbf{v}_0, i), \quad v_{0\mu}^2 = -\alpha^2, \quad \xi_\mu = (\boldsymbol{\xi}, i\xi), \\ & \eta_\mu = (\boldsymbol{\eta}, i\nu), \quad E = E_0(1 - \xi - \alpha\nu), \\ & \varkappa_1 = m^{-2}[(p_{0\mu} - q_\mu)^2 + m^2], \\ & \varkappa_2 = m^{-2}[(p_{0\mu} - k_\mu)^2 + m^2], \end{aligned} \tag{10}$$

The tensor $S^{\mathrm{br}}_{\mu\nu}$ is symmetrical in μ and ν, just like $S^{\mathrm{sc}}_{\mu\nu}$.

Formulas (8) and (9) contain both the ordinary Bethe–Heitler bremsstrahlung and the already noted processes with multiparticle transfer. These formulas can be generalized also to include the case when the bremsstrahlung quantum is modified by polarization, so that $\varepsilon_t(\mathbf{k}, \omega_k) = 1 - \omega_p^2/\omega_k^2$ (the results are not presented here, in view of their complexity). They include in the classical limit $k \ll E_0$ also those results of Ter-Mikaelyan[15] which do not pertain to multiple scattering during the radiation process.

The expression for the production probability of a pair e^+e^- with momenta $\mathbf{p}_+$, $\mathbf{p}_-$ and with energies E_+, E_- by a hard quantum with momentum $\mathbf{k}$ and energy k, summed and averaged as above, is obtained by replacing in the expression for the bremsstrahlung matrix element $p_{0\mu}$ by $-p_{+\mu}$ and k_μ by $-k_\mu$, and by appropriately modifying the statistical factor

$$dw_{\mathbf{p}_+\mathbf{q}} = e^4(N_\omega + 1) S^{\mathrm{pair}}_{\mu\nu} T_{\mu\nu}(\mathbf{q}, \omega) \frac{\Omega d\mathbf{p}_+ d\mathbf{q}}{(2\pi)^5 m^2 k}, \tag{11}$$

where $T_{\mu\nu}(\mathbf{q}, \omega)$ is again given by formula (7), and

$$S^{\mathrm{pair}}_{\mu\nu}(p_{+\mu}, p_{-\mu}, k_\mu, \mathbf{q}, \omega) = S^{\mathrm{br}}_{\mu\nu}\;(-p_{0\mu}, p_\mu, -k_\mu, \mathbf{q}, \omega). \tag{12}$$

The medium receives here an energy $\omega = k - E_+ - E_-$ and a momentum $\mathbf{q} = \mathbf{k} - \mathbf{p}_+ - \mathbf{p}_-$. The contribution $\omega < 0$ in (5), (8), and (11) corresponds to the probability of scattering, bremsstrahlung, and pair production, which result from the absorption by the particle of an excitation $|\omega|$ with momentum $-\mathbf{q}$ taken from the medium. The coefficient $N_\omega + 1$ then goes over into the coefficient $N_{|\omega|}$.

3. PROPERTIES OF THE ELECTROMAGNETIC FORM FACTOR

From (7) we get the symmetry properties of the tensor $T_{\mu\nu}$:

$$\begin{aligned} T_{\mu\nu}^*(\mathbf{q}, \omega) &= g_\mu g_\nu T_{\nu\mu}(\mathbf{q}, \omega), \\ T_{\mu\nu}(-\mathbf{q}, -\omega) &= -T_{\nu\mu}(\mathbf{q}, \omega). \end{aligned} \tag{13}$$

In the 4-tensor $T_{\mu\nu}$ we can separate components $T^s_{\mu\nu}(\mathbf{q}, \omega)$ and $T^a_{\mu\nu}(\mathbf{q}, \omega)$ which are symmetrical and antisymmetrical respectively in the indices μ and ν. The formulas (5), (8), and (11) for the probabilities contain only the symmetrical part $T^s_{\mu\nu}(\mathbf{q}, \omega)$, since the spin sums (6), (9), and (12) are symmetrical in μ and ν. This guarantees that the probabilities are real.

The electromagnetic form factor $T_{\mu\nu}(\mathbf{q}, \omega)$ is connected with the retarded Green's function $D^R_{\mu\nu}$ of the electromagnetic field that interacts with the substance; the Fourier component of this function with respect to the two spatial coordinates and the time are

$$\begin{aligned} D_{\mu\nu}{}^{\mathrm{m}R}(\mathbf{q}, \mathbf{q}', \omega) = {} & -i \lim_{\delta \to +)} \iiint \frac{d\mathbf{x}d\mathbf{x}'}{\Omega^2} dt\, \theta(t)\, e^{-i\mathbf{q}\mathbf{x} - i\mathbf{q}'\mathbf{x}' + i(\omega + i\delta)t} \\ & \times \mathrm{Sp}\{e^{(F - H_{\mathrm{cp}})/T}[A_\mu{}^{\mathrm{m}}(\mathbf{x}, t) A_\nu{}^{\mathrm{m}}(\mathbf{x}', 0) - A_\nu{}^{\mathrm{m}}(\mathbf{x}', 0) A_\mu{}^{\mathrm{m}}(\mathbf{x}, t)]\} \\ = {} & g_\mu \sum_{if} (A_\mu{}^{\mathrm{m}}(-\mathbf{q}))_{fi}{}^* (A_\nu{}^{\mathrm{M}}(\mathbf{q}'))_{fi} (\rho_i - \rho_f)(E_i - E_f + \omega + i\delta)^{-1}. \end{aligned} \tag{14}$$

We are interested only in the symmetrical part of the function (14 for $\mathbf{q}' = -\mathbf{q}$.

$$D^{\mathrm{m}RS}_{\mu\nu}(\mathbf{q}, \omega) = {}^1/_2[D_{\mu\nu}{}^{\mathrm{m}R}(\mathbf{q}, -\mathbf{q}, \omega) + D_{\nu\mu}{}^{\mathrm{m}R}(\mathbf{q}, -\mathbf{q}, \omega)]. \tag{15}$$

A direct comparison of (14), (15), and (7) yields

$$\begin{aligned} & T_{ij}{}^s(\mathbf{q}, \omega) = \mathrm{Im}\, D^{\mathrm{m}Rs}_{ij}(\mathbf{q}, \omega), \\ & T_{44}(\mathbf{q}, \omega) = \mathrm{Im}\, D^{\mathrm{m}Rs}_{44}(\mathbf{q}, \omega), \quad T_{i4}(\mathbf{q}, \omega) = \mathrm{Re}\, D^{\mathrm{m}Rs}_{i4}(\mathbf{q}, \omega), \end{aligned} \tag{16}$$

where i, j = 1, 2, 3. In the homogeneous case the Green's function in the coordinate representation depends only on the difference $\mathbf{x} - \mathbf{x}'$. With this

$$D_{\mu\nu}{}^{\mathrm{m}R}(\mathbf{q}, \mathbf{q}', \omega) = \delta_{\mathbf{q}, -\mathbf{q}'} D_{\mu\nu}{}^{\mathrm{m}R}(\mathbf{q}, \omega), \tag{17}$$

and the Green's function $D^{\mathrm{m}R}_{\mu\nu}(\mathbf{q}, \omega)$ of the electromagnetic field in the homogeneous medium can be expressed in terms of the dielectric constant of the medium.

If the medium is not only homogeneous but also isotropic, the expression for $D^{\mathrm{m}R}_{\mu\nu}(\mathbf{q}, \omega)$ in terms of the longitudinal and transverse dielectric constants $\varepsilon_l(\mathbf{q}, \omega)$ and $\varepsilon_t(\mathbf{q}, \omega)$, which can be found in [16, 17], leads, when

(16) and (17) are used, to the formula

$$T_{\mu\nu}(\mathbf{q},\ \omega) = 4\pi\Omega^{-1}\ \mathrm{Im}\ \{\delta_{\mu4}\delta_{\nu4}\ /\ [\mathbf{q}^2\varepsilon_l(\mathbf{q},\ \omega)] - (1-\delta_{\mu4})(1-\delta_{\nu4})(\delta_{\mu\nu}-q_\mu q_\nu/\mathbf{q}^2)[\omega^2\varepsilon_t(\mathbf{q},\omega)-\mathbf{q}^2]^{-1}\} \quad (18)$$

for the electromagnetic form factor. The first and second terms of (18) describe the transfer of relatively small momenta $q^{-1} \gg 10^{-8}$ cm to the medium via the longitudinal and transverse field, respectively. When (18) is substituted in (5) we obtain for the scattering probability the well known expressions [12-14] for the long-wave longitudinal and transverse energy and momentum transverse to the medium from a fast charged particle (longitudinal polarization losses, Cerenkov radiation). On the other hand, when (18) is substituted in (8), we obtain the corresponding corrections to the bremsstrahlung probability, which were derived in [4, 10, 11]. A similar correction to the pair production probability is obtained when (18) is substituted in (11).

The contribution made to the probabilities (5), (8), and (11) by the transverse (μ, ν = 1, 2, 3) or mixed (μ or ν = 4) parts of $T_{\mu\nu}(\mathbf{q}, \omega)$ should generally speaking be small (in a ratio of the order of v_{at}/c, where v_{at} is the velocity of the electrons of the substance) compared with the contribution from the longitudinal part

$$T_l(\mathbf{q},\omega) \equiv T_{44}(\mathbf{q},\omega) = \pi(N_\omega+1)^{-1}\sum_{if}\rho_i(T)\,|(\varphi(-\mathbf{q}))_{fi}|^2\delta(E_i+\omega-E_f), \quad (19)$$

where

$$(\varphi(-\mathbf{q}))_{fi} = \frac{4\pi}{\Omega\mathbf{q}^2}\Big(\sum_a e_a e^{-i\mathbf{q}\mathbf{x}_a}\Big)_{fi}, \quad \rho(\mathbf{x}) = \sum_a e_a\delta(\mathbf{x}-\mathbf{x}_a),$$

the sums are taken over all charges of the substance. The probabilities of (5), (8), and (11) are proportional to $T_l(\mathbf{q}, \omega)$. The scattering probability is

$$dw_{\mathbf{q}}^{\mathrm{sc}} = e^2\frac{N_\omega+1}{v_0}S_l^{\mathrm{sc}}\ (\mathbf{p}_0,\mathbf{q},\omega)\,T_l(\mathbf{q},\omega)\frac{\Omega\,(d\mathbf{q})}{(2\pi)^3}, \quad (20)$$

$$S_l^{\mathrm{sc}}\ (\mathbf{p}_0,\mathbf{q},\omega) \equiv S_{44} = 1+\mathbf{v}_0\mathbf{v}+m^2/E_0E. \quad (21)$$

The bremsstrahlung probability is

$$dw_{\mathbf{kq}}^{\mathrm{br}} = e^4\frac{N_\omega+1}{v_0}S_l^{\mathrm{br}}\ \ T_l(\mathbf{q},\omega)\frac{\Omega\,(d\mathbf{q})\,(d\mathbf{k})}{(2\pi)^5m^2k}, \quad (22)$$

$$\begin{aligned} S_l^{\mathrm{br}} &= [(1-\xi-\alpha v)\varkappa_1^2\varkappa_2^2]^{-1}\{-8[\varkappa_1(1-\xi)+\varkappa_2]^2+8(\varkappa_1+\varkappa_2)^2\alpha v \\ &+ 4\varkappa_1\varkappa_2(\varkappa_2-\varkappa_1)\alpha v - 8\varkappa_1(\varkappa_1+\varkappa_2)\alpha v\xi - 4\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)\alpha v\xi \\ &+ \varkappa_1\varkappa_2(\varkappa_1^2+\varkappa_2^2)\alpha^2 - 4\varkappa_1\varkappa_2^2\alpha^2v^2 + (\eta^2-v^2)[-4(1+(1-\xi)^2)\varkappa_1\varkappa_2 \\ &+ 8\varkappa_1\varkappa_2\alpha v + 2(\varkappa_1+\varkappa_2)^2\alpha^2 - 2\varkappa_1\varkappa_2(\varkappa_1+\varkappa_2)\alpha^2] + (\eta^2-v^2)^2\varkappa_1\varkappa_2\alpha^2\}. \end{aligned} \quad (23)$$

The pair production probability is obtained from (22) and (23) by making the ordinary substitutions, just as in the derivation of (11) and (12).

The probability ratio of any two of the three processes under consideration does not depend, in the Coulomb approximation, on the properties of the medium, since the form factor T_l cancels out, for example,

$$\frac{dw_{\mathbf{kq}}^{\mathrm{br}}}{dw_{\mathbf{q}}^{\mathrm{sc}}} = \frac{S_l^{\mathrm{br}}}{S_l^{\mathrm{sc}}}\frac{e^2(d\mathbf{k})}{(2\pi)^2m^2k} \equiv F(\mathbf{p}_0,\mathbf{p},\mathbf{k},\mathbf{q},\omega). \quad (24)$$

Here F is a function which is determined only by the properties of the hard system, and also by the energy and momentum transferred to the medium. However, it does not depend at all on either the concrete properties of the medium or on the type of the transfer process. An explicit expression for this function is obtained with the aid of (21) and (23) without any additional approximations—unlike the expression for the form factor $T_l(\mathbf{q}, \omega)$.

In a medium consisting of N independent atoms we have

$$T_l^{\mathrm{indep}} = T_l^{\mathrm{el}} + T_l^{\mathrm{inel}}, \quad (25)$$

$$T_l^{\mathrm{el}} = \frac{16\pi^3e^2Z^2}{(N_\omega+1)\Omega^2\mathbf{q}^4}[1-F(\mathbf{q})]^2C(\mathbf{q})\delta(\omega), \quad (26)$$

$$T_l^{\mathrm{inel}} = \frac{16\pi^3e^2Z^2N}{(N_\omega+1)\Omega^2\mathbf{q}^4}{\sum_f}'|F_f(\mathbf{q})|^2\delta(\Delta E_f-\omega), \quad (27)$$

where $F(\mathbf{q})$ and $F_f(\mathbf{q})$ are the elastic and inelastic atomic form factors, ΔE_f is the excitation energy of the f-th atomic level, and the structure factor

$$C(\mathbf{q}) = \overline{|L|^2} = \Big|\sum_{\alpha=1}^{N}e^{i\mathbf{q}\mathbf{R}_\alpha}\Big|^2 \quad (28)$$

takes into account the correlation of the spatial arrangement of the atoms of the medium (averaging over the positions $\mathbf{R}_\alpha$ of the nuclei). In a crystal, $C(\mathbf{q})$ is the Laue factor, and in an amorphous medium

$$C(\mathbf{q}) = N\Big[1+(4\pi n/q)\int_0 [g(r)-1]\,r\sin qr\,dr\Big], \quad (29)$$

where g(r) is the binary correlation function characterizing the probability of finding the nuclei at a distance r from one another, $n = N/\Omega$ is the concentration of the atoms. In a gas $C(\mathbf{q}) = N$.

Substitution of (25)–(27) in (5) leads to the main expressions for the probabilities of the elastic and inelastic scattering by the atoms; the structure diffraction pattern is superimposed on these probabilities. On the other hand, substitution in (8) and (11) leads to the formulas for the Bethe-Heitler and the Wheeler-Lamb[5, 6] bremsstrahlung and pair production in the case of a gas, and to the formulas of [7, 8] in the case of a crystal. The case of amorphous condensed media has so far not yet been considered (see Sec. 5).

4. PARALLELISM OF THE PROCESSES OF SCATTERING, BREMSSTRAHLUNG, AND PAIR PRODUCTION

The function $T_{\mu\nu}(\mathbf{q}, \omega)$ differs from zero for $(\mathbf{q}, \omega)$ pairs that correspond to the possible excitations of the medium. If we ignore the dependence of $T_{\mu\nu}(\mathbf{q}, \omega)$ on the direction of $\mathbf{q}$, then we obtain two-dimensional or one-dimensional excitation regions on the $(|\mathbf{q}|, \omega)$ plane. A system of fast charged particles and hard photons can transfer to the medium only $(\mathbf{q}, \omega)$ pairs that are allowed by the kinematics of the medium. The kinematics of the scattering, bremsstrahlung, and pair production is very similar—the allowed $(\mathbf{q}, \omega)$ pairs lie in the $(\mathbf{q}, \omega)$ plane on the lower branches of hyperbolas whose symmetry axes are parallel to the ω axis, and the details of the arrangement depend on the energies and directions of the fast particles and the hard photons. Transfers to the medium, of $(\mathbf{q}, \omega)$ values corresponding to the intersection of these hyperbolas with the regions of excitation of the medium on the $(\mathbf{q}, \omega)$ plane are also possible. A measure of the transfer intensity is the function $S_{\mu\nu}T_{\mu\nu}(\mathbf{q}, \omega)$, which is specified at all points of the $(\mathbf{q}, \omega)$ plane.

In the case of bremsstrahlung, the equation of the kinematic hyperbolas is

$$(\omega - E_1)^2 - (q - p_1 \cos \theta_1)^2 = m^2 + p_1^2 \sin \theta_1, \qquad (30)$$

where

$$\mathbf{p}_1 = \mathbf{p}_0 - \mathbf{k}, \quad E_1 = E_0 - k, \quad \theta_1 = \measuredangle(\mathbf{p}_1, \mathbf{q}),$$
$$\vartheta = \measuredangle(\mathbf{p}_0, \mathbf{k}), \quad p_1 = E_0 v_1 = [p_0^2 + k^2 - 2p_0 k \cos\vartheta]^{1/2},$$
$$0 < \theta_1 < \arccos[\sqrt{2\xi(1 - v_0 \cos\vartheta)} / v_1].$$

When $k = 0$ Eq. (30) describes the kinematics of the scattering. All the bremsstrahlung hyperbolas are inside and below the hyperbolas corresponding to scattering with the same E_0 and $\theta_1 = \theta$, $\theta = \measuredangle(\mathbf{p}_0, \mathbf{q})$ (the larger k, the lower they lie); the hyperbola corresponding to the maximum $k = E_0 - m$ is only tangent to the q axis.

It is obvous that the kinematics of the bremsstrahlung is close to the kinematics of the scattering: (i) at small $k \ll p_0$ and arbitrary p_0, or (ii) in the ultrarelativistic case $E_0 \gg m$ ($\alpha \ll 1$), when the bremsstrahlung hyperbola differs from the scattering hyperbola only in a downward shift by a small amount $\delta E \sim m^2 k/E_0^2 = k\alpha^2$ everywhere except $\xi \approx 1$.

In both indicated cases, the same excitations of the medium will take part, and all the scattering mechanisms (in the broad sense which was explained earlier following formula (7)) appear also in the bremsstrahlung. The hyperbola for pair production is obtained from the bremsstrahlung hyperbola (28) by replacing $p_{1\mu} = p_{0\mu} - k_\mu$ by $k_{1\mu} = k_\mu - p_{+\mu}$. The kinematics of pair production will be close to the scattering kinematics only in the ultrarelativistic limit, when k, E_+, $E_- \gg m$.

It follows from the foregoing that a clear cut parallelism exists between the three processes, and is manifest in the energy and momentum transfer by the same excitation of the medium. It is of interest to trace experimentally this parallelism in those of its links where it has hitherto not been observed, for example, to confirm the existence of hard Cerenkov radiation, bremsstrahlung with transfer to plasmons,[4, 9-11] excitons, magnons, etc., to observe the structure effect in bremsstrahlung in an amorphous body, etc. Hard Cerenkov radiation has low intensity, but is of interest in principle as an example of hard-photon emission stimulated by a flux of soft quanta (as a result of the factor N_ω in (8) and (11)). The latter has apparently not been obtained so far by any method.

5. MULTIATOMIC TRANSFERS IN BREMSSTRAHLUNG AND PAIR PRODUCTION

In the case of large momentum transfers $\eta \gg \eta_{at} = q_{at}/m \sim e^2 Z^{1/3}$, any substance behaves like an aggregate of independent nuclei and electrons. At momentum transfers $\eta \le \eta_{at}$, all possible multiatomic effects are significant in the transfer (the structure of the arrangement of the atoms of the medium, the change in the screening of the nuclear charge by the electrons compared with the case of independent atoms, the appreciable change in the excitation spectrum of the medium), as a result of which the electromagnetic form factor differs in condensed media from the gas expressions (25)–(27) with $C(\mathbf{q}) = N$. These effects are well known for the case of scattering and, in part,[7, 8] for bremsstrahlung and pair production. Actually, all should become manifest in bremsstrahlung and pair production, as noted above.

Observation of these effects is hindered by the fact that in the study of the spectrum one usually fixes with low accuracy only one of the angles, and this causes a large interval $\eta_{min} < \eta \lesssim 1$ to come into play (the minimum momentum transfer is

$$\eta_{min} = \frac{\xi}{2(1-\xi)} \frac{\alpha^2 + \vartheta^2}{\alpha}$$

in the ultrarelativistic limit) and to a strong smearing of the multiatomic effects. Exceptions are the well known cases of bremsstrahlung in pair production in crystals,[7, 8] in which the presence of long-range order leads to participation of not too small transfers (multiples of the reciprocal momentum of the lattice), and as a result, to a very appreciable change in the spectra.

A certain role can be played at small values of Z also by changes in the screening as a result of a generalization of the valence electrons and a change in the spectrum of the inelastic transfers. Let us consider, for example, bremsstrahlung in the ultrarelativistic limit $\alpha \ll 1$, $\vartheta \lesssim \alpha$, taking into account only longitudinal transfers and calling attention to the fact that the multiatomic effects are significant at small momentum transfers $\eta < \eta_0$, where η_0 is somewhat larger than $\eta_{at} \sim e^2 Z^{1/3}$. It is clear that we should have $\eta_{min} \le \eta_0$, and consequently $\xi \lesssim \eta_0/\alpha$. All these small quantities lead to an appreciable simplification of the spin sum (23), and we obtain for the correction to the bremsstrahlung spectrum as a result of the multiatomic effect

$$\Delta w_{\xi\vartheta} = \frac{2e^4 m^3}{(2\pi)^3} G(\xi, \vartheta)\, d\xi\, d\vartheta^2 \int_{\eta_{min}(\xi,\vartheta)}^{\eta_0} \eta^3 d\eta \int d\nu\, (N_\omega + 1)[T_l^{mult.} - T_l^{indep}], \quad (31)$$

$$G(\xi, \vartheta) = \frac{1 + (1-\xi)^2}{\xi} \frac{\alpha^4}{(\alpha^2 + \vartheta^2)^2} - 4\frac{1-\xi}{\xi} \frac{\alpha^6 \vartheta^2}{(\alpha^2 + \vartheta^2)^4}, \qquad (32)$$

where the integration is carried out over all the kinematically admissible ν. The form factor T_l^{indep} is represented by formulas (25)–(27, and T_l^{mult} is represented by the general formula (19). The corresponding expressions for the case of pair production differs from (31) and (32) by the usual replacements.

The lack, in the general case, of sufficient information concerning T_l^{mult} makes it possible to present at the present only rough estimates. Let us estimate first the role of the multiparticle effects in inelastic transfers. We approximate the dielectric constant by the function used by Fermi in his theory of polarization losses:[12]

$$\varepsilon_l(\mathbf{q}, \omega) = 1 + \nu_p^2(\nu_0^2 - \nu^2)^{-1} + i\delta, \qquad (33)$$

where $\delta \to +0$, $\nu_p = (4\pi n Z e^2/m^3)^{1/2}$ is the plasma frequency, n is the number of nuclei per unit volume, $\nu_0 = E_{at}/m = \sigma Z^{4/3} e^4$ the average excitation energy of the atom ($\sigma \sim 1$). The quantities ν_0 and ν_p have in condensed media the same order of magnitude. According to (18)

$$T_l^{mult.\,inel} = \frac{2\pi^2 \nu_p^2}{\Omega m^2 \eta^2 \nu_1} \delta(\nu - \nu_1), \qquad (34)$$

where $\nu_1 = \sqrt{\nu_0^2 + \nu_p^2}$. Formulas (33) and (34) do not take into account the spatial dispersion and are applicable, strictly speaking, only when $\eta \ll \eta_{at}$ but we shall extend them as far as $\eta \sim \eta_{at}$. We proceed in similar fashion with the single-atom expression (27) for F_f, considering it in the dipole approximation:

$$\Sigma_f' |ZF_f(\mathbf{q})|^2 = sq^2 a_{at}^2 Z = s\eta^2 Z^{1/3} / e^4,$$

where

$$a_{at} = (me^2 Z^{1/3})^{-1}, \quad s \sim 1.$$

Substituting this approximate single-atom expression and (34) in (27) and (31), and integrating with respect to η, we obtain the correction $\Delta dw_{\xi}^{br.inel}$ to the bremsstrahlung probability, which we shall compare with the Bethe-Heitler probability $dw_{\xi\vartheta}^{B-H}$. The ratio

$$\frac{\Delta dw_{\xi\vartheta}^{\text{br. inel}}}{dw_{\xi\vartheta}^{\text{B-H}}} = \frac{(\nu_0/2\nu_1) - s\sigma}{\sigma Z^{5/3} \ln(183 z^{-1/3})}, \tag{35}$$

is of the order of $10^{-2}-10^{-1}$ when Z is small and $\xi < \eta_{at}/\alpha$. The sign of the correction is not certain, and its magnitude is of the same order as the Wheeler-Lamb inelastic single-atom correction.

Statistical screening in a crystalline body is changed as a result of the sharing of the valence electrons; the change is the larger, the larger the fraction of the valence electrons, that is, the smaller Z. We can take this change into account by replacing in formula (25) the atomic elastic form factors (26) by the Bloch form factors

$$F_{\text{Bl}} = (1/\Omega Z) \sum_{\nu p} F_{\nu p}(\mathbf{q}), \tag{36}$$

where

$$F_{\nu p}(\mathbf{q}) = \int_{\Delta} u_{\nu p}^* e^{i\mathbf{qx}} u_{\nu p}(dx),$$

$\Delta \equiv (4\pi/3)R^3$ is the volume of the crystal cell, $u_{\nu p}(\mathbf{x})$ is the Bloch factor in the Bloch function $\Omega^{-1/2} u_{\nu p}(\mathbf{x}) \times \exp(i\mathbf{p}\cdot\mathbf{x})$, and ν is the number of the band.

Considering elements with $3 \le Z \le 6$ we assume that the states of the internal electrons do not change when the crystal is formed, and the states of the external $Z-2$ electrons is approximated by plane waves, taking $u_{\nu p} = 1$—the weak-coupling approximation, which should give too high a value for the correction. Using further simple Slater wave functions for [18] for the description of the atomic states, we obtain the elastic multiparticle correction to the bremsstrahlung probability when $\xi < \eta_{at}/\alpha$ (for simplicity we disregard the structure factor $C(\mathbf{q})$:

$$\Delta dw_{\xi\vartheta}^{\text{br. inel}} = \frac{4e^6 n(Z-2)^2}{m^2} G(\xi, \vartheta) \frac{d\xi\, d\vartheta^2}{\alpha^2} \left[\ln \frac{\lambda_2 R}{2a_0} - 2.24 + \frac{0{,}167\lambda_2^2}{(Z-2)\lambda_1^2} \right],$$

where $a_0 = 1/me^2$ is the Bohr radius and λ_1 and λ_2 are the Slater parameters. The correction (37) amounts to -4.2% (Li), -9.5% (Be), -11.6% (B), -12% (graphite), and -13.5% (diamond) of the Bethe-Heitler probability, calculated with the Slater functions in the elastic form factor (26). The estimates (35) and (37) pertain also to multiparticle corrections to the pair production probability. There are apparently not enough exact experimental data pertaining to the soft part of the spectrum, making it desirable to further work aimed at comparing theory with experiment.

Multiatomic effects in bremsstrahlung and pair production become much more noticeable if transfers with small η are separated. This can be done in two ways. The first method is obvious and consists in using a coincidence scheme with simultaneous registration of both produced particles. Let us consider again the case of bremsstrahlung from an ultrarelativistic electron in an amorphous medium, and let us consider for concreteness only the influence of the structure factor $C(\eta)$ in (25). Recognizing that $\mathbf{k}$ and $\mathbf{p}$ lie in one plane with $\mathbf{p}_0$, but on opposite sides of $\mathbf{p}_0$, denoting by χ the angle between $\mathbf{p}$ and $\mathbf{p}_0$, and assuming $\vartheta, \chi \lesssim \alpha \ll 1$, we obtain from the kinematics the following values for the longitudinal and transverse components of η with respect to $\mathbf{p}_0$:

$$\eta_{\parallel} = \frac{\alpha\xi}{1-\xi} \frac{\alpha^2 + \vartheta^2}{\alpha^2} + \frac{\xi\vartheta^2 + (1-\xi)\chi^2}{2\alpha}, \quad \eta_{\perp} = \frac{\xi\vartheta - (1-\xi)\chi}{\alpha}. \tag{38}$$

The order of magnitude $\eta_\perp$ is in general larger than $\eta_\parallel$ by a factor $1/\alpha$, but $\eta_\perp$ can be made to vanish at any arbitrary point $\xi = \xi_0$ of the spectrum, by taking $\chi = \xi_0(1-\xi_0)^{-1}$. Then it turns out that $\eta_\perp \lesssim \eta_\parallel$ in some interval $\Delta\xi$ near ξ_0, and

$$\eta = \sqrt{\eta_\parallel^2 + \eta_\perp^2} = \frac{\vartheta}{\alpha(1-\xi_0)} \left[(\xi - \xi_0)^2 + \frac{1}{4}\xi_0^2 \left(\frac{\alpha^2 + \vartheta^2}{\vartheta} \right)^2 \right]^{1/2}. \tag{39}$$

From (39) we see that $\Delta\xi \sim \xi_0(\alpha^2 + \vartheta^2)/\vartheta$.

The structure factor $C(\eta)$ has a number of pronounced maxima and minima.[19, 20] The position of the principal maximum is connected with the interatomic distance R and amounts to $\eta_{str} \approx 2\pi/mR \sim 10^{-2}$. Since the bremsstrahlung probability $dw_{\xi\vartheta\chi} \propto C(\eta) = C[\eta(\xi)]$, and also, according to (24), it is proportional to the scattering probability dw_η, a structure picture which duplicates the analogous picture (see, for example, [20]) of the angular distribution in electron diffraction (but in the scale of ξ), should be seen against the background of the Bethe-Heitler spectrum, in the interval $\Delta\xi$ near ξ_0. The relative height of the maxima and minima will be the same (for example, fivefold), as in electron diffraction. However, unlike in electron diffraction, the structure picture in the bremsstrahlung spectrum will be repeated twice, symmetrically with respect to ξ_0 (see (39)).

In order for the structure picture to appear on the bremsstrahlung spectrum it is necessary to have $\eta_{min} < \eta_{str}$. When $E_0 = 5.1$ MeV ($\alpha = \vartheta = 0.1$) this condition corresponds to $\xi_0 < 0.1$ ($k_0 = E_0\xi < 500$ keV), $\Delta\xi \sim 2\alpha\xi_0 \sim 100$ keV, and $\chi \sim 10^{-2}$. To reveal the structure picture clearly it is necessary to have sufficiently good resolution. For example, with η fixed at an absolute accuracy $\delta\eta \sim 0.1$, $\eta_{str} \sim 10^{-3}$, the scatter in the angles should be $\delta\vartheta \sim (\alpha/\xi)\delta\eta \sim 10^{-3}$ and $\delta\chi \sim \alpha\delta\eta \sim 10^{-4}$, the scatter in $\delta\xi \sim (\Delta\xi)^{-1}\eta_{str}\,\delta\eta \sim 10^{-3}$, and $\delta k = \delta E \sim 5$ keV.

Attention must be called to the fact that in the present case, just as in [8], and unlike in [7], there is no need at all for an energy of hundreds of MeV to reveal the structure singularities in the bremsstrahlung spectrum. Unlike both [7] and [8], the coincidence method makes it possible to observe a strong structure effect not only in the case of a crystalline body, but even in the case of an amorphous one.

Another possible method of eliminating large momentum transfers is to study the bremsstrahlung spectrum due to an electron moving in a substance through a channel of sufficiently small diameter D. The only transfers that will arise here are $\eta \lesssim 2\pi/mD$, for example transfers to long-wave plasmons or to soft transverse quanta (hard Cerenkov radiation), to long-wave phonons, etc. Let us estimate, for example, the probability of transfer in a channel to plasmons, using expression (18), which pertains in fact to a homogeneous medium. Using again formula (33) for $\varepsilon(\mathbf{q}, \omega)$, substituting (34) in (31) and putting $\eta_0 = 2\pi/mD$, we get

$$dw_{\xi\vartheta} = \frac{me^4 d\xi d\vartheta^2}{4\pi a^2}(N_{\omega_p}+1)G(\xi, \vartheta)\left(\frac{4\pi^2}{m^2D^2} - \eta_{\min}^2\right). \quad (40)$$

When $D \sim 10^{-6}$ and $2\pi/mD \sim 3 \times 10^{-4}$ the probability (40) of bremsstrahlung in a channel, with transfer to a longitudinal field, is $\sim 10^{-4}(N_{\omega_p} + 1)/Z^2$ of the Bethe-Heitler probability, so that this effect can be regarded as observable.

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