AVERAGED EQUATIONS OF A NONSTATIONARY RESONANT MEDIUM

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Averaged equations are deduced under nonstationary conditions for an electromagnetic field in a one-dimensional resonant medium, without expanding the field in eigen functions of an empty resonator. For weak nonlinearity or small coherence, the equations are identical with those of the balance theory. Equations for stationary conditions are derived as a special case.

1. INTRODUCTION

THE operating features of quantum amplifiers and generators are frequently analyzed by using balance-theory equations derived under the assumption that the transitions in the atoms are determined only by the number of photons of given frequency. Such an analysis is valid if there are no phase relations between the photons. These conditions are usually satisfied in quantum amplifiers (see, for example, [1,2]). In generators, the interference between the direct and backward waves is significant, and allowance for it is essential.

In the exact theory of quantum generators it is necessary to start from equations that take into account the presence and the interaction of many natural oscillation modes that fall within the width of the emission line of the active medium. The fields satisfying Maxwell's macroscopic equations can be expanded in the eigenfunctions of the unperturbed resonator, as is done in the theory of the two-mode generator [3–5], and one can start from the system of ordinary differential equations for the amplitudes of the proper waves. Such a method is useful only in those cases when the number of modes in the resonator is small, and can hardly lead to the desired purpose in a multimode generator.

Another possibility of simplifying Maxwell's equations lies in the high monochromaticity of the laser emission. By representing the field in the form of monochromatic waves with slowly varying amplitudes, propagating in opposite directions, it is possible to lower the order of the differential equations and rewrite them in simpler form for these amplitudes. The slow dependence of the indicated amplitudes on the coordinates and on the time correspond to allowance for the multimode character. For a generator operating in the stationary regime, such an approach has been developed for dipole transitions in [6] and for arbitrary transitions in [1]. However, operation of solid-state lasers is essentially nonstationary, so that it is important to have equations for the nonstationary regime.

The present paper is devoted to the derivation of such equations for a one-dimensional generator. The initial system is made up of the equations of quantum electrodynamics [7] averaged over an infinitesimally small physical value [8]. By representing the field in the form of a superposition of waves traveling in opposite directions and by averaging over the spatial period of the wave, we obtain first-order equations for the slowly varying amplitude. These equations take explicitly into account the coherence effect, and when the amplitudes are small they coincide with the balance equation. On the other hand, if the time dependence of the amplitudes is neglected, these equations go over into the equations of the stationary generator [1,6]. We use the derived equations to analyze certain examples and to determine the corrections that must be introduced into balance theory to allow for the coherence effects.

2. DERIVATION OF THE AVERAGED EQUATIONS OF A GENERATOR IN THE NONSTATIONARY REGIME

Quasiclassical equations for a resonant medium with arbitrary transition multipolarity were derived in [8]. If allowance is made, in the usual fashion, for the relaxation of the transition current Γ for the nonresonant and spontaneous losses β and $1/\tau$, and also for the pump W, then the initial system of equations for the projection of the potential A in the direction of the matrix element of the transition current ρ and for the level population Δ in the one-dimensional case takes the form

$$\left(\frac{1}{v}\frac{\partial^2}{\partial t^2}+\beta\frac{\partial}{\partial t}-v\frac{\partial^2}{\partial x^2}\right)A=-4\pi\left[M\left(i\frac{\partial}{\partial x}\right)\rho^* + M^*\left(-i\frac{\partial}{\partial x}\right)\rho\right], \tag{1}$$

$$\frac{\partial\rho}{\partial t}+i\omega_0\rho+\frac{\Gamma}{2}\rho=\frac{i}{c\hbar}\Delta M\left(-i\frac{\partial}{\partial x}\right)A, \tag{2}$$

$$\frac{\partial\Delta}{\partial t}+\Delta\left(W+\frac{1}{\tau}\right)-n\left(W-\frac{1}{\tau}\right) =\frac{2i}{c\hbar}\left[\rho M^*\left(i\frac{\partial}{\partial x}\right)-\rho^* M\left(-i\frac{\partial}{\partial x}\right)\right]A, \tag{3}$$

where v is the wave velocity in the medium, ω_0 the transition frequency, n the density of the active atoms, and M the transition matrix element. We represent, as usual, the potential and the current in the form

$$A=A'e^{-i\omega t}+A'^*e^{i\omega t}, \quad \rho=\rho' e^{-i\omega t}, \tag{4}$$

where ω is the emission frequency ($\omega-\omega_0 \ll \omega_0$), and the amplitudes A′ and ρ' vary slowly with time compared with $\exp(-i\omega t)$. In terms of the new variables, the equations (1)—(3) of the active medium take the form

$$\frac{2i\omega}{v}\frac{\partial A'}{\partial t}+\frac{\omega^2}{v}A'+i\omega\beta A'+v\frac{\partial^2 A'}{\partial x^2}=4\pi M^*\left(-i\frac{\partial}{\partial x}\right)\rho'. \tag{5}$$

$$\frac{\partial\rho'}{\partial t}+i\varepsilon\rho'+\frac{\Gamma}{2}\rho'=\frac{i}{c\hbar}\Delta M\left(-i\frac{\partial}{\partial x}\right)A', \tag{6}$$

$$\frac{\partial\Delta}{\partial t}+\Delta\left(W+\frac{1}{\tau}\right)-n\left(W-\frac{1}{\tau}\right)=\frac{2i}{c\hbar}\rho' M^*\left(i\frac{\partial}{\partial x}\right)A'^* -\frac{2i}{c\hbar}\rho'^* M\left(-i\frac{\partial}{\partial x}\right)A', \tag{7}$$

where $\epsilon=\omega_0-\omega$.

To simplify these equations we take into account the fact that in real solid-state generators the relaxation time is much shorter than the characteristic generation times and we neglect the derivative $\kappa\rho'/\kappa t$ in (6) compared with $\Gamma\rho'/2$. It then follows from (6) that

$$\rho'=\frac{i\Delta}{c\hbar(\Gamma/2+i\varepsilon)}M\left(-i\frac{\partial}{\partial x}\right)A'. \tag{8}$$

Substituting this value of ρ' in (5) and (7), we get

$$\frac{2i\omega}{v}\frac{\partial A'}{\partial t}+\frac{\omega^2}{v}A'+i\omega\beta A'+v\frac{\partial^2 A'}{\partial x^2} =-\frac{4\pi i}{c\hbar(\Gamma/2+i\varepsilon)}M^*\left(-i\frac{\partial}{\partial x}\right)\Delta M\left(-i\frac{\partial}{\partial x}\right)A', \tag{9}$$

$$\frac{\partial\Delta}{\partial t}+\Delta\left(W+\frac{1}{\tau}\right)-n\left(W-\frac{1}{\tau}\right) =-\frac{2\Gamma\Delta}{c^2\hbar^2(\Gamma^2/4+\varepsilon^2)}\left[M^*\left(i\frac{\partial}{\partial x}\right)A'^*\right]\left[M\left(-i\frac{\partial}{\partial x}\right)A'\right]. \tag{10}$$

We resolve the potential into forward and backward waves in the following manner:

$$A'=A_1e^{ikx}+A_2e^{-ikx} \tag{11}$$

and assume, as usual, that

$$|\partial A_{1,2}/\partial x|\ll k|A_{1,2}|. \tag{12}$$

In order to be able to separate the waves A_1 and A_2 from each other in (9), we average over the spatial period of the wave. We then get

$$\frac{\partial A_1}{\partial x}+\frac{1}{v}\frac{\partial A_1}{\partial t}+\frac{\beta}{2}A_1=\frac{2\pi}{c\hbar\omega(\Gamma/2+i\varepsilon)}\left\langle e^{-ikx}M^*\left(-i\frac{\partial}{\partial x}\right) \times\Delta[M(k)A_1e^{ikx}+M(-k)A_2e^{-ikx}]\right\rangle, \tag{13}$$

$$-\frac{\partial A_2}{\partial x}+\frac{1}{v}\frac{\partial A_2}{\partial t}+\frac{\beta}{2}A_2=\frac{2\pi}{c\hbar\omega(\Gamma/2+i\varepsilon)}\left\langle e^{ikx}M^*\left(-i\frac{\partial}{\partial x}\right) \Delta[M(k)A_1e^{ikx}+M(-k)A_2e^{-ikx}]\right\rangle, \tag{14}$$

$$\frac{\partial\Delta}{\partial t}+\Delta\left(W+\frac{1}{\tau}\right)-n\left(W-\frac{1}{\tau}\right) =-\frac{2\Gamma\Delta}{c^2\hbar^2(\Gamma^2/4+\varepsilon^2)}|M(k)A_1e^{ikx}-M(-k)A_2e^{-ikx}|^2. \tag{15}$$

The angle brackets in the right sides of (13) and (14) denote the averaging indicated above. To carry out this averaging, we represent Δ in the form

$$\Delta=\sum_{n=-\infty}^{+\infty}\Delta_n e^{2ikxn}, \tag{16}$$

the validity of which will be demonstrated later. For the time being it is important only that the Δ_n, as functions of x, vary much more slowly than exponentials. Substituting (16) in (13) and (14) we get

$$\frac{\partial A_1}{\partial x}+\frac{1}{v}\frac{\partial A_1}{\partial t}+\frac{\beta}{2}A_1 =\frac{2\pi M^*(k)}{\hbar\omega c(i\varepsilon+\Gamma/2)}[M(k)\Delta_0A_1+M(-k)\Delta_1A_2], \tag{17}$$

$$-\frac{\partial A_2}{\partial x}+\frac{1}{v}\frac{\partial A_2}{\partial t}+\frac{\beta}{2}A_2 =\frac{2\pi M^*(-k)}{\hbar\omega c(i\varepsilon+\Gamma/2)}[M(-k)\Delta_0A_2+M(k)\Delta_{-1}A_1]. \tag{18}$$

We introduce the following notation for the cross sections:

$$\sigma_{1,2} = 2\pi\Gamma |M(\pm k)|^2 / \hbar c\omega (\Gamma^2/4 + \varepsilon^2), \quad (19)$$

and in lieu of the fields $A_{1,2}$ the potential normalized for the flux:

$$A_{1,2} = (2\pi\hbar c/\omega)^{1/2} a_{1,2} \qquad |a_{1,2}|^2 = J_{1,2}. \quad (20)$$

In the mixed products of the type $M^*(k)M(-k)$, we neglect the phase difference of the matrix elements $M(k)$ and $M(-k)$, since they are cancelled out in final analysis by the same phase factors which are contained in $\Delta_{\pm 1}$. For fields $a_{1,2}$, the equations take the form

$$\frac{\partial a_1}{\partial x} + \frac{1}{v}\frac{\partial a_1}{\partial t} + \frac{\beta}{2} a_1 = \left(\frac{1}{2} - i\frac{\varepsilon}{\Gamma}\right)(\sigma_1 \Delta_0 a_1 + \sqrt{\sigma_1\sigma_2}\Delta_1 a_2),$$
$$-\frac{\partial a_2}{\partial x} + \frac{1}{v}\frac{\partial a_2}{\partial t} + \frac{\beta}{2} a_2 = \left(\frac{1}{2} - i\frac{\varepsilon}{\Gamma}\right)(\sigma_2 \Delta_0 a_2 + \sqrt{\sigma_1\sigma_2}\Delta_{-1} a_1); \quad (21)$$

$$\frac{\partial \Delta}{\partial t} + \Delta\left(W + \frac{1}{\tau}\right) - n\left(W - \frac{1}{\tau}\right) = -\Delta\,[2\sigma_1 J_1 + 2\sigma_2 J_2 + 2\sqrt{\sigma_1\sigma_2}(a_1^* a_2 e^{-2ikx} + a_1 a_2^* e^{2ikx})]. \quad (22)$$

Equation (22) is of first order in the time and can be formally solved with respect to Δ:

$$\Delta = \Delta(0, x)\gamma(t, x)\exp\{-4\sqrt{\sigma_1\sigma_2}|z(t, x)|\cos[\arg z(t, x) - 2kx]\} + n\left(W - \frac{1}{\tau}\right)\gamma(t, x)\int_0^t \frac{dt'}{\gamma(t', x)}\exp\left\{-4\sqrt{\sigma_1\sigma_2}|z(t, x) - z(t', x)|\cos(\arg[z(t, x) - z(t', x)] - 2kx)\right\}; \quad (23)$$

We have introduced here the notation

$$\gamma(t, x) = \exp\left\{-\left(W + \frac{1}{\tau}\right)t - 2\int_0^t [\sigma_1 J_1(t', x) + \sigma_2 J_2(t', x)]\,dt'\right\}, \quad (24)$$

$$z(t, x) = \int_0^t a_1^*(t', x) a_2(t', x)\,dt'. \quad (25)$$

To determine the coefficients Δ_n we make use of the formula

$$e^{-z\cos\varphi} = \sum_{n=-\infty}^{+\infty} (-1)^n I_n(z) e^{-in\varphi},$$

where I_n are Bessel functions of imaginary argument. Taking this expansion into account, we determine from (23) the populations Δ_0 and $\Delta_{\pm 1}$:

$$\Delta_0 = \Delta(0, x)\gamma(t, x) I_0(4\sqrt{\sigma_1\sigma_2}|z(t, x)|) + n\left(W - \frac{1}{\tau}\right) \times \gamma(t, x)\int_0^t \frac{dt'}{\gamma(t', x)} I_0(4\sqrt{\sigma_1\sigma_2}|z(t, x) - z(t', x)|), \quad (26)$$

$$\Delta_{\pm 1} = -\Delta(0, x)\gamma(t, x) I_1(4\sqrt{\sigma_1\sigma_2}|z(t, x)|) \times \exp\{\mp i \arg z(t, x)\} - n\left(W - \frac{1}{\tau}\right)\gamma(t, x)\int_0^t \frac{dt'}{\gamma(t', x)} I_1(4\sqrt{\sigma_1\sigma_2}|z(t, x) - z(t', x)|)\exp\{\mp i\arg[z(t, x) - z(t', x)]\}. \quad (27)$$

Thus, (21), with the notation (24)–(27), are the equations of a resonant medium in the nonstationary regime. Unlike the balance equation, they take explicit account of the coherence effects via the interference between the forward and backward waves [the factor $z(t, x)$]. This interference, as can be readily seen, is significant, first, when the nonlinearity of the medium is large and, second, when the coherence is good. In the opposite case, when

$$4\sqrt{\sigma_1\sigma_2}|z(t, x)| \ll 1, \quad (28)$$

we can put $\Delta_{\pm 1} = 0$ in (21). We then get the equations

$$\frac{\partial J_1}{\partial x} + \frac{1}{v}\frac{\partial J_1}{\partial t} + \beta J_1 = \sigma_1 \Delta_0 J_1,$$
$$-\frac{\partial J_2}{\partial x} + \frac{1}{v}\frac{\partial J_2}{\partial t} + \beta J_2 = \sigma_2 \Delta_0 J_2, \quad (29)$$
$$\frac{\partial \Delta_0}{\partial t} + \Delta_0\left(W + \frac{1}{\tau}\right) - n\left(W - \frac{1}{\tau}\right) = -2\Delta_0(\sigma_1 J_1 + \sigma_2 J_2),$$

which coincide with the balance equations. This shows that (28) should be regarded as the condition for the applicability of the balance equations. Inasmuch as the forward and backward waves are coupled with each other by the boundary conditions, the magnitude of the interference integral z will be determined by the coherence of one of the waves.

3. STATIONARY REGIME

Let us ascertain the conditions under which (21) have stationary solutions. If a_1 and a_2 do not depend on t, then it follows from (24) and (25) that

$$z = a_1^* a_2 t, \qquad \gamma = \exp\{-(W + 1/\tau)t - 2(\sigma_1 J_1 + \sigma_2 J_2)t\}. \quad (30)$$

In order for the right sides of (21) to be independent of t in this case, it is necessary to assume that

$$(W + 1/\tau)t \gg 1. \quad (31)$$

When this condition is satisfied, the first terms in the populations Δ_0 and $\Delta_{\pm 1}$ can be neglected, and the integration with respect to t can be extended in the second terms to infinity. By the same token, we neglect in fact the exponentially small terms. Taking into account also the formula

$$\int_0^\infty e^{-\alpha x} I_\nu(\beta x)\,dx = \frac{\beta^\nu}{\sqrt{\alpha^2-\beta^2}(\alpha+\sqrt{\alpha^2-\beta^2})} \quad (\alpha > \beta),$$

we get

$$\Delta_0 = n(W - 1/\tau)[(W + 1/\tau + 2\sigma_1 J_1 + 2\sigma_2 J_2)^2 - 16\sigma_1\sigma_2 J_1 J_2]^{1/2},$$

$$\Delta_{+1} = \frac{n}{4J_1J_2\sqrt{\sigma_1\sigma_2}}\left(W - \frac{1}{\tau}\right) \times \left\{1 - \frac{W + 1/\tau + 2\sigma_1 J_1 + 2\sigma_2 J_2}{[(W + 1/\tau + 2\sigma_1 J_1 + 2\sigma_2 J_2)^2 - 16\sigma_1\sigma_2 J_1 J_2]^{1/2}}\right\} a_1 a_2^*, \quad (32)$$

and Δ_{-1} is obtained from Δ_{+1} by replacing the factor $a_1 a_2^*$ by $a_1^* a_2$.

Substituting (32) in (21) and rewriting the equations for the fluxes $J_{1,2}$, we get

$$\frac{\partial J_1}{\partial x} + \beta J_1 = \frac{n}{4}\left(W - \frac{1}{\tau}\right) \times \left\{\frac{4\sigma_1 J_1 - (W + 1/\tau + 2\sigma_1 J_1 + 2\sigma_2 J_2)}{[(W + 1/\tau + 2\sigma_1 J_1 + 2\sigma_2 J_2)^2 - 16\sigma_1\sigma_2 J_1 J_2]^{1/2}} + 1\right\},$$

$$-\frac{\partial J_2}{\partial x} + \beta J_2 = \frac{n}{4}\left(W - \frac{1}{\tau}\right) \times \left\{\frac{4\sigma_2 J_2 - (W + 1/\tau + 2\sigma_1 J_1 + 2\sigma_2 J_2)}{[(W + 1/\tau + 2\sigma_1 J_1 + 2\sigma_2 J_2)^2 - 16\sigma_1\sigma_2 J_1 J_2]^{1/2}} + 1\right\}, \quad (33)$$

which coincide with the stationary equation equations obtained in [1,6]. Thus, if a stationary regime exists at all, it sets in for the time $t \gg (W + 1/\tau)^{-1}$ and is described by Eqs. (33).

4. CLOSED RESONATOR

Let us consider a closed resonator with instantaneous Q switching. Since generation develops in such resonators very rapidly, the influence of the pump and of the spontaneous losses can be neglected ($W = 1/\tau = 0$). We assume also that the initial conditions do not depend explicitly on x and are such that $a_1 = a_2$ (this corresponds in fact to excitation of one longitudinal mode in the resonator at the initial instant). Let us consider the case $\sigma_1 = \sigma_2 = \sigma$. It is then easy to show that at any instant of time $a_1 = a_2$ (total interference). Equations (21) for the fluxes $J_1 = J_2 = J$ take under these conditions the form

$$\frac{1}{v}\frac{dJ}{dt} + \beta J = -\frac{\sigma\Delta(0)}{4} \times \frac{d}{dt}\left\{\exp\left[-4\sigma\int_0^t J(t')\,dt'\right] I_0\left[4\sigma\int_0^t J(t')\,dt'\right]\right\}. \quad (34)$$

Integrating this equation with respect to t and introducing the symbol

$$U(t) = 4\sigma\int_0^t J(t')\,dt', \quad (35)$$

we obtain the equation

$$dU/dt = 4\sigma J_0 + \sigma v\Delta(0)[1 - e^{-U} I_0(U)] - v\beta U \quad (36)$$

($J_0 = J(0)$), which can be readily integrated:

$$t = \int_0^U \frac{dx}{[4\sigma J_0 + \sigma v\Delta(0)] - \sigma v\Delta(0)e^{-x}I_0(x) - v\beta x}. \quad (37)$$

Balance theory corresponds to putting unity in lieu of the function $I_0(x)$ under the integral sign. Let us determine the total output during the generation time (i.e., $U(\infty)$). The value of $U(\infty)$ is determined by the poles of the integrand, i.e., by the condition

$$4\sigma J_0 + \sigma v\Delta(0) - \sigma v\Delta(0)e^{-U(\infty)} I_0[U(\infty)] - v\beta U(\infty) = 0. \quad (38)$$

Assuming that $4\sigma J_0 \ll \sigma v\Delta(0)$ and putting

$$\alpha = \Delta(0)/\Delta_{\text{thr}} \qquad \Delta_{\text{thr}} = \beta/\sigma, \quad (39)$$

we get the following equation for the determination of $U(\infty)$:

$$\alpha = \frac{U(\infty)}{1 - e^{-U(\infty)} I_0[U(\infty)]}. \quad (40)$$

On the other hand, if we solve the same problem in the balance equation, we get

$$\alpha = \frac{U_6(\infty)}{1 - \exp\{-U_6(\infty)\}}. \quad (41)$$

Comparison of these formulas shows that when $\alpha \ll 1$ and $\alpha \gg 1$ the values of $U(\infty)$ and $U_b(\infty)$ are practically the same. The maximum difference between these quantities takes place at $\alpha \sim 3$, when $U(\infty)/U_b(\infty)$ is approximately 0.75. The foregoing example shows that even the crudest measurements (energy measurements) make it possible to determine the extent to which a theory that takes coherence into account deviates from the balance theory.

5. PASSAGE OF STRONG RECTANGULAR PULSE THROUGH A RESONANT MEDIUM

As another example, let us consider the passage of a strong rectangular pulse of duration T through an active medium, the right end face of which is reflecting with a reflection coefficient r. We shall assume that the input signal is so strong that in first approximation its variation as a result of the passage can be neglected. We then substitute for a_1 and a_2 in the right sides of (21) their limiting values, and obtain the corrections to a_1 and a_2 by perturbation theory. The limits of applicability of this method are determined by the condition $2\sigma U_0 r \gg 1$, where U_0 is the total number of quanta initially incident on the left end face. We denote the radiation output on the right and on the left by U_1 and U_2 respectively. The same quantities calculated by balance theory will be denoted U_{1b} and U_{2b}. We note that for a rectangular pulse of duration T, the role of coherence length is played by the quantity $L_{coh} = vT$.

We shall not present the general formulas here. Detailed results of the calculations, together with the computations, will be published separately. We note only the following: If $L_{coh} \ll L$ (L – length of the active medium), then the results, as expected, do not differ in practice from the results of balance theory. In the opposite limiting case, when $4\sigma U_0 L/L_{coh} \ll 1$ and r is not very close to unity ($2\sigma U_0(1 - r) \gg 1$), we obtain the following simple formulas:

$$U_1 = (1-r)\left(U_0 + \frac{\Delta L}{2}\right), \quad U_{1b} = (1-r)\left[U_0 + \frac{\Delta L}{2(1+r)}\right],$$

$$U_2 = r\left(U_0 + \frac{\Delta L}{2}\right), \quad U_{2b} = r\left[U_0 + \frac{\Delta L}{(1+r)}\right], \quad (42)$$

where Δ is the initial level population. As seen from these formulas

$$U_1 + U_2 = U_{1b} + U_{2b} = U_0 + \frac{\Delta L}{2}, \quad (43)$$

i.e., the total output from the left and from the right is the same as in balance theory. However, this output is distributed between U_1 and U_2 differently. The differences between $U_{1,2}$ and $U_{1,2b}$ may turn out to be appreciable.

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