FINITE-ORBIT PLASMA INSTABILITY IN TOROIDAL SYSTEMS

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It is shown that the displacement of the guiding centers of plasma particles in an inhomogeneous magnetic field (finite orbits) can play a role equivalent to that of the transverse ion viscosity. The effective viscosity due to the finite orbits can exceed significantly the usual collisionless magnetic viscosity of the ions. When this situation is taken into account it is found that the plasma can be unstable and this instability has been called the finite-orbit instability. This instability can appear at much lower collision frequencies than the usual dissipative instabilities.

1. INTRODUCTION

It is well known that a broad class of drift instabilities can appear in a straight magnetic field. These instabilities are modified appreciably in a more complex toroidal system. For example, the presence of trapped particles can lead to the excitation of oscillations[1, 2] while a minimum H configuration tends to provide stabilization.[3] In the present work we direct attention to the fact that periodic deviations of the guiding centers from a line of force in a toroidal field[4] can lead to the excitation of a special kind of instability which we have called the finite-orbit instability. This instability is closely related to the drift-dissipative instability.[5, 6] It is well known that the growth rate for the drift-dissipative instability is proportional to the square of the Larmor radius ρ^2. However, in the more complicated field characteristic of toroidal devices a particle can be displaced by a distance Λ from a line of force, this distance being much larger than the Larmor radius ρ; this deviation is due to the drift of the guiding center in the inhomogeneous magnetic field. It is reasonable to suppose that the replacement of the Larmor radius in the drift-dissipative instability by the quantity Λ will lead to an instability with a higher growth rate for a given collision frequency or to the appearance of instabilities at lower collision frequencies. The periodic motion of the guiding centers around the line of force leads to a collisionless viscosity (due to the finite orbits) in precisely the same way as the motion of charged particles in Larmor orbits leads to a magnetic viscosity.[7]

Evidently this instability mechanism has not been noted by Coppi and Rosenbluth[3] who have considered a similar problem of plasma stability in a periodic magnetic field. The general solution of an equation with periodic coefficients is of the form $\exp(ik_z z)\,\varphi(z)$ where the period $\varphi(z)$ coincides with the period of the field. The results in [3] refer to the $k_z = 0$ case where the curvature that produces the drift plays the role of a force that tends to make the plasma deviate from equilibrium. Hence the corresponding results in [3] might be called the dissipative balloon mode. In the present work we consider the more dangerous case $k_z \neq 0$ in which, as we have noted above, the drift leads to an effect similar to collisionless viscosity.

2. GENERAL RELATIONS

The basic factor responsible for the drift of guiding centers in toroidal systems is the inhomogeneity of the magnetic field. For example, in a circular torus such as Tokamak the magnetic field at the inside surface of the chamber is stronger than at the outside surface because of the toroidal effect. As a result, when one considers motion along the line of force the absolute magnitude of the total field is found to be variable. This effect is equivalent to an effective corrugation of the longitudinal magnetic field. Similarly, in toroidal systems such as the stellarator the corrugation arises because of the winding that produces the rotational transform. Hence, the magnetic field in systems of this kind can be written in the form

$$H = H_0\left\{1 - \frac{r}{R_{0c}}\left[\cos\left(\frac{2}{L_R}z\right) - h\right]\right\}. \qquad (2.1)$$

Here, z is the coordinate measured along the line of force, R_{0c} characterizes the depth of modulation of the field and L_R characterizes the period. The small quantity $h < 1$ ($h \sim r/R_{0c}$) is introduced in order to take account of the minimum H feature.

The instability considered below can be important for systems with small shear. For this reason, in deriving equations for the small oscillations we shall in general neglect the small azimuthal component of the magnetic field $H_\varphi \ll H_0$. Expressing the perturbation quantities in the form $\varphi(z)\exp\{-i\omega t + im\varphi\}$ and integrating the kinetic equation over the unperturbed trajectories of the particles (assuming that collisions are important and that we can neglect trapped particles[2])

$$r = r_0 - \frac{v_\perp}{\Omega}[\sin(-\Omega t + \alpha) - \sin\alpha],$$

$$\varphi = \varphi_0 + \frac{v_\perp}{r\Omega}[\cos(-\Omega t + \alpha) - \cos\alpha]$$

$$-\frac{1}{2}\frac{L_R}{rR_{0c}\Omega}\left(v_\| + \frac{1}{2}\frac{v_\perp^2}{v_\|}\right)$$

$$\times\left[\sin\frac{2}{L_R}(z_0 + v_\| t) - \sin\frac{2}{L_R}z_0 - ht\frac{2v_\|}{L_R}\right],$$

$$z = z_0 + v_\| t, \tag{2.2}$$

we obtain the following expression for the perturbed ion density:

$$n' = -\frac{e}{T^0}n^0\varphi - i\frac{e}{T^0}n^0\sum_{k_z}\varphi(k_z)e^{ik_z z}a_i\omega\left\langle\int_{-\infty}^{0}\exp\left\{-i\omega t\right.\right.$$

$$+ ik_z v_\| t - ik\frac{L_R}{2R_{0c}\Omega}\left(v_\| + \frac{1}{2}\frac{v_\perp^2}{v_\|}\right)\left[\sin\frac{2}{L_R}(v_\| t + z)\right.$$

$$\left.\left.\left. - \sin\frac{2}{L_R}z - ht\frac{2v_\|}{L_R}\right]\right\}dt\right\rangle, \tag{2.3}$$

where $a_i = (1 - \omega_i^*/\omega)$, $k_z = 2\pi n/L$, $k = m/r$, L is the length of the system and

$$\omega_i^* = \frac{kT^0}{M\Omega n^0}\frac{dn^0}{dr}$$

is the ion drift frequency while the brackets $\langle\,\rangle$ denote averages over a Maxwellian distribution. In (2.3) we have taken an average over the fast cyclotron gyration of the particles and have neglected effects due to the finite ion-Larmor radius (drift approximation) because we shall consider long wave perturbations below $k\rho_i \ll 1$.

The most dangerous instability is characterized by the longest wavelength $k \to 0$. In treating these instabilities, in (2.3) we can expand in terms of the quantity k, retaining terms of order k^2 inclusively. After integration over velocity the density expression assumes the form

$$n' = -\frac{e}{T^0}n^0\varphi + \frac{e}{T^0}n^0\varphi\sum_{k_z}\varphi(k_z)e^{ik_z z}a_i\left[Y(k_z) + k\Lambda\left(h\right.\right.$$

$$\left.\left. - \cos\frac{2}{L_R}z\right)F_1 + (k\Lambda)^2F_2\right], \tag{2.4}$$

where

$$\Lambda = \frac{1}{2}\frac{L_R}{R_{0c}}\frac{v_T}{\Omega},$$

$$F_1 = -\frac{1}{2}\left[x_+ - x_- - \left(x_+ + \frac{1}{2x_+}\right)Y\left(k_z + \frac{2}{L_R}\right)\right.$$

$$\left. + \left(x_- + \frac{1}{2x_-}\right)Y\left(k_z - \frac{2}{L_R}\right)\right],$$

$$F_2 = \frac{1}{2}\left[x^2 - \left(x^2 + 1 + \frac{1}{2x^2}\right)Y(k_z) - \frac{1}{2}x_+^2\right.$$

$$+ \frac{1}{2}\left(x_+^2 + 1 + \frac{1}{2x_+^2}\right)Y\left(k_z + \frac{2}{L_R}\right) - \frac{1}{2}x_-^2$$

$$\left. + \frac{1}{2}\left(x_-^2 + 1 + \frac{1}{2x_-^2}\right)Y\left(k_z - \frac{2}{L_R}\right)\right].$$

Here, we have introduced the notation

$$x = \frac{\omega}{k_z v_T}, \quad x_\pm = \frac{\omega}{(k_z \pm 2/L_R)v_T}, \quad v_T = \sqrt{\frac{2T}{m}},$$

$$Y(k_z) = \left\langle\frac{\omega}{\omega - k_z v_\|}\right\rangle,$$

and the brackets $\langle\,\rangle$, as above, denote an average over a Maxwellian distribution. The function $Y(k_z)$ can be expressed in terms of the Kramp function, for example

$$Y(k_z) = -i\sqrt{\pi}\frac{\omega}{|k_z|v_T}W\left(\frac{\omega}{k_z v_T}\right).$$

In order to simplify the analysis we assume that in the time corresponding to a period of these oscillations the particles do not traverse a length L_R, that is to say $v_\| t/L_R \ll 1$; we also assume that $k_z v_T/\omega < 1$ so that (2.4) can be simplified considerably:

$$n' = -\frac{e}{T^0}n^0\varphi + \frac{e}{T^0}n^0a_i\sum_{k_z}\varphi(k_z)e^{ik_z z}\left[1 + k\eta\left(h\right.\right.$$

$$\left.\left. - \cos\frac{2}{L_R}z\right) + \frac{7}{8}(k\eta)^2\right];$$

$$\eta = \Lambda\frac{\omega_R}{\omega}, \quad \omega_R = \frac{2v_T}{L_R}. \tag{2.5}$$

Here, Λ characterizes the dimension of the orbit, that is to say, the distance over which the guiding center of the particle can be displaced due to its drift in the inhomogeneous magnetic field and ω_R is the frequency associated with the transit time between the corrugations L_R for particles characterized by the thermal velocity. The quantity η characterizes the displacement of a particle in a time $t \sim 1/\omega$.

We have assumed above that the ions move across the magnetic field without collisions. However, as is shown in the Appendix, the relation in (2.5) holds to an accuracy within a numerical factor of order unity even when $\nu_{ii} \to \infty$.

When the electron mean free path is much smaller than the longitudinal dimensions of the system, the electrons can be described by the hydrodynamic equations. In this approximation the perturbation in electron density is given by[8]

$$n_e' = -\frac{ck}{\omega H}\frac{dn^0}{dr}\varphi + \frac{e}{T^0}n^0\varphi(k_z)\chi(k_z)a_e,$$

$$a_e = \left(1 - \frac{\omega_e^*}{\omega}\right), \quad \omega_e^* = -\frac{kcT_e^0}{eHn^0}\frac{dn^0}{dr},$$

$$\chi(k_z) = i\frac{k_z^2 v_e^2}{\omega\nu_e}\Big/\left(1 + i\frac{k_z^2 v_e^2}{\omega\nu_e}\right). \tag{2.6}$$

3. FINITE-ORBIT INSTABILITY

The equation for the potential, which describes the small oscillations of a plasma, can be obtained from the neutrality condition $n' = n_e'$. Using (2.5) and (2.6) we find

$$\frac{T_e^0}{T_i^0}a_i\left[\frac{7}{8}(k\eta)^2\varphi(k_z) - \frac{1}{2}k\eta\left(\varphi\left(k_z - \frac{2}{L_R}\right)\right.\right.$$
$$\left.\left. + \varphi\left(k_z + \frac{2}{L_R}\right)\right) + hk\eta\right] - a_e\varphi(k_z)\chi(k_z) = 0. \tag{3.1}$$

It is of interest to compare this equation with the dispersion equation for the inertial drift instability:[5, 6]

$$-\frac{T_e^0}{T_i^0}a_i(k\rho_i)^2 - a_e\chi(k_z) = 0. \tag{3.2}$$

It is evident that (3.1) differs from (3.2) in that ρ_i is replaced by η. The latter quantity, which represents the distance traversed by the guiding center of an ion in one period of the oscillations being considered here, plays the role of an effective orbit dimension.

Now let us consider the conditions for which the finite orbit effect can be important. It follows from (2.4) that the term containing $\cos(2z/L_R)$ in the square brackets cannot exceed the term which is quadratic in $k\Lambda$, that is to say, the wavelength along the system must be sufficiently long so that the term containing $\cos(2z/L_R)$ can be averaged. This condition can be written $k_z \ll 2/L_R$. In the other limiting case of short wavelengths $k_z \gg 2/L_R$, the oscillations can be regarded as localized at each point and the $(k\Lambda)^2$ term in (2.4) is unimportant; taking account of the dissipation we then obtain the usual "gravitational" dissipative instability[8] with a "gravitational force" which is a periodic function of z.

In a high-temperature plasma, in which the dissipation is small ($\nu_e \to 0$), the off-diagonal terms in (3.1) disappear rapidly. Hence we can consider a system which contains three harmonics: $\varphi(k_z)$, $\varphi(k_z - 2/L_R)$, $\varphi(k_z + 2/L_R)$. Setting the determinant of this system equal to zero we obtain the following dispersion relation:

$$\left\{\frac{T_e^0}{T_i^0}a_i\left[\frac{7}{8}(k\eta)^2 + hk\eta\right] - a_e\chi(k_z)\right\}$$
$$\times\left\{\frac{T_e^0}{T_i^0}a_i\left[\frac{7}{8}(k\eta)^2 + hk\eta\right]\right.$$
$$\left. - a_e\chi\left(\frac{2}{L_R}\right)\right\} = \frac{1}{2}\left(\frac{T_e^0}{T_i^0}\right)^2 a_i^2(k\eta)^2. \tag{3.3}$$

Using this expression for the case $\omega \ll \omega^*$, $k_z^2 v_e^2/\nu_e < \omega < 4v_e^2/L_R^2\nu_e$ we obtain the following value for the frequency:

$$\omega = i\frac{3}{8}\frac{T_e^0}{T_i^0}(k\Lambda\omega_R)^2\left[\frac{k_z^2 v_e^2}{\nu_e} - ih\frac{T_e^0}{T_i^0}(k\Lambda\omega_R)\right]^{-1}. \tag{3.4}$$

It is evident from this expression that minimum H does not stabilize these oscillations, but only reduces the growth rate to some extent.

Let us now consider in greater detail the case in which there is no minimum H, h = 0. The maximum growth rate for this instability $\omega \sim \omega^*$ is achieved for a value of k_z [as follows from (3.4)] given by

$$k_z^2 \sim \left(\frac{m}{M}\frac{T_i^0}{T_e^0}\right)^{1/2}\left(\frac{a}{R_{0c}}\right)^2\left(\frac{\rho_i}{a}\right)^2\frac{(ka)}{S}\frac{1}{a^2}. \tag{3.5}$$

Here, we have introduced the parameter that characterizes the importance of collisions in the plasma: $S = \lambda_e\rho_i/a^2$.[9] If k_z as given by (3.5) is found to be smaller than π/L then ω does not reach ω^* and its maximum value can be found from (3.4) by taking $k_z \sim \pi/L$.

If we assume that in this instability the usual dimensional estimate holds for the diffusion coefficient $D \sim \gamma/k_\perp^2$, determining (ka) from (3.5) ($k_z \sim \pi/L$) we find

$$D \sim D_B\left(\frac{m}{M}\frac{T_i^0}{T_e^0}\right)^{1/2}\left(\frac{\rho_i L}{\pi R_{0c}a}\right)^2\frac{1}{S}, \tag{3.6}$$

where $D_B = cT_e^0/eH$. The factor that multiplies D_B in (3.6) cannot be larger than unity. If some choice of the parameters in an actual device in (3.6) does make this quantity larger than unity it should be replaced by unity and the corresponding value of k_z will not be π/L, but will be larger, in accordance with (3.5).

Let us now consider plasma parameters and the limits of applicability of the preceding analysis. In order for the finite-orbit instability to develop the length of the system must be great enough to satisfy the condition

$$L\rho_i/\pi a^2 > R_{0c}/a; \tag{3.7}$$

If this does not hold the longitudinal motion of the ions becomes important and the instability vanishes.

It is also assumed that the electron dissipation is strong enough so that the electrons can be regarded as hydrodynamic. In order for this criterion to hold we require that two inequalities be satisfied: $\omega < \nu_e$ and $k_z\lambda_e < 1$. However, the strongest limitation on collisions derives from the requirement that there be no trapped particles:[2]

$$S < R_{0c}/a. \tag{3.8}$$

If the inequality in (3.8) is not satisfied it is possible to excite an instability due to the dissipative instability of trapped ions up to $S < (L\rho_i/\pi a^2)$. If this condition is not satisfied, that is to say, if the mean free path of the electrons exceeds the length of the system the dissipation becomes so weak that it cannot prevent the electrons from reaching equilibrium along the lines of force, in which case the electrons establish a Boltzmann distribution and the diffusion vanishes.[9]

Thus, the necessary condition for the system to exhibit a strong instability $\gamma \sim \omega$ is that both inequalities (3.7) and (3.8) be satisfied. It follows from (3.6) that under these conditions $D > D_B(mT_i^0/MT_e^0)^{1/2}$. We note that the analysis given above depends on the assumption $L_R \ll L$. If this assumption is not satisfied the instability does not appear.

It is interesting to compare the condition for the appearance of this finite-orbit instability with the corresponding conditions for the drift-dissipative instability and the thermal-force instability. It is well known that the thermal-force instability appears when $S < (m/M)^{1/2}$; on the other hand, the finite-orbit instability can appear when $S \sim 1$, that is to say, the required collision frequency can be much smaller. For a given collision frequency the drift-dissipative instability can only develop in a much longer system. Specifically, it is easy to show that the appropriate length relation is

$$L_{\mathrm{dd}}/L_{\mathrm{fo}} \sim (a/R_{0c})\,(a/\rho_i) \gg 1.$$

Let us now consider how this instability is modified in systems that exhibit large angular deviations between the lines of force, such as Tokamak. The expression for the frequency ω (3.4) also holds for this case if we make the substitutions $k_z \to k_{\|} = kx\theta/r$, $R_{0c} \to R$ where R is the major radius of the torus and $k = m/r$ while θ is the angular deviation, which, in order-of-magnitude terms, is equal to the angle between the lines of force of the magnetic field separated by a distance a, the characteristic scale size for the inhomogeneity. For a circular torus such as Tokamak we have

$$\theta = \frac{r}{R}\frac{rq'}{q^2}, \quad q = \frac{rH_z}{RH_\varphi}.$$

In view of the fact that Tokamak exhibits a minimum H,[10] $h \sim r/R$, it is easy to show that the instability growth rate in this case is reduced appreciably as compared with the case in which there is no shear. This can be shown most clearly from the expression for the diffusion coefficient

$$D \sim D_{\mathrm{B}}\left[\left(\frac{m}{M}\right)^{1/2}\left(\frac{a}{R}\right)^4\frac{1}{h^2}\left(\frac{\rho_i}{a}\right)^2\frac{1}{S}\right]^{1/3}, \tag{3.9}$$

in which the factor that multiplies D_B is less than 10^{-2} for values of the parameters typical of Tokamak.

4. CONCLUSION

We have shown in the present work that taking account of so-called mixing effects, i.e., the displacement of particle guiding centers by virtue of the drift in the inhomogeneous magnetic field, can lead to an instability that we have called the finite-orbit instability.[8] In this case, taking account of the ion motion in the finite orbits is equivalent to taking account of an effective transverse collisionless viscosity. In principle this instability can develop in any system with a bumpy magnetic field that contains a sufficiently dense plasma (dense enough so that collisional effects are important). However, the largest growth rate $\gamma \sim \omega$ is not achieved in a system with small shear or with large modulation of the field ($R_{0c} \sim a$). A distinguishing feature of this instability, as compared with other familiar dissipative instabilities, is that it can develop for much lower collision frequencies. In systems with small shear a minimum H configuration does not provide stabilization. However, the joint effect of a large shear and minimum H does reduce the growth rate appreciably.

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APPENDIX

Here we will obtain an expression for the ion density for an arbitrary collision frequency. In the case considered below, long wavelengths $k \to 0$ and the absence of temperature gradients (∇T^0), it is qualitatively correct to take account of ion collisions by introducing a collision integral in the Bhatnagar-Gross-Krook form.[11, 12] In this case the ion kinetic equation can be written in the form

$$\frac{df'}{dt} + \nu f' = \frac{e}{M}\nabla\varphi\cdot\nabla f_0 + \nu f_0\left[\frac{n'}{n^0} + 2\alpha(v_xu_x + v_yu_y)\right.$$

$$+\left(\frac{mv^2}{2T^0}-\frac{3}{2}\right)\frac{T'}{T^0}\Big]. \qquad (A.1)$$

Here we have taken account of the fact that the ion motion is across the lines of force. It is assumed that the wavelengths of the oscillations are long enough $k\rho_i \ll 1$ so that the effect of the finite Larmor radius can be neglected. Under these conditions f coincides with the distribution function for the guiding centers and the second term in the rectangular brackets, which guarantees momentum conservation, vanishes. This means that the motion of the guiding centers in finite orbits (mixing effect) does not lead to a collisional viscosity of the finite orbits, that is to say, the motion being considered only leads to a collisional thermal conductivity. This is understandable physically. We are considering collisions for which the quantities $V_\parallel$ and $V_\perp$ are conserved, that is to say, the momentum of the particles is simply rotated around the axis that coincides with the direction of H_0. It is known that such collisions make a contribution to the ordinary viscosity but that they cannot make a contribution to the finite-orbit viscosity because the orbits do not intersect (because of the fact that the orbits, as follows from (2.2), depend on $v_\parallel$ and $v_\perp$ but not on the angles in velocity space). A contribution to the transport process comes only from those collisions which change $v_\parallel^2$ and $v_\perp^2$; these effects, in some sense, are equivalent to a perturbation of the temperature and are taken account of in (A.1).

Integrating (A.1) along the trajectories it is easy to obtain the following expressions for the perturbed ions densities:

$$n'=-\frac{e}{T^0}n^0\varphi+a_i\frac{e}{T^0}n^0\varphi\frac{\Delta_1}{\Delta_2},$$

$$\Delta_1=\mathrm{Det}\begin{vmatrix}-i\omega P[(1)], & -\nu P\left[\left(\frac{mv^2}{2T^0}-\frac{3}{2}\right)\right]\\ -i\omega P\left[\left(\frac{1}{3}\frac{mv^2}{T^0}\right)\right], & 1-\nu P\left[\left(\frac{1}{3}\frac{mv^2}{T^0}\right)\left(\frac{mv^2}{2T^0}-\frac{3}{2}\right)\right]\end{vmatrix},$$

$$\Delta_2=\mathrm{Det}\begin{vmatrix}1-\nu P[(1)], & -\nu P\left[\left(\frac{mv^2}{2T^0}-\frac{3}{2}\right)\right]\\ 1-\nu P\left[\left(\frac{1}{3}\frac{mv^2}{T^0}\right)\right], & 1-\nu P\left[\left(\frac{1}{3}\frac{mv^2}{T^0}\right)\left(\frac{mv^2}{2T^0}-\frac{3}{2}\right)\right]\end{vmatrix}, \qquad (A.2)$$

Here we have introduced the notation

$$P[A(v)]=\left(\frac{m}{2T^0\pi}\right)^{3/2}\int e^{-mv^2/2T^0}d\mathbf{v}A(v)$$
$$\times\int_{-\infty}^{0}\exp\left\{-i(\omega+i\nu)t-\frac{k}{2}\frac{L_R}{R_{0c}\Omega}\left(v_\parallel+\frac{1}{2}\frac{v_\perp^2}{v_\parallel}\right)\right.$$
$$\left.\times\left[\sin\frac{2}{L_R}(v_\parallel t+z)-\sin\frac{2}{L_R}z-ht\frac{2v_\parallel}{L_R}\right]\right\}. \qquad (A.3)$$

If the ion mean free path is much smaller than the distance between the corrugations of the magnetic field $\lambda_i \ll L_R$, in (A.3) we can expand the sine function and the expression is simplified appreciably.

We consider only two relatively simple limiting cases below: the case of small k (it is assumed, however, that ω or $\nu \gg v_T/L_R$), and the case of high collision frequencies $\nu_{ii}\to\infty$.

In the first case

$$n'=-\frac{e}{T^0}n^0\varphi+\frac{e}{T^0}n^0\varphi a_i\left[1+k\eta\left(\cos\frac{2}{L_R}z-h\right)\right.$$
$$\left.+\frac{7}{8}(k\eta)^2\frac{1}{1+i\nu_i/\omega}+i\frac{\nu_i}{\omega}\frac{5}{6}(k\eta)^2\frac{1}{1+i\nu_i/\omega}\right]. \qquad (A.4)$$

In the second case

$$n'=-\frac{e}{T^0}n^0\varphi+\frac{e}{T^0}n^0\varphi a_i$$
$$\times\frac{1-{}^4/_3k\eta(\cos 2zL_R^{-1}-h)}{1-{}^7/_3k\eta(\cos 2zL_R^{-1}-h)+{}^1/_3(k\eta)^2}. \qquad (A.5)$$

It follows from (A.4) that for $\nu_{ii}\gg\omega$ and $\omega\gg(k\Lambda)^2\omega_R^2/\nu_i$ the expression that has been obtained coincides essentially with the expression for the ion density (A.2) in the collision-free case. This same result naturally follows from (A.5) when $\omega\gg(k\Lambda)\omega_R$. The term $i(k\Lambda)^2\omega_R^2/\nu_i$ in (A.4) takes account of the thermal conductivity due to the finite orbits as indicated above.[1)]

[1)]It has been brought to our attention that the similar result has been obtained by A. B. Mikhaĭlovskiĭ by means of a hydrodynamic analysis.

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187