RADIATION IN TWO-PARTICLE ANNIHILATION OF AN ARBITRARILY POLARIZED ELECTRON-POSITRON PAIR

D. F. MOROZOV

Submitted to JETP editor September 22, 1966

J. Theoret. Phys. (U.S.S.R.) 52, 1212-1220 (May, 1967)

The polarization contribution to the exact differential photon emission cross section in two-particle annihilation of an arbitrarily polarized electron-positron pair is calculated by the method of invariant tensor integration[1]. The polarization contributions to the integral cross sections are calculated for a longitudinally and transversely polarized electron-positron pair. The effect of polarization on the cross section for photon emission by initial particles annihilating into a pair of any other particles is considered by taking into account the form factors of the latter particles[2]. A simple method is suggested for calculating the tensor contractions used in the Baĭer-Khoze calculations of the cross sections for some processes carried out by technique[1], obviating the need for calculating the tensors themselves and thus enhancing the usefulness of this method.

1. INTRODUCTION

BAĬER and Khoze[1,2] calculated the integral photon emission cross sections in two-particle annihilation of unpolarized e^- and e^+, by using a method of invariant integration of the tensors[1]. In this paper we calculate by the same method the polarization contribution to the exact differential photon emission cross section in two-particle annihilation of an arbitrarily polarized electron-positron pair. We calculate the polarization contributions to the integral cross sections for longitudinally and transversely polarized electron-positron pairs. We show that the probabilities of processes accompanied by annihilation or creation of relativistic pairs e^-, e^+ (μ^-, μ^+) whose particles have identical helicities is of the order of m^2/E^2 (μ^2/E^2) compared with the probabilities of the corresponding processes in which the pair particles have different helicities[3-10]. Account is taken of the effect of the polarization on the cross section for the emission of photons from initial particles that annihilate into a pair of arbitrary particles, with allowance for the form factors of the latter[3]. In determining the cross sections of certain processes by method of Baĭer and Khoze[1], we propose to calculate, by means of a simple procedure, the necessary contractions of the tensors without calculating the latter; this improves the usefulness of the method.

In Sec. 2 we consider annihilation into a pair of fermions, and in Sec. 3 annihilation into a pair of scalar particles; in Sec. 4 we analyze the formulas for the integral cross section; in the Appendix we indicate a method of obtaining the contractions of the tensors. We use the notation of[1,2].

2. ANNIHILATION INTO A FERMION PAIR

We consider the case when the final particles are fermions, say muons[1]. We consider only the contributions made to the cross sections (differential and integral) by the polarization of the initial particles; the total cross sections can be readily obtained by using the results of[1].

For the contribution made by the arbitrary polarization of the initial particles to the differential cross sections for photon emission from these particles we obtain the expression

$$\frac{d^3\sigma_e(\xi_1,\xi_2)}{d\Omega_k d\omega}=\frac{m^2\alpha^3\omega\beta_0}{6\pi E^2\beta}\frac{2\mu^2+\Delta^2}{\Delta^4}\times\Big\{\Big(\frac{1}{\varkappa^2}+\frac{1}{(\varkappa')^2}\Big)(m^2A_1-A_2)+\frac{A_4}{(\varkappa')^2}+\frac{A_3}{\varkappa^2}+\frac{2A_1}{m^2}-\frac{1}{m^2\varkappa\varkappa'}[2(p_1p_2^+)(-m^2A_1+A_2)+A_3(m^2+\varkappa')+A_4(m^2-\varkappa)]\Big\}. \tag{2.1}$$

We have introduced here the notation

$$A_1=2(1-\gamma^2)(\xi_1\mathbf{n})(\xi_2\mathbf{n})-(\xi_1\xi_2),$$

$$A_2=\omega^2\{(\xi_1\mathbf{n}_1)(\xi_2\mathbf{n}_1)-[\beta^2\gamma^2-(\gamma-1)^2\cos^2\vartheta_k]\times(\xi_1\mathbf{n})(\xi_2\mathbf{n})-[\beta\gamma-(\gamma-1)\cos\vartheta_k](\xi_1\mathbf{n})(\xi_2\mathbf{n}_1)+[\beta\gamma+(\gamma-1)\cos\vartheta_k](\xi_1\mathbf{n}_1)(\xi_2\mathbf{n})\},$$

$$A_3=2m\beta\gamma^2\omega(\xi_2\mathbf{n})\{[(\gamma-1)\cos\vartheta_k-\beta\gamma](\xi_1\mathbf{n})+(\xi_1\mathbf{n}_1)\},$$

$$A_4 = 2m\beta\gamma^2\omega(\xi_1\mathbf{n})\{[(1-\gamma)\cos\vartheta_k - \beta\gamma](\xi_2\mathbf{n}) - (\xi_2\mathbf{n}_1)\};$$
$$\gamma = \frac{E}{m}, \quad \frac{\mathbf{k}}{\omega} = \mathbf{n}_1, \quad \frac{\mathbf{p}_1}{|\mathbf{p}_1|} = \mathbf{n} \ (\text{ось } z);$$
$$(s_r)_0 = (-1)^{r+1}\beta\gamma(\xi_r\mathbf{n}), \quad \mathbf{s}_r = \xi_r + (\gamma-1)(\xi_r\mathbf{n})\mathbf{n}, \tag{2.2}$$

where s_1 and s_2 are the 4-vectors of polarization of the electron and positron respectively, ξ_r is a unit vector in the polarization direction in the rest system of the particle, and r = 1 or 2 (β, $\varkappa$, L, Λ, and Δ are defined in[1]).

We consider the case of annihilation of longitudinally-polarized particles. In this case, the contribution made to the photon emission differential cross section, with respect to the angle ϑ_k between the initial electron and the photon, is

$$\frac{d^2\sigma_e(s_-, s_+)}{d\cos\vartheta_k d\omega} = \frac{m^2\alpha^3\omega\beta_0}{3E^2\beta}\frac{2\mu^2+\Delta^2}{\Delta^4}(s_-s_+)\cdot$$
$$\times\left\{\left(\frac{1}{\varkappa^2}+\frac{1}{(\varkappa')^2}\right)[m^2(1-2\gamma^2)+\omega^2\gamma^2(\beta^2-\cos^2\vartheta_k)\right.$$
$$-2m\beta^2\gamma^3\omega] - \frac{2}{\varkappa'^2}m\beta\gamma^3\omega\cos\vartheta_k - \frac{2}{m^2}(2\gamma^2-1)$$
$$+\frac{2}{\varkappa^2}m\beta\gamma^3\omega\cos\vartheta_k + \frac{1}{m^2\varkappa\varkappa'}[2(p_1p_2{}^+)(m^2(1-2\gamma^2) \tag{2.3}$$
$$\left.+\omega^2\gamma^2(\beta^2-\cos^2\vartheta_k)) + 4m\beta\gamma^3\omega(m^2\beta - \varkappa'\cos\vartheta_k)]\right\},$$

where s_- (s_+) is the helicity quantum number of e^- (e^+). Integrating with respect to the photon emission angle we get

$$d\sigma_e(s_-, s_+) = \frac{2\alpha^3 d\omega\beta_0}{3E^2\beta\omega}\frac{2\mu^2+\Delta^2}{\Delta^4}(s_-s_+)$$
$$\times\left\{2m^2\left[(2\gamma^2-1) - \frac{1}{m^2}\gamma^2\beta^2\omega(\omega-2m\gamma)\right.\right.$$
$$+\frac{\omega^2\gamma^2}{\beta^2E^2}(1-L+\gamma^2) - \frac{2m\gamma^3\omega}{E^2}\left(\gamma^2-\frac{L}{2}\right)$$
$$\left.+\frac{(2\gamma^2-1)\omega^2}{m^2}\right] + L\left[\left(2-\frac{m^2}{E^2}\right)\right.$$
$$\left.\times(m^2-2E^2+\omega^2(\gamma^2-1)) + 2\omega\left(E-\frac{m^2}{E}\right)\right]$$
$$\left.+\frac{\omega^2}{\beta^2}(L-2)(2\gamma^2-1)\right\}. \tag{2.4}$$

the integral cross section for the emission of a photon ($\omega < m$) by arbitrarily polarized initial particles is

$$d\sigma_e'(s_-, s_+) = [1-(s_-s_+)(1-m^2/E^2)]d\sigma_e, \tag{2.5}$$

where $d\sigma_e$ is the cross section for the emission of a photon by initially unpolarized particles and is given by (2.55) of[1].

Thus, the allowed process is the annihilation of a particle with its own antiparticle (the signs of the helicities of e^- and e^+ are different); the probability of annihilation of a particle with a foreign antiparticle (identical signs of helicities of e^- and e^+) is smaller by a factor E^2/m^2 than the probability of the allowed annihilation.

Zel'dovich[3] indicates a connection between the polarizations in the created relativistic electron-positron pairs, namely, a left (right) e^- must be created with a right (left) e^+. The probability of the processes in which left (right) e^- are created with left (right) e^+ is smaller by a factor E^2/m^2 than the probability of the allowed processes.

Starting from the fact that relativistic e^- with positive and negative helicities are described independently by two-component spinors[11,12], we can conclude that the allowed processes should include also those in which relativistic left (right) e^- annihilate with right (left) e^+. Processes in which relativistic left (right) e^- annihilate with left (right) e^+ have a probability smaller by E^2/m^2.

Such a ratio of the cross sections for two-pion (two-fermion, two-vecton) annihilation of longitudinal polarized e^- and e^+ can be obtained from expressions (4) of[4] ((7) of[4]) for the cross sections of two-particle annihilation of arbitrarily polarized e^- and e^+, from expressions (6) and (10) of[5] for the cross sections of two-pion and two-fermion annihilation of longitudinally polarized e^- and e^+, or from expressions (7) and (9) of[6] for the cross section for the production of pseudoscalar mesons in the annihilation of longitudinally polarized e^- and e^+, and also from (12) of[6] for the cross sections of the inverse processes. It is possible to show with the aid of the same formulas that for the allowed processes the direction of motion of the pseudoscalar meson does not coincide with the direction of motion of the relativistic electron[3].

It follows also from the paper of Shakhnazaryan[5] that the helicity of the relativistic Dirac e^- and e^+ does not change upon scattering. In scattering of e^- by e^- (e^+), the matrix element of the transition from the initial state, in which the particle helicities are different, to a final state in which the particle helicities are reversed, is different from zero, but a contribution to it is made by the exchange (annihilation) diagram, that is, the particle helicity is conserved in the scattering. Thus, in scattering of e^- by e^- (e^+) the contribution of the direct (annihilation) diagrams differs from zero only when the helicity of the particle does not change (the helicity of the particle is different both before and after the collision). This again confirms that relativistic e^- and e^+ can be created and annihilated only when they have different helicities. A similar relation between the cross sections in the relativistic case follows from expression (10) of[7] for the cross section of two-photon annihilation of arbitrarily polarized e^- and e^+.

Sokolov et al.[8] have found that only e^- and e^+ with opposite helicities can annihilate in the reaction of $e^- + e^+ \to \nu + \bar{\nu}$, owing to the direct weak interaction in the ultrarelativistic case (the same holds for μ^- and μ^+ having opposite helicities, with creation of a muonic neutrinos). In the inverse process, only e^- and e^+ with opposite helicities are producted. From the results of Sadykhov et al.[9], and also from[4,5] it follows that proton-antiproton pairs are produced in e^- and e^+ annihilation only with opposite helicities of the latter (the initial particles are assumed to be Dirac particles). Using formula (4) of[10], we can show that in the annihilation of proton-antiproton pairs the cross section for the production of pairs of particles e^-, e^+ (μ^+, μ^+) with opposite helicities is larger by a factor E^2/m^2 (μ^2) than the cross section for the production of a pair of particles with identical helicities, regardless of the helicities of the annihilating nucleons or of their structures.

Analyzing the results of Sokolov et al.[13], we find that in the relativistic case ($\mu \ll E$) in the reaction $e^- + e^+ \to \nu + \bar{\nu}$, regardless of the polarization of the initial particles, the probability of the creation of muons with identical helicities is smaller by a factor E^2/μ^2 than the probability of creation of particles with different helicities. Therefore the cross section of the processes in which relativistic e^- and e^+ (μ^- and μ^+) having different helicities are annihilated or created is approximately the double the cross section of these processes for unpolarized particles.

Let us consider the cross section for photon emission by muons. The contribution made by the arbitrary polarization of the initial particles to the differential cross section is

$$\frac{d^3\sigma_\mu(\xi_1,\xi_2)}{d\Omega_k d\omega} = \frac{\alpha^3\omega}{128\pi^2E^6\beta}\Big[2m^2a_1A_1 - \frac{2(a_1+\Lambda^2a_2)}{\omega^2}(2E^2A_2 - A_1\varkappa\varkappa' - A_4\varkappa + A_3\varkappa')\Big]. \quad (2.6)$$

In the case of annihilation of longitudinal polarized particles, the contribution to the photon-emission differential cross section with respect to the angle between the initial electron and the photon is

$$\frac{d^2\sigma_\mu(s_-,s_+)}{d\cos\vartheta_k d\omega} = \frac{\alpha^3\omega}{64\pi E^6\beta}(s_-s_+)\Big\{2m^2a_1(1-2\gamma^2) + \frac{2(a_1+\Lambda^2a_2)}{\omega^2}[2E^2\omega^2\gamma^2(\beta^2-\cos^2\vartheta_k) - (2\gamma^2-1)\varkappa\varkappa' - 4\beta^2\gamma^2\omega^2E^2 - 2m\beta\gamma^3\omega\cos\vartheta_k(\varkappa+\varkappa')]\Big\}. \quad (2.7)$$

The integral cross section for the emission of a photon ($\omega < m$) by muons is

$$d\sigma_\mu'(s_-,s_+) = [1-(s_-s_+)(1-m^2/E^2)]d\sigma_\mu, \quad (2.8)$$

where $d\sigma_\mu$ is the integral cross section for photon emission by muons in annihilation of unpolarized particles and is given by (2.36) of[1]. It is seen from the expression for the cross section that the muons emit only when the initial particles have different helicities.

Thus, the summary integral cross section for the emission of a photon ($\omega < m$) in the annihilation of longitudinal polarized particles is

$$d\sigma'(s_-,s_+) = [1-(s_-s_+)(1-m^2/E^2)]d\sigma, \quad (2.9)$$

where $d\sigma$ is the summary integral cross section in the annihilation of unpolarized particles and is given by (2.41) of[1].

Let us consider the case of annihilation of transversely polarized particles. The contribution to the differential cross section of photon emission by initial particles is of the form (the azimuthal angle φ is reckoned from the plane of the vectors $\mathbf{p}_1$ and $\boldsymbol{\xi}_1$ lie)

$$\frac{d^3\sigma_e'(\xi_1,\xi_2)}{d\Omega_k d\omega} = -\frac{\alpha^3\omega\beta_0}{12\pi E^2\beta}\frac{2\mu^2+\Delta^2}{\Delta^4}\cdot \times\Big\{h\Big[2m^2\Big(\frac{1}{\varkappa^2}+\frac{1}{\varkappa'^2}\Big)+\frac{4(2E^2-m^2)}{\varkappa\varkappa'}\Big] + (\xi_1\xi_2)\Big[2m^4\Big(\frac{1}{\varkappa^2}+\frac{1}{\varkappa'^2}\Big)+\frac{4m^2(2E^2-m^2)}{\varkappa\varkappa'}+4\Big]\Big\}, \quad (2.10)$$

$$h \equiv \omega^2\sin^2\vartheta_k\cos\varphi\cos[\varphi-\arccos(\xi_1\xi_2)]. \quad (2.11)$$

After integrating over the azimuthal photon emission angle φ, we obtain the contribution made to the differential cross section with respect to the angle between the electron and the photon:

$$\frac{d^2\sigma_e'(\xi_1,\xi_2)}{d\cos\vartheta_k d\omega} = \frac{\alpha^3\omega\beta_0}{6E^2\beta}\frac{2\mu^2+\Delta^2}{\Delta^4}(\xi_1\xi_2) \times\Big[(2m^4+m^2\omega^2\sin^2\vartheta_k)\Big(\frac{1}{\varkappa^2}+\frac{1}{\varkappa'^2}\Big) + \frac{2}{\varkappa\varkappa'}(2E^2-m^2)(\omega^2\sin^2\vartheta_k+2m^2)+4\Big]. \quad (2.12)$$

Integrating with respect to the photon emission angle φ_k we get

$$d\sigma_e'(\xi_1,\xi_2) = \frac{2\alpha^3d\omega\beta_0}{3E^2\beta\omega}\frac{2\mu^2+\Delta^2}{\Delta^4}(\xi_1\xi_2) \times\Big\{2m^2\Big[L\Big(1-\frac{m^2+\omega^2}{2E^2}\Big)-1\Big]-4\omega^2\Big\}. \quad (2.13)$$

Let us examine the contribution made by the polarization to photon emission by muons. The contribution to the differential cross section is written in the form

$$\frac{d^3\sigma_\mu'(\xi_1,\xi_2)}{d\Omega_k d\omega} = -\frac{\alpha^3\omega}{32\pi^2E^4\beta}\left\{\frac{m^2a_1}{2E^2}(\xi_1\xi_2) + \frac{a_1+\Lambda^2a_2}{\omega^2}\left[h + \frac{\varkappa\varkappa'}{2E^2}(\xi_1\xi_2)\right]\right\}. \quad (2.14)$$

Integrating over the azimuthal angle, we obtain the contribution to the differential cross section with respect to the angle between the electron and the photon

$$\frac{d^2\sigma_\mu'(\xi_1,\xi_2)}{d\cos\vartheta_k d\omega} = \frac{\alpha^3\omega}{16\pi E^4\beta}(\xi_1\xi_2) \times\left[\frac{m^2a_1}{2E^2} + \frac{1}{2}(a_1+\Lambda^2a_2)\left(\sin^2\vartheta_k + \frac{\varkappa\varkappa'}{\omega^2E^2}\right)\right]. \quad (2.15)$$

Integrating with respect to the photon emission angle, we get

$$d\sigma_\mu'(\xi_1,\xi_2) = \frac{m^2}{2E^2}\left(1+\frac{m^2}{2E^2}\right)^{-1}(\xi_1\xi_2)\,d\sigma_\mu, \quad (2.16)$$

where $d\sigma_\mu$ is the cross section for the emission of the photon in the annihilation of unpolarized particles and is given by (2.36) of[1]. The interference term in this case (annihilation into a pair of fermions) is equal to[1] 1).

3. ANNIHILATION INTO A PAIR OF SCALAR PARTICLES

Just as in the creation of a fermion pair, we shall consider only the contributions made to the cross section by the polarization. The contribution to the differential photon emission cross section from initially arbitrarily polarized particles, in the case of creation of a pair of scalar particles, can be expressed in terms of the corresponding contribution for the case of creation of a fermion pair:

$$\frac{d^3\sigma_e{}^s(\xi_1,\xi_2)}{d\Omega_k d\omega} = \frac{\Delta^2\beta_0}{4(\Delta^2+2\mu^2)}\frac{d^3\sigma_e(\xi_1,\xi_2)}{d\Omega_k d\omega}. \quad (3.1)$$

This relation is satisfied for contributions to the differential cross sections of photon emission with respect to the angle between the electron and photon and for contributions to integral cross sections, since Δ and β_0 do not depend on the angles.

Inasmuch as relation (3.1) is not violated when the polarization contribution is replaced by the corresponding cross section for photon emission by initially unpolarized particles, the integral cross section for the emission of a photon ($\omega < m$) by initially longitudinally-polarized particles has a form similar to (2.5):

$$d\sigma_e{}^s(s_-,s_+) = [1-(s_-s_+)(1-m^2/E^2)]d\sigma_e, \quad (3.2)$$

where $d\sigma_e$ is the cross section for the emission of a photon by initially unpolarized particles in the creation of a pair of scalar particles, and is given by expression (2.12) of[2].

The contribution to the differential cross section of photon emission by the final scalar particles can be expressed in terms of the corresponding contribution for the cross section of emission by final fermions

$$\frac{d^3\sigma_s(\xi_1,\xi_2)}{d\Omega_k d\omega} = -\frac{R}{4}\left\{\frac{d^3\sigma_\mu(\xi_1,\xi_2)}{d\Omega_k d\omega}\right\}, \quad (3.3)$$

where R denotes the substitution operation

$$a_1\to h_1, \quad a_2\to h_2. \quad (3.3')$$

Relations of the type (3.3) are satisfied also for polarization contributions to differential (with respect to the angle between the electron and the photon) and integral cross sections, since the quantities a_1, a_2, h_1, and h_2 do not depend on the angles. Inasmuch as in annihilation of unpolarized particles the cross sections for the photon emission by scalar particles are fermions are connected by a relation similar to (3.3), the contribution to the integral cross section of photon emission by scalar particles in the annihilation of longitudinal polarized particles is similar to (2.8):

$$d\sigma_s(s_-,s_+) = [1-(s_-s_+)(1-m^2/E^2)]d\sigma_s, \quad (3.4)$$

where $d\sigma_s$ is the integral cross section of photon emission by scalar particles in the annihilation of unpolarized particles and is given by (2.22) of[2]. We see that the final scalar particles, like the initial ones, emit only when the latter have different helicities. The interference term is equal to zero[2].

Thus, in the annihilation of the longitudinally-polarized particles with creation of a pair of scalar particles, the integral photon emission cross section is

$$d\sigma'(s_-,s_+) = [1-(s_-s_+)(1-m^2/E^2)]d\sigma, \quad (3.5)$$

where $d\sigma$ is given by (2.26) of[2].

In order to take into account the effect of the longitudinal (transverse) polarization of the initial particles on their emission in the case of particle-antiparticle pair production, with allowance of the form factors of the final pair, it is necessary to replace Z in (3.6) of[2] by the quantity Z″ (Z′) and Y in (3.7) of[2] by Y″ (Y′). We use here the notation

$$Z'' = Z + 2m^2(s_-s_+)Z_0'' = [1-(s_-s_+)(1-m^2/E^2)]Z,$$
$$Z' = Z + (\xi_1\xi_2)Z_0,$$
$$Y'' = Y + (s_-s_+)Y_0' = [1+(s_-s_+)(1-m^2/E^2)]Y,$$
$$Y' = Y + (\xi_1\xi_2)Y_0; \quad (3.6)$$

1)Expression (2.6) of [1] should contain the tensor $K_{2\nu\nu'}$ in lieu of $K_{2\nu'\nu}$.

Z is given by expression (2.11) of[2], Z_0'' is the expression in the curly brackets in (2.3), Z_0 the expression in the square brackets of (2.12), Y the expression (2.13) in[2], Y_0 the expression in the curly brackets of (2.13), and Y_0' the expression in the curly brackets of (2.4).

4. ANALYSIS OF INTEGRAL CROSS SECTIONS

Let us consider the case of annihilation into a pair of fermions. As already noted, in the annihilation of particles having different helicities the cross section for the emission of a photon ($\omega < m$) is approximately double the emission cross section in annihilation of unpolarized particles. The cross section for the emission of a photon in annihilation of particles with identical helicities is of the order of m^2/E^2 relative to the emission cross section in the annihilation of unpolarized particles ($\omega < m$). The cross sections for photon emission near the threshold of muon production and far from it can be readily obtained from expressions (2.5), (2.8), and (2.9) of the present paper and from the formulas of Sec. 3 of[1]. It must only be noted that formula (3.1) of[1] is incorrect. This formula was derived in[1] (with accuracy to terms of first order in ω/E) using the relation $\mu^2/E^2 \sim 1$ (near threshold), whereas it is actually necessary to use the relation $4\mu^2/\Delta^2 \sim 1$ (near threshold). Consequently, the cross section for the emission of a photon by initially unpolarized particles near threshold is given not by (3.1) of[1], but by

$$d\sigma_e{}^{th} = \frac{2\alpha^3 d\omega\beta_0}{E^2\omega}\left(\ln\frac{2E}{m} - \frac{1}{2}\right). \tag{4.1}$$

A factor $1 + m^2/2E^2$ was left out from expression (3.2) of[1]. When this factor is taken into account, the cross section for photon emission by muons near threshold becomes

$$d\sigma_\mu{}^{th} = \frac{4}{3}\frac{\alpha^3 d\omega}{E^2\omega}\beta_0{}^3\left(1 + \frac{m^2}{2E^2}\right). \tag{4.2}$$

It is seen from (2.13) and (2.16) that the contribution to the cross section from allowance for the transverse polarization of the initial particles is of the order of m^2/E^2 relative to the cross section for the emission of the photon by the corresponding particles in annihilation of unpolarized particles[1].

In the case of annihilation of transversely polarized particles, the cross sections near threshold are given by

$$d\sigma'^{th}_{e(\mu)}(\xi_1, \xi_2) = \left[1 + \frac{m^2}{2E^2}(\xi_1\xi_2)\right] d\sigma^{th}_{e(\mu)}, \tag{4.3}$$

where $d\sigma^{th}_{e(\mu)}$ is given by (4.1) (or (4.2)). The cross sections far from threshold are obtained by replacing $d\sigma^{th}_{e(\mu)}$ in (4.3) by expression (3.3) from[1] (or (3.4) of[1]).

In the case of production of scalar particles, the formulas for the cross sections are similar in limiting cases to the corresponding formulas for the case of production of fermions (except that now, of course, it is necessary to use in the formulas the expressions obtained for the cross sections in[2] 2)).

APPENDIX

In calculating of the cross sections of certain processes[1,2,14,15] by method of invariant integration of tensors, it is advantageous to calculate immediately the required contractions of different types of tensors (Compton, current, etc.) with second-rank tensors (metric tensors and tensors made up of products of vector components), without explicitly calculating the tensors themselves. This reduces greatly the volume of the calculation.

In[1] they calculated in explicit form the Compton tensor of the initial particles $M_e^{\nu\nu'}$ ((2.11)–(2.14) of[1]), which was then contracted with the tensors $g_{\nu'}$ and $k_\nu k_{\nu'}$, and these contractions are used to obtain the cross sections. At the same time, to find the contractions it is sufficient to have the structural form (2.7)[1] of the tensor $M_e^{\nu\nu'}$, viz., to contract the tensor $M_e^{\nu\nu'}$ (with the tensors $g_{\nu\nu'}$ and $k_\nu k_{\nu'}$ and then calculate the contractions (scalars). Indeed, it is easy to find the scalar directly than to calculate a second rank tensor, for example $\mathrm{Sp}\,\gamma^\nu \hat{a}\hat{b}\hat{c}\hat{d}\gamma^{\nu'}$, and then contract it with the tensors $g_{\nu\nu'}$ and $k_\nu k_{\nu'}$ and moreover sum fifteen terms for each case. The scalars can be obtained directly:

$$(\mathrm{Sp}\,\gamma^\nu\hat{a}\hat{b}\hat{c}\hat{d}\gamma^{\nu'})\,g_{\nu\nu'} = 4\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{d};$$
$$(\mathrm{Sp}\,\gamma^\nu\hat{a}\hat{b}\hat{c}\hat{d}\gamma^{\nu'})\,k_\nu k_{\nu'} = k^2\,\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{d}; \tag{A.1}$$

$$(\mathrm{Sp}\,\gamma^\nu\hat{a}\hat{b}\hat{c}\gamma^{\nu'}\hat{d})\,g_{\nu\nu'} = -2\,\mathrm{Sp}\,\hat{d}\hat{a}\hat{b}\hat{c},$$
$$(\mathrm{Sp}\,\gamma^\nu\hat{a}\hat{b}\hat{c}\gamma^{\nu'}\hat{d})\,k_\nu k_{\nu'} = 2\,(kd)\,\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{k} - k^2\,\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{d}; \tag{A.2}$$

$$(\mathrm{Sp}\,\gamma^\nu\hat{a}\hat{b}\gamma^{\nu'}\hat{c}\hat{d})\,g_{\nu\nu'} = 16\,(ab)\,(cd),$$
$$(\mathrm{Sp}\,\gamma^\nu\hat{a}\hat{b}\gamma^{\nu'}\hat{c}\hat{d})\,k_\nu k_{\nu'} = 2\,(ak)\,\mathrm{Sp}\,\hat{b}\hat{k}\hat{c}\hat{d}$$
$$-\,2\,(bk)\,\mathrm{Sp}\,\hat{a}\hat{k}\hat{c}\hat{d} + k^2\,\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{d}. \tag{A.3}$$

In the paper of Baĭer and Galitskiĭ[14], besides contracting the tensor $N_{1\mu\nu}$ with the tensor $g_{\mu\nu}$, it is in fact necessary to find its contraction only

---

2)Formulas (2.27) and (2.28) of [2] are obtained by using near threshold the relation $4\mu^2/\Delta^2 \sim 1$ and not $\mu^2/E^2 \sim 1$ as in [2].

with a tensor of the type $p_\mu n_\nu + p_\nu n_\mu$. In this case the use of this procedure is no less effective.

We present an example:

$$(\mathrm{Sp}\,\gamma^\mu \hat{a}\hat{b}\hat{c}\hat{d}\gamma^\nu)(p_\mu n_\nu + p_\nu n_\mu) = 2(pn)\,\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{d}, \quad (A.4)$$

$$(\mathrm{Sp}\,\gamma^\mu \hat{a}\hat{b}\hat{c}\gamma^\nu\hat{d})(p_\mu n_\nu + p_\nu n_\mu) = 2(dn)\,\mathrm{Sp}\,\hat{p}\hat{a}\hat{b}\hat{c} + 2(dp)\,\mathrm{Sp}\,\hat{n}\hat{a}\hat{b}\hat{c} - 2(pn)\,\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{d}, \quad (A.5)$$

$$(\mathrm{Sp}\,\gamma^\mu \hat{a}\hat{b}\gamma^\nu\hat{c}\hat{d})(p_\mu n_\nu + p_\nu n_\mu) = 2(ap)\,\mathrm{Sp}\,\hat{b}\hat{n}\hat{c}\hat{d} + 2(an)\,\mathrm{Sp}\,\hat{b}\hat{p}\hat{c}\hat{d} - 2(bp)\,\mathrm{Sp}\,\hat{a}\hat{n}\hat{c}\hat{d} - 2(bn)\,\mathrm{Sp}\,\hat{a}\hat{p}\hat{c}\hat{d} + 2(pn)\,\mathrm{Sp}\,\hat{a}\hat{b}\hat{c}\hat{d}. \quad (A.6)$$

Thus, the employed method makes it possible to avoid summation of thirty terms, of which twenty-four vanish in the case of (A.4), and the remaining six yield only three different terms. With the aid of our device this result is obtained automatically.

[1] V. N. Baĭer and V. A. Khoze, JETP 48, 946 (1965), Soviet Phys. JETP 21, 629 (1965).

[2] V. N. Baĭer and V. A. Khoze, JETP 48, 1708 (1965), Soviet Phys. JETP 21, 1145 (1965).

[3] Ya. B. Zeldovich, JETP 41, 912 (1961), Soviet Phys. JETP 14, 651 (1962).

[4] V. N. Baĭer and V. S. Fadin, DAN SSSR 161, 74 (1965), Soviet Phys. Doklady 10, 204 (1965).

[5] Yu. G. Shaknazaryan, JETP 47, 1503 (1964), Soviet Phys. JETP 20, 1010 (1965).

[6] I. G. Dzhafarov, YaF 1, 665 (1965), Soviet JNP 1, 477 (1965).

[7] A. A. Tkachenko, JETP 44, 1668 (1963), Soviet Phys. JETP 17, 1123 (1963).

[8] A. A. Sokolov, Yu. P. Ivanov, and D. V. Gal'tsov, YaF 1, 507 (1965), Soviet JNP 1, 361 (1965).

[9] F. S. Sadykhov, I. G. Dzhafarov, and R. Sh. Yakh'yaev, JETP 47, 1463 (1964), Soviet Phys. JETP 20, 984 (1965).

[10] A. A. Sokolov, B. K. Kerimov, F. S. Sadykhov, and R. Sh. Yakh'yaev, DAN SSSR 161, 1317 (1965), Soviet Phys. Doklady 10, 353 (1965).

[11] M. Cini and B. Touschek, Nuovo Cimento 7, 422 (1958).

[12] S. K. Bose, A. Gamba, and E. C. G. Sudarshan, Phys. Rev. 113, 1661 (1959).

[13] A. A. Sokolov, B. K. Kerimov, I. G. Dzhafarov, and R. Sh. Yakh'yaev, DAN SSSR 166, 337 (1966), Soviet Phys. Doklady 11, 57 (1966).

[14] V. N. Baĭer and V. M. Galitskii, JETP 49, 661 (1965), Soviet Phys. JETP 22, 459 (1966).

[15] V. N. Baĭer, V. S. Fadin, and V. A. Khoze, JETP 50, 156 (1966), Soviet Phys. JETP 23, 104 (1966).

Translated by J. G. Adashko
149