SHOWERS PRODUCED IN LEAD BY 0.1 TO 1 BeV PHOTONS

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The characteristics of showers produced in lead by 0.1, 0.2, 0.3, 0.5, and 1 BeV photons have been determined on the basis of experimental data on the electron distributions of showers produced in lead by electrons.

1. INTRODUCTION

THE experimental study of showers produced by photons of a measured energy (hundreds of MeV) in materials with high Z permits us to obtain information necessary both for constructing a detailed theory of the electromagnetic cascade and for solving a number of complex technical problems of experimental physics (determination of γ-ray detector efficiency, of the resolution of total-absorption spectrometers, and so forth). Shower calculations have been made in this energy region by the method of moments[1] and the Monte Carlo method.[2] In the first case the averaged characteristics of a cascade were calculated over a wide range of secondary particle energies. In the second case the probability characteristics of the showers were also determined; however, the data were obtained for shower particle energies greater than 10 MeV, which is considerably above the detection threshold for γ rays in detectors.

The experimental study of showers produced by photons encounters considerable technical difficulties associated with the necessity of using a monoenergetic γ-ray beam and detecting showers produced by individual γ rays.[3] The experimental studies made up to the present time have been able to provide information only on the averaged characteristics of the showers.[4]

The purpose of the present work was to determine the characteristics of showers produced in lead by photons with energy E_0 = 0.1—1 BeV, by a theoretical means based on experimental data on showers produced by electrons. Studies of electron-produced showers involve less technical difficulty than showers produced by photons, and in the last few years considerable experimental information has been accumulated on the subject.[5-10]

In the energy region above 100 MeV the interaction of photons with atoms of lead reduces to the conversion of the photon to an electron-positron pair,[1)] a process whose characteristics are known with high accuracy, and the subsequent passage of the electrons[2)] through matter. In this case the pair moves practically in the same direction as the primary photon. The average separation angle of the pair $\theta \approx m_e c^2/E$ is a fraction of degree and is two orders of magnitude smaller than the multiple scattering angle of the pair components in a radiation length.

2. CALCULATION OF THE SHOWERS

We will designate by $V_K(E_e, E, t')$ the probability that in a shower produced by an electron with energy E_e, K electrons with energy greater than E will be produced at a depth t′. The probability of interest $W_N(E_0, E, t)$ for production of N electrons at a depth t in a shower produced by a photon with energy E_0 is related to $V_K(E_e, E, t')$ by the integral relation

$$W_N(E_0, E, t) = \int_0^t dt' \exp[(t'-t)\sigma(E_0)] \int_0^{E_0} dE_e \varphi(E_0, E_e) \times \sum_{K=0}^{N} V_K(E_e, E, t') V_{N-K}(E_0 - E_e, E, t'). \quad (1)$$

Here $\sigma(E_0)$ is the cross section for absorption of photons with energy E_0 in the material, and $\varphi(E_0, E_e)$ is the distribution function of the pair components in energy.

In computing the absorption cross sections we used data on the cross section for pair production in lead.[11-16] For the energy range E_0 = 100—200 MeV where the Compton scattering cross sec-

1)The fraction of Compton scattering and photoelectric effect is only 2% for E_0 = 100 MeV and drops rapidly with increasing energy.

2)We will not distinguish between electrons and positrons in the shower.

Values of $W_N(E_0, E = 1 \text{ MeV}, t)$

W_N	t, g/cm²									
	5	10	15	20	25	30	35	40	45	50
E_0=100 MeV										
W_0	0.638	0.467	0,449	0.520	0,610	0.701	0,772	0.834	0.882	0.913
W_1	0,084	0.207	0.243	0.235	0,202	0.162	0.126	0.098	0,077	0.060
W_2	0.217	0.220	0,215	0,177	0.139	0.102	0.084	0.055	0,034	0.022
W_3	0,031	0.069	0.064	0,047	0,037	0.020	0.013	0.009	0,006	0,004
W_4	0,019	0,032	0,025	0.017	0,011	0.007	0.004	0.003	0.002	0,001
W_5	0,002	0,007	0,005	0.003	0.002	0,001	0.001	0,001	0.000	0,000
E_0=200 MeV										
W_0	0.601	0,378	0,283	0,278	0.337	0,425	0.524	0,614	0,710	0.772
W_1	0,053	0,138	0,200	0.243	0.251	0.235	0,210	0.186	0.165	0.152
W_2	0,255	0.278	0.276	0.266	0,241	0.211	0,170	0,122	0.078	0,047
W_3	0,054	0,107	0,132	0,126	0.110	0,091	0.072	0.061	0,038	0.025
W_4	0,029	0.059	0.069	0.059	0,043	0,027	0,017	0,010	0.006	0.004
W_5	0,007	0.024	0.026	0.019	0,012	0,007	0.005	0,003	0.002	0,001
$W_{>5}$	0,003	0.012	0.015	0.009	0.006	0,003	0,002	0.001	0.001	0.000
E_0=300 MeV										
W_0	0,583	0.347	0,226	0.185	0.202	0,261	0.341	0.434	0,531	0,614
W_1	0.035	0.092	0.140	0.176	0,207	0.223	0,224	0.225	0,222	0.215
W_2	0.257	0,269	0.261	0,254	0.254	0,236	0.217	0,178	0.134	0,094
W_3	0,070	0.134	0.167	0.191	0,175	0.163	0.140	0,115	0,087	0,061
W_4	0.038	0,083	0,107	0.108	0,095	0,070	0.046	0,028	0,016	0,009
W_5	0,012	0,042	0,055	0,051	0.041	0,030	0.020	0,012	0,007	0,005
$W_{>5}$	0.006	0,034	0.044	0,036	0.027	0,018	0,011	0,007	0.004	0.002
E_0=500 MeV										
W_0	0.572	0,331	0,197	0.130	0.112	0,131	0,178	0,245	0,326	0,420
W_1	0.022	0.052	0,066	0.080	0,102	0,134	0,172	0,202	0.224	0.234
W_2	0.252	0.246	0,205	0,194	0,200	0.211	0.220	0,213	0.195	0,167
W_3	0.076	0,140	0,162	0.176	0.184	0.183	0.176	0.156	0,130	0,100
W_4	0,049	0.093	0,140	0.155	0.155	0,139	0.115	0,083	0,061	0,040
W_5	0,017	0.067	0,101	0.116	0,112	0,096	0.076	0,055	0.037	0,023
W_6	0.008	0.043	0.067	0.076	0,072	0.059	0,043	0,029	0,017	0.010
W_7	0.003	0.021	0.037	0,042	0,037	0.025	0.016	0,009	0.005	0,003
W_8	0,001	0.010	0.016	0,019	0,018	0.015	0,010	0,006	0,004	0.002
W_9	0,000	0.004	0.007	0,008	0,006	0.005	0.004	0,002	0.001	0.001
W_{10}	0,000	0.001	0,002	0,003	0,002	0,001	0,001	0.001	0,000	0,000
E_0=1000 MeV										
W_0	0,563	0,317	0,179	0.103	0.064	0.049	0,051	0,069	0.101	0.154
W_1	0,012	0,020	0.021	0.021	0.026	0.040	0,062	0.094	0.132	0.170
W_2	0,243	0,190	0,135	0,098	0,084	0.086	0,107	0,131	0.162	0.189
W_3	0,080	0.112	0,111	0,099	0.099	0.117	0,137	0.156	0,161	0.154
W_4	0,056	0,106	0.121	0,121	0,125	0,134	0,144	0.148	0,136	0,110
W_5	0,024	0.077	0,109	0,124	0.137	0.144	0.141	0.128	0,106	0,084
W_6	0.012	0.060	0,100	0,124	0.135	0.131	0.118	0.094	0,077	0.054
W_7	0.006	0,043	0,082	0,106	0.113	0,108	0.091	0,074	0,055	0.039
W_8	0,003	0.043	0.058	0,080	0.087	0,080	0.063	0.048	0.033	0.020
W_9	0,001	0,026	0.038	0.054	0,058	0.051	0.040	0,028	0,018	0,010
W_{10}	0.001	0.008	0.023	0.034	0,035	0,031	0,024	0.017	0,011	0,006
$W_{>10}$	0.001	0.007	0.023	0.036	0.037	0,030	0.022	0.014	0,009	0.005

tion amounts to 1—2% of the pair production cross section, we made an additional calculation taking into account Compton scattering. At higher energies Compton scattering was not included. The values of photon absorption cross sections used in the calculation agree with the experimentally determined values[12-14,17] within 1.5%.

Values of $\varphi(E_0, E_e)$ were determined according to Bethe and Heitler.[18] The corrections for the inaccuracy due to use of the Born approximation are small,[12] as is confirmed by the experimental data.[12,19,20]

In calculating the probabilities W_N we used the experimental data[6,8,10] for values of $V_K(E_e, E, t)$ obtained in the energy region E_e = 45—1000 MeV for a cutoff energy E = 1 MeV.[5] The integral in Eq. (1) was evaluated by successive summation with a step of 1 g/cm² in t′.

3. FLUCTUATIONS OF THE NUMBER OF ELECTRONS

The values of the probability $W_N(E_0, E = 1 \text{ MeV}, t)$ obtained as the result of the calculation are listed in the table. The errors in the probabilities obtained are determined mainly by errors in the experimental data on fluctuations in showers produced by electrons. For N = 0 these amount to 1% at small depths t and increase to 3% with increasing t. For N = 1—5 the errors are 5—10% at $E_0 \leq 300$ MeV. For $E_0 \geq 500$ MeV the errors are 5—10% for N = 1—5 and increase to 10—20% for N = 5—10.

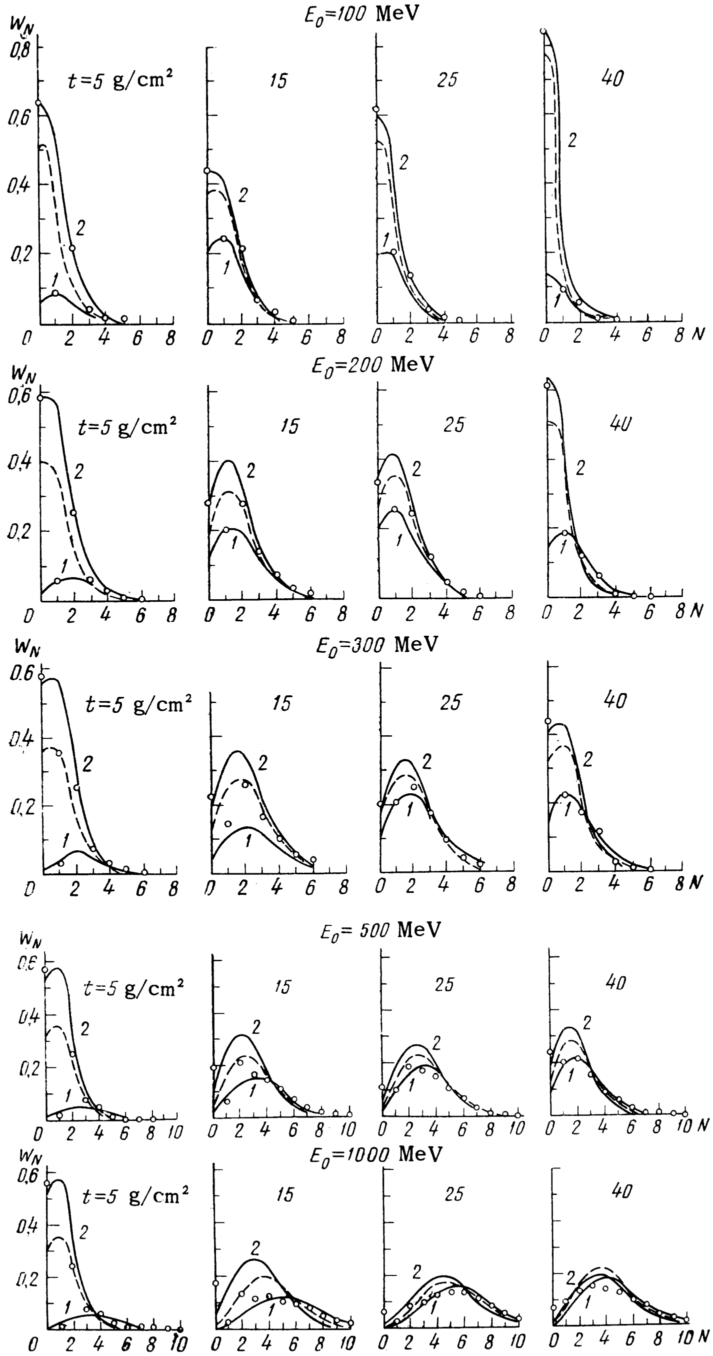

FIG. 1. Probabilities $W_N(E_0, E = 1\text{ MeV}, t)$. Points—calculated probabilities. For ease in comparison of the theoretical results with the approximating distributions, we have shown the generalized distributions (2) in which the factoral of the integer N is replaced by the generalized factorial—the gamma function $\Gamma(N + 1)$. Curves 1—distribution (2) corresponding to odd N, 2—even N. Dashed curves—generalized Poisson distributions.

As can be seen from Fig. 1, which shows the values of W_N as a function of N, in the region up to the peak of the cascade curves we can see a distinct "even-odd" effect[21]—an enhancement of the probabilities for appearance of showers with an even number of electrons. This effect, which is due to the fact that in the photon conversion the electrons are produced in pairs, is observed also in the case of showers produced by electrons, where showers with an odd number of electrons N predominate for small values of t. With increasing t the energy of the shower is rapidly subdivided, the production of high-energy electron pairs loses its predominant role, and the even-odd disappears.

To approximate the functions obtained we used a distribution[10] describing the shower by means of two Poisson distributions (for even and odd N):

$$W_N = \begin{cases} a\alpha_1^N/N!\,\mathrm{sh}\,\alpha_1, & N - \text{odd} \\ (1-a)\,\alpha_2^N/N!\,\mathrm{ch}\,\alpha_2, & N - \text{even} \end{cases} . \quad (2)$$

Here α_1 and α_2 are related to the average values $\overline{N}_1$ and $\overline{N}_2$ of the number of electrons N in showers with odd and even numbers of electrons:

$$\overline{N}_1 = \alpha_1\,\mathrm{cth}\,\alpha_1, \quad \overline{N}_2 = \alpha_2\,\mathrm{th}\,\alpha_2. \quad (3)$$

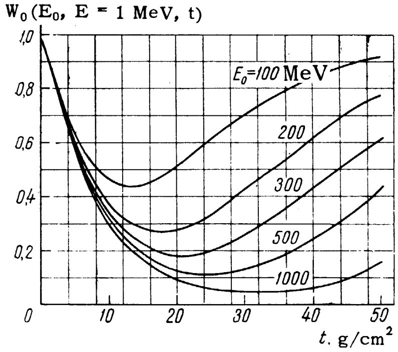

FIG. 2. The probability $W_0(E_0, E = 1$ MeV, t).

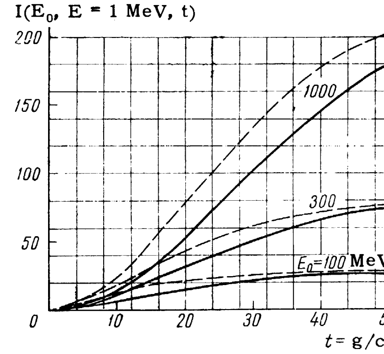

FIG. 4. Integral cascade curves. The designations are the same as in Fig. 3.

For energies $E_0 < 500$ MeV distribution (2) accurately describes the fluctuations of showers at all depths t. At higher energies the agreement becomes less satisfactory, particularly for even N.

Figure 2 shows values, calculated for various energies, of the probability $W_0(E_0, E, t)$ that electrons with energy $E > 1$ MeV are absent in the shower at depth t. A knowledge of this quantity enables one to determine the efficiency for detection of photons by γ telescopes using a lead converter, and also to choose the optimum converter thickness.

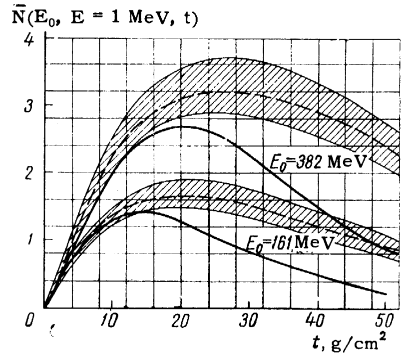

FIG. 5. Cascade curves obtained in the present work (solid lines) and calculated by the method of moments [1] (dashed curves). The shaded region corresponds to the uncertainty in the values of E, which is ± 0.5 MeV.

4. CASCADE CURVES

The distributions of W_N obtained allow us to determine the cascade curves $\overline{N}(E_0, E, t)$—the average number of electrons in the shower as a function of the depth t. These curves are shown in Fig. 3, together with cascade curves for showers produced by electrons. The errors in the values of $\overline{N}$ obtained are 3—5%.

The development of showers produced by photons, in contrast to showers produced by electrons, is characterized by a smaller height and greater width of the peak in the cascade curve, which shifts towards higher values of t by roughly one radiation length. The integral cascade curves

$$I(E_0, E, t) = \int_0^t \overline{N}(E_0, E, t')\,dt', \qquad (4)$$

which determine the energy loss of the shower in the material, also differ for showers initiated by photons and electrons (Fig. 4).

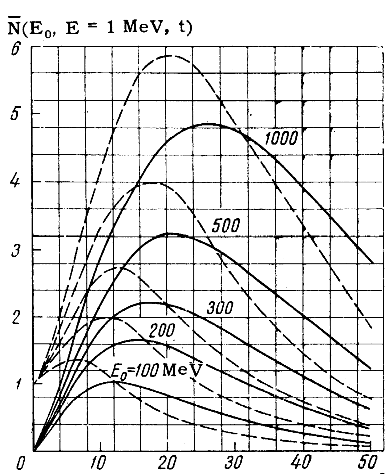

FIG. 3. Cascade curves for showers produced by photons (solid curves) and electrons (dashed curves).

The cascade curves found in the present work have been compared in Fig. 5 with curves obtained by the method of moments.[1] The radiation length and critical energy for lead have been taken as 6.4 g/cm^2 and 7.4 MeV.[22] The comparison is made for primary electron energies of 161 and 382 MeV. The shaded region in Fig. 5 is the corridor of values corresponding to the uncertainty in the value of the cutoff energy E, which in our case is 1.0 ± 0.5 MeV. Agreement between the cascade curves occurs only in the region of small t.

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1 I. P. Ivanenko, JETP **32**, 491 (1957), Soviet Phys. JETP 5, 413 (1957).

[2] D. F. Crawford and H. Messel, Phys. Rev. **128**, 2352 (1962).

[3] V. P. Agafonov, B. B. Govorkov, S. P. Denisov, and E. V. Minarik, PTÉ, **5**, 47 (1962).

[4] W. Blocker, R. W. Kenney, and W. K. H. Panofsky, Phys. Rev. **79**, 419 (1950).

[5] O. A. Zaĭmidoroga, Yu. D. Prokoshkin, and V. M. Tsupko-Sitnikov, JETP **51**, 749 (1966), Soviet Phys. JETP **24**, 498 (1967).

[6] H. Lengeler, W. Tejessy, and M. Deutschmann, Z. Physik **175**, 283 (1963).

[7] E. E. Beklin and J. A. Earl, Phys. Rev. **136**, B237 (1964).

[8] H. Thom, Phys. Rev. **136**, B447 (1964).

[9] C. A. Heusch and C. Y. Prescott, Phys. Rev. **135**, B772 (1964).

[10] O. A. Zaĭmidoroga, Yu. D. Prokoshkin, and V. M. Tsupko-Sitnikov, JETP **52**, 79 (1967), Soviet Phys. JETP **25**, 51 (1967).

[11] H. Davies, H. A. Bethe, and L. C. Maximon, Phys. Rev. **93**, 788 (1954).

[12] E. Malamud, Phys. Rev. **115**, 687 (1959).

[13] J. L. Lawson, Phys. Rev. **75**, 433 (1949).

[14] J. W. DeWire, A. Ashkin, and L. A. Beach, Phys. Rev. **83**, 505 (1951).

[15] A. Sørenssen, Nuovo Cimento **38**, 745 (1965).

[16] G. W. Grodstein, X-ray Attenuation Coefficients from 10 keV to 100 MeV, NBS Cir. 583, 1957.

[17] J. D. Anderson, R. W. Kenney, and C. A. McDonald, Jr., Phys. Rev. **102**, 1626 (1956).

[18] H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).

[19] J. W. DeWire and L. A. Beach, Phys. Rev. **83**, 476 (1951).

[20] C. R. Emigh, Phys. Rev. **86**, 1028 (1952).

[21] R. R. Wilson, Phys. Rev. **86**, 261 (1952).

[22] O. I. Dovzhenko and A. A. Pomanskiĭ, JETP **45**, 268 (1965), Soviet Phys. JETP **18**, 187 (1964).

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