INVESTIGATION OF PLASMA TURBULENCE DUE TO THE ION-ACOUSTIC INSTABILITY

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We have investigated the turbulent plasma state and diffusion across a magnetic field in the presence of an ion-acoustic instability caused by current flow. The amplitude of the oscillations in density is found to be comparable with the density itself $\tilde{n}/n \sim 1$. The plasma is in a turbulent state and the phase correlation is maintained for 10—15 cycles. The diffusion coefficient increases sharply when the instability is excited. At amplitudes such that $\tilde{n}/n \sim 1$ the diffusion coefficient is inversely proportional to the magnetic field, being $(1.5 \pm 0.3) \times 10^3\ \mathrm{cm^2/sec}$ for $H = 1000$ Oe; this value is close to the Bohm value $2.2 \times 10^3\ \mathrm{cm^2/sec}$.

It has been shown earlier[1-3] that a potassium plasma with an inhomogeneous density distribution is subject to a drift instability in the absence of a current. The nature of the sheath at the surface of the ionizing plate is an important factor in the development of this instability. The instability is observed only in the presence of an ion sheath. In the presence of an electron sheath the azimuthal perturbations are evidently inhibited by an effect related to the Simon short-circuit effect[3] and the instability does not develop.

An instability that is excited in the presence of a current flow and an ion sheath has been described earlier.[4] It is found that in this case the drift branch is still observed and that at the critical velocity of the electrons with respect to the ions $U_c \sim 2 \times 10^6$ cm/sec a new ion-acoustic branch appears; in the latter case the oscillation frequency is independent of the magnetic field and varies inversely with the length of the system, while the phase velocity is essentially that of the ion-acoustic wave. In the presence of an electron sheath, in which case the drift instability is not present, only the ion-acoustic branch is excited.

It is the purpose of the present work to investigate the plasma state and diffusion across the magnetic field in the presence of the ion-acoustic instability.

It is convenient to describe the plasma state in terms of the space-time correlation function $\langle \varphi(\mathbf{r}, t)\varphi(0, 0)\rangle$ which is related to the so-called spectral density function $I_{\mathbf{k}\omega}$ by the following expression:[5]

$$\langle \varphi(\mathbf{r}, t)\varphi(0,0)\rangle = \int\int e^{-i\omega t + i\mathbf{k}\mathbf{r}} I_{\mathbf{k}\omega}\, d\mathbf{k}\, d\omega.$$

The space-time correlation function can be determined by the time correlation function as measured at different points in space. The normalized correlation function $\rho(\tau)$ is given by[6,7]

$$\rho(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T u(t)\vartheta(t+\tau)\, dt/\bar{u}^2\bar{\vartheta}^2,$$

where $u(t)$ and $\vartheta(t)$ are the random signals that are being investigated, for instance the oscillations of potential or density as measured by a probe; the bar denotes a time average over the interval $T \to \infty$.

The experimental measurement of the quantity $\rho(\tau)$ is extremely difficult technically; it is sufficient, however, to measure the so-called polarity correlation function $R(\tau)$; this function relates the signs of two functions $u(t)$ and $\vartheta(t)$ rather than the functions themselves:[8]

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \operatorname{sign} u(t) \operatorname{sign} \vartheta(t+\tau)\, dt,$$

where

$$\operatorname{sign} x \begin{cases} 1 \text{ for } x > 0 \\ -1 \text{ for } x < 0 \end{cases}.$$

If a normal distribution of amplitudes obtains, the quantity $R(\tau)$ is related to $\rho(\tau)$ by

$$R(\tau) = 2\pi^{-1} \arcsin \rho(\tau).$$

In practice, the equivalent polarity correlation function $F(\tau)$ is given by

$$F(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \theta[u(t)]\,\theta[\vartheta(t+\tau)]\, dt,$$

where

$$\theta(x) = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x < 0 \end{cases}.$$

The polarity function is related to $\rho(\tau)$ and $R(\tau)$ by the expressions[7]

$$\rho(\tau) = -\cos 2\pi F(\tau), \quad R(\tau) = 4F(\tau) - 1.$$

It will be shown below that to obtain qualitative results it is sufficient to carry out a qualitative correlation analysis rather than the measurement of the correlation functions themselves.[2,9]

If the plasma is not in a turbulent state and the waves being studied reduce to a stationary wave, one should observe a regular pattern which is maintained in the course of time. If the plasma is in a turbulent state the oscillations are random in nature. Under these conditions we may define a weakly turbulent state as one in which the interaction between waves is small. As a consequence, a given frequency is associated with a single wave number (the spectral density function exhibits a delta-function nature) while the "lifetime" of the wave packets[9] (the time between significant phase changes) is large compared with the oscillation period. Thus, the phase correlation for waves at a given point will be maintained over a large number of cycles. Similarly, the distance over which the phase correlation is maintained is large compared with the wavelength.

In the highly turbulent state the interaction between waves increases and the lifetime is reduced, becoming comparable with the oscillation period; the spectral density distribution becomes smeared out (with respect to $\mathbf{k}$) and a given $\mathbf{k}$ is now associated with an ensemble of values of ω. In this case the correlation time at a given point becomes small, as does the correlation distance.

In the present work we investigated the plasma state and transverse diffusion in the presence of the ion-acoustic instability under conditions for which the oscillation amplitude is large: $\tilde{n}/n \sim 1$ ($\tilde{n}$ and n are the plasma density and its fluctuating component respectively). A qualitative correlation analysis has been carried out and the time correlation function (polarity) has been measured; it is found that the plasma is in a turbulent state and that the phase correlation of the oscillations is maintained for approximately 10–15 cycles. It is shown that the plasma diffusion coefficient across the magnetic field increases sharply in the presence of the instability, reaching a value of $(1.5 \pm 0.3) \times 10^3\,\text{cm}^2/\text{sec}$ (H = 1000 Oe) when $\tilde{n}/n \sim 1$; this value is two or three orders of magnitude greater than the classical value. The diffusion coefficient is inversely proportional to the magnetic field and is close to the Bohm value in absolute magnitude. We note that an increase in the diffusion coefficient due to the excitation of a current-driven instability in an alkali plasma has been reported earlier.[10,11]

DESCRIPTION OF THE APPARATUS AND EXPERIMENTAL METHOD

This work has been carried out on an apparatus described earlier[12] shown in Fig. 1. The plasma is produced by thermal ionization of potassium on a tungsten plate (ionizer) of radius R = 2 cm which is heated to temperatures of approximately 2000° K. A second plate (electrode) is located at a distance L = 36 cm from the first but is not heated. The magnetic field, which is along the axis of the system, can be varied from 600 to 3500 Oe. The plasma density is a maximum at the axis and falls off in the radial direction; the experiments are carried out at densities of 10^9–10^{10} cm^{-3} and a residual pressure of $(2-8) \times 10^{-7}$ mm Hg. so that the plasma is highly ionized and collisionless. The plasma density and the amplitude of the density oscillations are measured with a Langmuir probe using the dc and ac components of the ion saturation current. The probe can be moved along the axis of the chamber and radially as well.

The oscillation spectrum is studied by means of a tuned selective voltmeter IUU-300 (bandwidth approximately 1 kHz) and a harmonic analyzer S5-3 (bandwidth approximately 200 Hz). We note that both devices have quadratic output detectors and thus make is possible to measure the effective amplitude, that is to say, these instruments measure the power spectrum characteristic of the random oscillations.

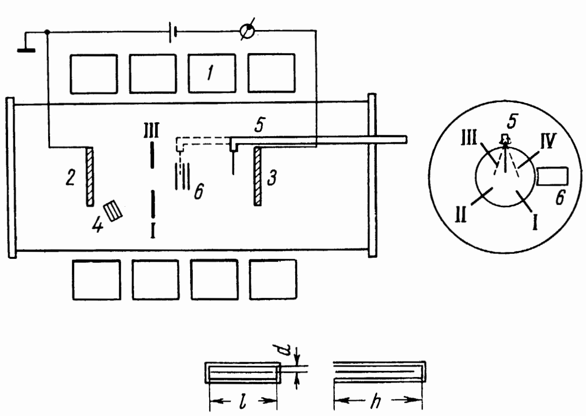

FIG. 1. Diagram of the apparatus: 1) solenoid; 2) ionizer, 3) electrode, 4) oven, 5) moveable probe, 6) diffusion measurement device (cf. section view below): l = 28 mm, d = 1.5 mm, h = 40 mm; I–IV) azimuthal probes.

The state of the plasma is investigated by a qualitative correlation analysis which is carried out as follows: the total signal from a probe or the signal at a given frequency are applied to a oscilloscope in the multiple trigger mode[2,9]. A selective amplifier is used to isolate the signal; this is a V6-2 amplifier, which has a bandwidth approximately 20% of the measured frequency, If the oscillations are coherent, a stationary pattern is observed on the oscilloscope; if the signal is noisy a smeared pattern is observed. In the weakly turbulent state the pattern can be maintained for many cycles, whereas in the highly turbulent state the pattern becomes random after a few cycles. By applying the signal to the oscilloscope in the single triggering mode it is possible to observe the change of amplitude and phase in time, that is to say, it is possible to observe the lifetime.

An automatic correlator is used to measure the correlation function;[14] this correlator measures the polarity correlation function $F(\tau)$. A block diagram of the apparatus is shown in Fig. 2. The delay time can be varied from 0 to 1500 μsec in steps of 20 μsec. The integration time is 500 msec. The amplitude of the input signal must lie within the limits 5—300 mV.

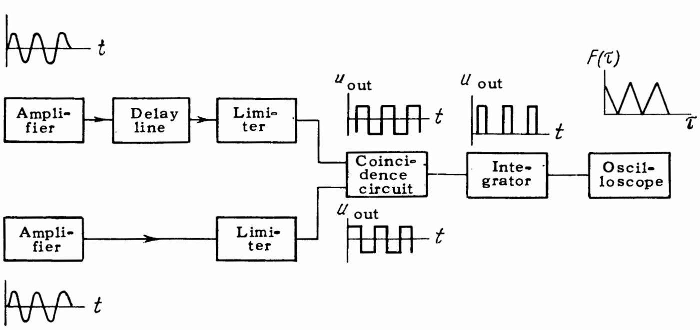

FIG. 2. Block diagram of the polarity coincidence correlator.

The diffusion across the magnetic field is measured from the transverse plasma flux.[13] The flux is measured by plates located at the boundaries of the plasma; a potential is applied between these plates (Fig. 1). The electric field is parallel to the magnetic field and is sufficient for complete separation of the charges so that the current in the plate circuit is determined by the plasma flux $(nv)_r$. Knowing the radial density radient one can then find the diffusion coefficient $D = (nv)_r (dn/dr)^{-1}$ We note that in working with this device one must carefully verify the absence of any spurious currents due to the possible effect of the measuring device on the lines of force of the magnetic field.

The diffusion coefficient is also estimated from the longitudinal density gradient (this method was used by A. Ivanov, E. Rakhimbabaev and V. Rusanov). We assume that the longitudinal drift velocity of the plasma is uniform over the cross-section of the column at a given z and that it is equal to the ion-thermal velocity.[15] Then, assuming that $n(r)$ is the same at different values of z and neglecting recombination, we find from the equation of continuity

$$(n_{z_1} - n_{z_2}) v_{Ti} S_{\perp\,\mathrm{eff}} = j_\perp S_\parallel .$$

Here, n_{z1} and n_{z2} represent the density at the points z_1 and z_2 respectively so that

$$n_{z_1} - n_{z_2} = \frac{dn}{dz}\Delta z; \qquad S_{\perp\,\mathrm{eff}} = \alpha\pi R^2,$$

where $\alpha < 1$ is a coefficient that takes account of the radial density distribution; $j_\perp = D_\perp\, dn/dr$ is the flux density across the magnetic field and $S_\parallel = 2\pi R \Delta z$. Thus,

$$D_\perp = \frac{\alpha R}{2} v_{Ti} \frac{dn/dz}{dn/dr}.$$

EXPERIMENTAL RESULTS

As already indicated, the experiments are carried out in the presence of an electron sheath at the surface of the ionizer. In this case, in the absence of current flow no individual frequencies are observed in the spectrum and the amplitude of the density fluctuations as measured with a broadband amplifier (total signal) is $(\tilde{n}/n)_{\mathrm{eff}} \sim 10^{-3}$.

When a current flow is initiated and the electron velocity with respect to the ions reaches a critical value $U_c \sim 2 \times 10^6$ cm/sec one observes that ion-acoustic oscillations are excited. In this case U_c is essentially the same as in the presence of an ion sheath. In making the transition from an ion sheath to an electron sheath in the presence of the excited instability, it is observed that the drift branch disappears while the ion-acoustic branch remains unchanged and that the final state is the same as that when one allows current flow in the presence of an electron sheath. The results of our investigation of ion-acoustic oscillations[4] apply to the case of an electron sheath. Thus, it may be assumed that a standing wave is established along the axis of the system; the fundamental mode is characterized by $\lambda \sim L$, the second by $L/2$ and so on, and the phase velocity is essentially the ion-acoustic velocity. No azimuthal component is observed under these conditions.

The excitation of the instability is of the class known as "hard" excitation, that is to say, as the value U_c is reached the amplitude increases abruptly. A characteristic oscillogram showing the development of the instability is given in Fig. 3 (a linearly increasing voltage is applied between electrodes). It is evident that the oscillation amplitude reaches a maximum value in 5—6 cycles whence it may be concluded that the growth rate of the instability is large (comparable with the frequency).

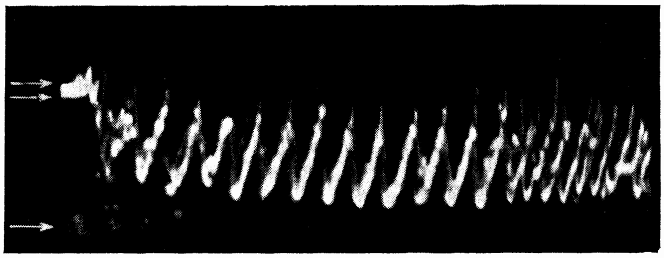

FIG. 3. Development of the instability. The first arrow indicates the zero level while the pair of arrows indicates the unperturbed density level. H = 1000 Oe and f = 5 kHz.

A typical oscillation spectrum is shown in Fig. 4. It is evident that the amplitude of the fundamental $(\tilde{n}/n)_{eff} \sim 0.25$. It is shown in [4] that the relation $\tilde{n}/n \approx e\tilde{\varphi}/T$ is maintained for the ion-acoustic oscillations. The total effective amplitude of the density oscillations is $(\tilde{n}/n)_{eff} \sim 0.5$—$0.6$.

In Fig. 5 we show oscillograms of the oscillations in density and indicate the position of the beam corresponding to zero density. It is clear that the density falls essentially to zero, that is to say, the total amplitude is such that $\tilde{n}/n \sim 1$. The oscillogram in Fig. 5 has been obtained in the single-triggering mode. It is evident that the oscillations are noisy. Oscillograms of this kind show that the phase becomes randomized in approximately 10 cycles.

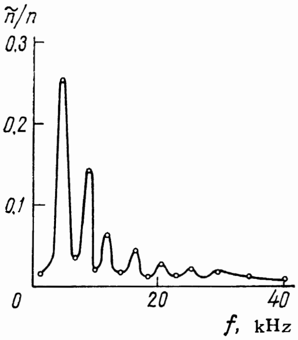

FIG. 4. Oscillation spectrum: $n = 5 \times 10^9$ cm^{-3}, H = 1000 Oe, the pass band is 0.2 kHz.

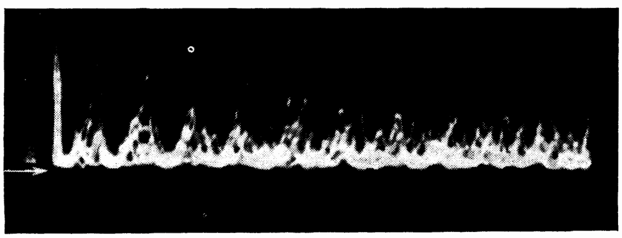

FIG. 5. Oscillogram of the density oscillations for the multiple-trigger mode. The arrow indicates the zero level of the density, $n = 3 \times 10^9$ cm^{-3}, H = 1000 Oe, the sweep speed is 200 μsec/cm.

In Fig. 6 we show oscillograms obtained with the single trigger mode. A case of sharp phase change can be noted. The lifetime is approximately 15 cycles.

In Fig. 7 we show the autocorrelation function (ACF) for the density oscillations as traced from the oscilloscope screen. For purposes of comparison we show the ACF for a sinusoidal signal. The ordinate scale for the ACF is determined by the amplitude of the ACF of the sinusoidal signal. It is evident that the amplitude of the ACF drops off, indicating a randomization of the oscillation phase. Unfortunately, the parameters of the device did not allow us to observe the correlation

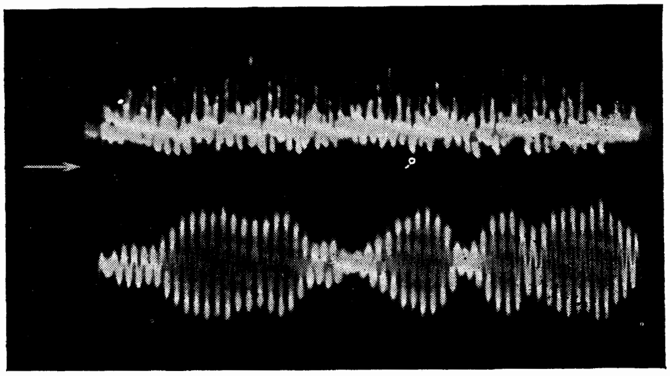

FIG. 6. Oscillogram of the density oscillations for the single trigger mode. The upper trace is the total signal (the arrow indicates the zero level of intensity); for the lower trace, f = 5 kHz (fundamental) $n = 3 \times 10^{-9}$ cm^{-3}, H = 1000 Oe, the sweep length is 1 msec/cm.

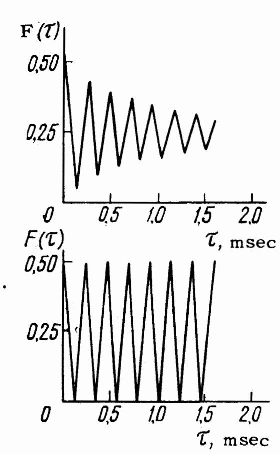

FIG. 7. Autocorrelation function. The upper curve is the total signal while the lower curve is a sinusoidal signal; $n = 5 \times 10^9$ cm^{-3} and H = 1000 Oe.

function for a long enough time interval to see complete decay. The correlation time has been estimated by linear extrapolation of the ACF to a factor of 10 below the initial value. In accordance with the results given above, this was found to be approximately 10 cycles.

Since these are long-wave oscillations ($\lambda \sim L$) one does not expect to see a change in the correlation function as a function of the distance between probes; indeed, the correlation function (ACF) is found to be essentially unchanged for probe separation distances up to 12 cm. We note that both the correlation function and the multiple-sweep mode record phase changes (including small phase changes) and spreading in **k** while the lifetime only characterizes the phase itself. The similarity of the correlation time as measured from the correlation function and the lifetime indicates that there is essentially no spread in **k**. Thus, we can assume that the plasma is in a weakly turbulent state in which the phase correlation is maintained for 10–15 cycles.

As indicated above, $(\tilde{n}/n)_{\text{eff}} \sim 10^{-3}$ in the absence of an instability. Under these conditions the diffusion coefficient as determined by the flux measuring device is 20 cm^2/sec when H = 1000 Oe. Since the classical diffusion coefficient for this case is approximately (1–10) cm^2/sec we may assume that the value that is found is determined by some spurious flux, that is to say, this is the lower limit of measurement of the diffusion coefficient.

When the instability is excited one observes, in addition to the growth in the amplitude of the oscillations, a sharp rise in the diffusion coefficient. Under these conditions the density at the axis is reduced and the flux to the measuring device is increased. Figure 8 shows that when the instability is excited the plasma is transported outward. Actually, at the axis of the column the oscillations cause a reduction in density whereas at r = 10 mm and especially at r = 25 mm (at the edge of the column) the density burst exceeds significantly the unperturbed level. In this case there is no phase shift between the bursts at azimuthal probes and the diffusion measurement devices located at angles of 180° with respect to each other, that is to say, the flux is symmetric in azimuth. No phase shift is observed in the bursts at different distances along the axis. It is evident from Fig. 9 that the flux measuring device also receives bursts which are correlated with the density maxima at the probe ($r \sim 15$ mm).

When the amplitude of the oscillations at the axis $\tilde{n}/n \sim 1$ the diffusion coefficient determined by the flux measurement device with H = 1000 Oe is $(1.5 \pm 0.3) \times 10^3$ cm^2/sec that is to say, approx-

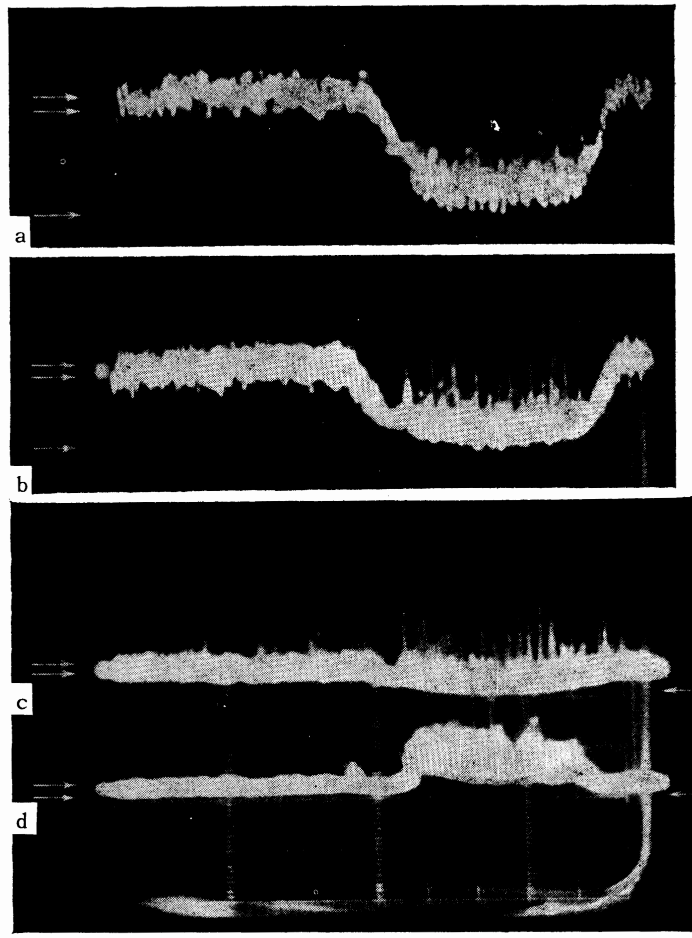

FIG. 8. Density oscillations: a) r = 0; b) r = 10 mm; c) r = 25 mm; d) the current in the diffusion measurement device. The single arrow indicates the zero level of the density while the double arrow indicates the unperturbed density level; H = 1000 Oe.

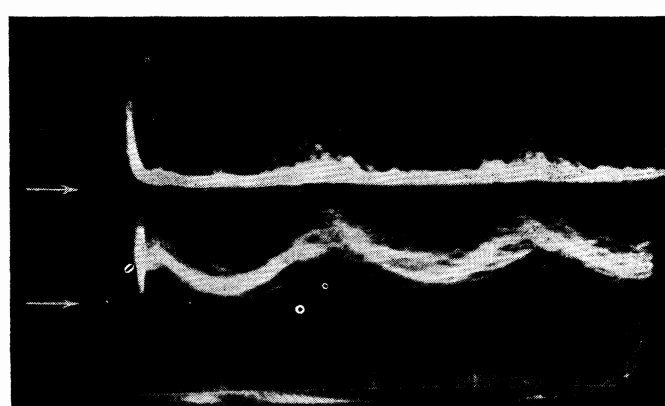

Fig. 9. Total signal (lower curve) and flux to the measurement device. The arrows indicate the zero levels for the density and the measured current. H = 1000 Oe.

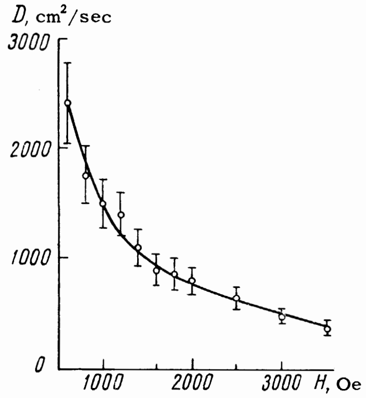

FIG. 10. The diffusion coefficient as a function of magnetic field.

imately the Bohm value[16] $D_B = \frac{1}{16} c T_e / eH = 2.2 \times 10^3$ cm²/sec (for $T_e = 0.35$ eV).[4] It is evident from Fig. 10 that the diffusion coefficient is inversely proportional to the magnetic field.

An estimate of the diffusion coefficient based on the longitudinal gradient for H = 1000 Oe yields a value of the same order of magnitude, $D \sim (4 \pm 0.8) \times 10^3$ cm²/sec. One expects that the value of D obtained this way will be too high since the value of the gradient is sensitive to recombination at the surface. Indeed, measurements carried out by means of a movable probe in the absence of all of the other measuring devices do yield a smaller value $D \sim (3 \pm 1) \times 10^3$ cm²/sec. Thus, the agreement between results is taken as satisfactory.

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