LASER MODE SYNCHRONIZATION BY DIELECTRIC CONSTANT MODULATION

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The radiation characteristics are investigated for a gas laser where the dielectric constant of an electro-optical crystal mounted inside the resonator is modulated at a frequency close to that of intermode beats. It is shown that, depending on the modulating voltage, the mode distribution of radiation intensity is determined chiefly either by the modulating voltage or by the properties of the laser's active medium. The first case was considered by Harris and others,[1-3] and the second case is considered in the present paper. Mode interaction due to Q switching of the laser resonator and mode interaction to dielectric constant modulation are compared.

In several recent investigations [1-3] of longitudinal laser mode interactions the dielectric constant of an element placed inside of the resonator was modulated at a frequency close to that of intermode beats. An especially careful theoretical and experimental study was made of laser operation in which the nonlinear properties of the amplifying medium influenced the generation of a given mode very much less than did the interaction of neighboring modes. It was shown that as a result of energy redistribution among the modes laser emission assumes the form of a frequency-modulated signal. This type of operation is interesting because of the possibility that a frequency-modulated signal could be converted into a monochromatic signal. The deviation of the radiation frequency is inversely proportional to the frequency difference between the modulating signal frequency and intermode beats. When this frequency separation is reduced, intermode beats are synchronized as in the case of resonator Q switching. We shall designate this as the strong-modulation operation of a laser.

Another limiting case occurs when, as the dielectric constant is modulated, a given mode interacts much less with its neighbors than with the amplifying medium. It is convenient to call this the case of weak-modulation operation, in which there is no appreciable redistribution of energy among the modes; regions of frequency-modulated radiation are absent, leaving a region of mode synchronization and regions of synchronous phase oscillations. This type of operation can prevail in lasers with high gain in the amplifying medium.

In the region of mode synchronization the radiation is pulsed. The generated spectral pulses are bell-shaped and of $2\pi\Delta\omega^{-1}$ duration, where $\Delta\omega$ is the Doppler line width at half maximum; the pulse repetition rate is $c/2l$, where l is the resonator length. The pulse amplitude is N times greater than the average intensity of laser emission (N is the number of modes). A laser having synchronized modes can thus be used as a source of the extremely powerful short pulses that are required for several applications.[4] Therefore the investigation of mode synchronization in a weak-modulation laser is of independent interest, like the case of strong modulation.

In the present experimental work mode synchronization was investigated by modulating the dielectric constant of an electro-optical crystal mounted inside of the laser resonator. Weak and strong modulation were studied, as well as the effects produced in laser oscillation by small frequency separations between the modulating signal and intermode beats.

Weak-modulation operation was studied by further simplifying the abbreviated equations for the cases of two and three laser modes, although this method can also be used to describe the behavior of any number of modes. Mode interaction in connection with weak modulation of the resonator dielectric constant is compared with mode synchronization for weak modulation of resonator loss.[5]

EXPERIMENT

A block diagram of the apparatus is shown in Fig. 1. In the He-Ne laser of $\lambda = 0.63\,\mu$ the mirror separation was 170 cm. A conventional z-cut KDP crystal was placed inside of the resonator near one plane mirror at the Brewster angle to transmitted light, so that the polarization plane of laser light formed a 45° angle with the x and y

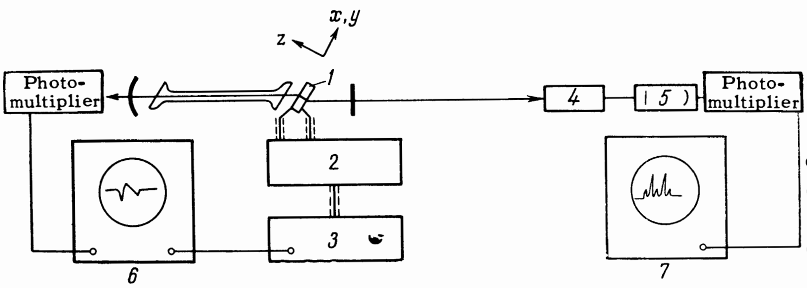

FIG. 1. Block diagram of apparatus. 1—electro-optical crystal, 2—amplifier, 3—rocking-frequency oscillator, 4—optical decoupling and matching lens, 5—scanning interferometer, 6,7—oscillographs (oscillograph 7 was synchronized with the oscillator 3).

crystal axes. This arrangement was convenient because: (1) The transmitted radiation is not amplitude-modulated; (2) the crystal introduces only small losses (the emission intensity was diminished to approximately one-half (1—0.75 mW) by introduction of the crystal; (3) laser generation depends only slightly on the accuracy of crystal adjustment. Only TEM_{00p} oscillations were present in the laser spectrum, which was examined with a Fabry-Perot scanning interferometer. In the absence of a modulating signal the laser generated 7 or 8 modes. With a 100—200 V potential applied to the crystal a frequency-modulated spectrum was observed. Figure 2 shows a typical distribution of the radiation intensity among the modes. With increased frequency separation the number of generated modes is reduced to 3; here the center mode is 4 or 5 times more intense than the side modes. This represents frequency modulation with a small modulation index. A similar effect is observed when the modulating voltage is reduced and the frequency separation is constant. Frequency modulation is extremely sensitive to changes in the positions of modes in the Doppler line; this results from thermal drift of resonator dimensions. The radiation spectrum can become very highly distorted when the modulation index is small. With a large index ($\gtrsim 2$) the change of resonator size can result in having the carrier mode replaced momentarily by a neighboring mode; then the pattern is restored. The same

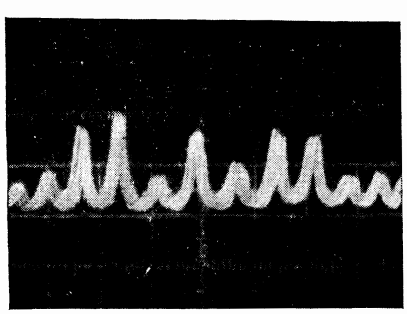

FIG. 2. Spectrum of laser with modulated dielectric constant (200 V applied to the crystal). The emission represents a frequency-modulated signal with ~ 3.3 modulation index.

occurs when one of the resonator mirrors is adjusted. When the frequency separation is reduced the number of laser modes increases and their intensities diminish, because a portion of the energy of modes lying within the amplifying curve of the mixture is transferred to modes that are not ordinarily amplified. A further reduction of the frequency separation leads to mutual synchronization of laser modes and to stabilization of intermode beat frequencies and of mode amplitudes.

Figure 3 is an oscillogram showing the dependence of laser light intensity on the frequency separation between the modulating signal and intermode beats. The dielectric constant of the crystal was modulated by a voltage of swinging frequency with which the oscillograph sweep was synchronized. The oscillator frequency deviation was 100 kc/sec. Figure 3 shows examples of strong modulation (a), weak modulation (c), and an intermediate case (b). Figure 3b was recorded with the same voltage applied to the crystal as in Fig. 3a, but in the latter the resonator was somewhat maladjusted. The middle of each oscillogram represents synchronized operation of the laser. On the right- and left-hand sides of Figs. 3a and b we see regions of frequency-modulated radiation; on both sides of the synchronized region in Fig. 3c we see regions exhibiting synchronized intensity oscillations of all modes at a frequency close to the aforementioned frequency separation. These oscillations caused ~10% variation of the laser intensity. Figure 4 shows the dependence of the synchronized region on the modulating voltage; the relation is linear at the lowest voltages. For the given intensity, $V_m \lesssim 30$ V applied to the crystal corresponds to weak modulation, and $V_m \gtrsim 100$ V to strong modulation. In the synchronized region the frequency of intermode beats is exactly equal to the modulating frequency. Therefore when the master oscillator frequency is changed by a frequency difference of δ within the synchronized region, the frequency of each mode is changed by an amount $\sim\delta \cdot 2l/\lambda$. This is manifested outwardly

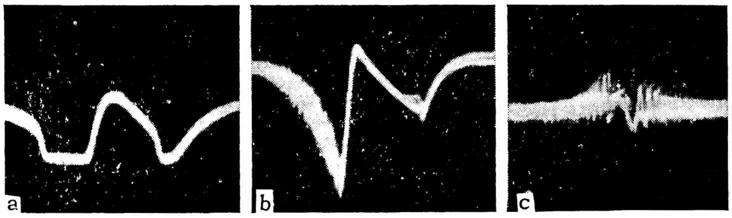

FIG. 3. Dependence of laser light intensity on frequency separation between the modulating signal and intermode beats; this was recorded by oscillograph 6. The modulating frequency increases from right to left. Voltage applied to crystal: a) and b) 200 V; c) 300 V. In case a) the laser was slightly maladjusted.

by a shift of the laser spectrum as registered with a scanning interferometer.

DISCUSSION OF RESULTS

There is very little difference between the dependence of the overall laser intensity on the aforementioned frequency difference, as well as the dependence of the synchronized region on the modulating signal strength, in the two different cases of weak dielectric-constant modulation and weak resonator-Q modulation.[5] To account for these results we apply the method of resimplification to the respective abbreviated equations. This is justified, as in the analysis of Q switching, when the following inequality, in the notation of [5], is satisfied:

$$\beta(K_n^{-1} + K_{n-1})[|\omega_n Q^{-1} - \sigma(E_n)|_{max}]^{-1} \ll 1, \quad (1)$$

Where Q_0 is the Q of the passive resonator, $K_n = E_n^0/E_{n+1}^0$ is the amplitude ratio of neighboring modes, $\sigma(E_n)$ defines the gain of the active resonator medium, $\beta = \omega_m \alpha$ (α characterizes the phase shift of light transmitted through the electro-optical crystal), ω_m is the modulating frequency, and ω_n is the oscillation frequency. We then obtain the following equations for the relative phases of neighboring modes and for the amplitude corrections:

$$\dot{\Phi}_n = -\varkappa_n + \delta + \beta[K_{n-1}\cos\Phi_{n-1} + (K_n^{-1} - K_n)\cos\Phi_n - K_{n+1}^{-1}\cos\Phi_{n-1}],$$

$$e_n = \frac{\beta}{|d\sigma/dE_n|_{E_n^0}}[K_n^{-1}\sin\Phi_n - K_{n-1}\sin\Phi_{n-1}]. \quad (2)$$

Here κ_m gives the laser frequency shift due to pulling that is associated with the shape of the gain curve, and δ is the frequency separation.

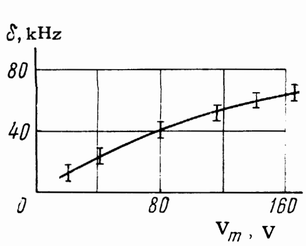

FIG. 4. Laser synchronization region vs. modulating voltage.

The relative phases and synchronized region for specific cases are obtained by solving the appropriate systems of equations and by investigating their stability. It is necessary only to investigate the phase equations, because the amplitude stability is determined by the nonlinear properties of the amplifying medium.

For two modes we obtain from (2) a single phase equation, which when $K_n^{-1} \neq K_n$ yields a stationary phase in the form

$$\cos\Phi_n = -\delta[\beta(K_{n-1} - K_n)]^{-1}$$

and the synchronized region

$$\delta = |\beta(K_n^{-1} - K_n)|.$$

As in the case of Q switching, outside of the synchronized region an oscillation equation exists for the phase. The equation describing mode synchronization of a two-mode laser coincides with the corresponding equation for the mutual synchronization of two autoheterodynes.[6] The stationary value of the phase is found in the region $\pi > \Phi_n > 0$ for $K_n^{-1} > K_n$ and in the region $\pi < \Phi < 0$ when $K_n^{-1} < K_n$. This proves that the weaker mode is captured by the stronger mode. The weak-mode amplitude is enhanced in the synchronized region, and the strong-mode amplitude is reduced, i.e., some equalization of the amplitudes occurs. When $K_n^{-1} \sim K_n$ the synchronizing process must be analyzed separately; [6] in this case phase and amplitude jumps can occur when the magnitude of the frequency separation is altered.

The phase equation for Q switching of a two-mode laser [5] corresponds to the equation that describes the synchronization of an autoheterodyne by an outside signal. The stationary phase values are contained in the region $-\pi/2 < \Phi_n < \pi/2$. Through synchronization each mode amplitude is enhanced, the weak mode more than the strong mode, and equalization of the amplitudes results.

For a three-mode laser the following system of phase equations is formed from (2):

$$\dot{\Phi}_{n-1} = \delta - \varkappa_{n-1} + \beta[(K_{n-1}^{-1} - K_{n-1})\cos\Phi_{n-1} - K_n^{-1}\cos\Phi_n],$$

$$\Phi_n = \delta - \varkappa_n + \beta[K_{n-1}\cos\Phi_{n-1} + (K_n^{-1} - K_n)\cos\Phi_n], \quad (3)$$

yielding the mode phases in the synchronized region. When the stability of this system is analyzed (subject to the entirely reasonable condition that the amplitude of the center mode exceeds the amplitudes of the side modes) we obtain $\sin\Phi_{n-1} > 0$, $\sin\Phi_n < 0$. It follows from (2) that in the synchronized region the amplitude of the center mode decreases as a result of the energy redistribution, while the amplitudes of the side modes increase. The stationary phases are given by

$$\cos\Phi_{n-1} = (D^{\varkappa}_{n-1} - \delta D_{n-1})(\beta D)^{-1},$$
$$\cos\Phi_n = (D_n^{\varkappa} - \delta D_n)(\beta D)^{-1},$$

where D is the determinant of (3), D_n is the determinant with the column containing $\cos\Phi_n$ replaced by a unity column, and D^K_n is the determinant with the same column replaced by the elements K_{n-1}, K_n. The conditions $|(D^K_{n-1} - \delta D_{n-1})(\beta D)^{-1}| < 1$ and $|(D^K_n - \delta D_n)(\beta D)^{-1}| < 1$ determine the region where a solution exists in the frequency difference coordinates of each mode. If the mode amplitudes are close, this region can contain phase jumps and regions of instability, as in the two-mode case.[7] If $|D^K_{n-1}/\beta D|$, and $|D^K_n/\beta D| \ll 1$, then $\delta \approx |\beta D/D_m|$, where D_m which is the smaller of the two values D_{n-1} and D_n, determines the region in which the laser can be synchronized.

The synchronization of three modes in connection with Q modulation proceeds somewhat differently. An analysis of the stability of the corresponding system of phase equations yields $\pi/2 > \Phi_n > -\pi/2$, $\pi/2 > \Phi_{n-1} > -\pi/2$. Here, as in the two-mode case, all mode amplitudes are enhanced through gain in the medium.

An increase of resonator loss, which may result from maladjustment of a mirror, will for constant β lead to violation of (1), the weak-modulation condition. An experimental confirmation is seen by comparing a and b of Fig. 3, which were derived for an identical modulating voltage but different values of the resonator Q.

In a $(2n + 1)$-mode laser the region of a stationary solution is determined by a system of 2n equations. If all modes have equal amplitudes and are equally spaced, the possible region of synchronization depends on the number of modes as n^{-1}, whereas for the synchronization of intermode beats by means of Q switching this dependence becomes $n^{-1}(n + 1)^{-1}$. It does not follow, however, that the synchronization is stable in this entire region.

We thus have seen that although the synchronization of intermode beats for weak Q switching and for weak dielectric-constant modulation are described by different equations, these two processes are manifested outwardly in an identical manner by a region of synchronization where the mode amplitudes are equalized to some degree, and outside of this region by a region where the relative phases oscillate synchronously with the separation frequency. In both cases the laser radiation is a superposition of separate modes and is pulsed in the synchronized region. In both cases, under strong modulation pulse formation is associated with periodic resonator losses. While losses are incurred in an entirely obvious manner for Q switching, in the case of dielectric-constant modulation the losses are associated with the formation of new modes lying far outside of the gain curve and deriving energy periodically from modes amplified by the gaseous mixture. In the case of weak modulation the losses pertaining to both cases are not governing factors; the pulsing results from mode interactions.

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