SOVIET PHYSICS JETP VOLUME 25, NUMBER 1 JULY, 1967

ORIENTATION OF HYDROGEN ATOMS BY RESONANCE RADIATION

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Submitted to JETP editor July 21, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 52, 242–254 (January, 1967)

Spin temperatures characterizing the populations of the magnetic sublevels of the $1S_{1/2}$ (F = 0, 1) state of HI atoms situated in a radio-frequency field (λ = 21 cm) and in ultraviolet radiation ($\lambda \sim$ 1216—912 Å) are calculated. The attenuation and amplification coefficient of 21-cm radio emission passing through a medium consisting of oriented HI atoms is determined. The conditions for the appearance of inverse population of the F = 1 ($1S_{1/2}$) level are determined.

1. INTRODUCTION

We consider the orientation of HI atoms under the influence of an arbitrary radiation flux. We determine the populations of the magnetic sublevels of the hyperfine multiplet of the ground state $1S_{1/2}$ of the HI atoms as functions of the intensity, spectrum, angular distribution, and polarization of the optical and radio emission incident on the atom. We assume that all the atoms are concentrated at the levels of the $1S_{1/2}$ multiplet and that the Zeeman splitting of these levels is much larger than their energy width, but much smaller than the hyperfine splitting. This is precisely the situation that obtains usually in astrophysics (clouds of interstellar hydrogen) and in radiospectroscopy (experiments with atomic beams). We are interested only in time-averaged quantities, and not in their fluctuations.

Under these conditions, the state of the HI atoms can be characterized by four quantities R_{FM}, which represent the populations of the levels F = 0, M = 0 and F = 1, M = 0, ±1 of the $1S_{1/2}$ state. ($\mathbf{F} = \mathbf{J} + \mathbf{I}$ is the total angular momentum of the atom (electron and proton) and M is the projection of F on the quantization axis, chosen in the direction of the magnetic field.)

The level populations R_{FM} define three spin temperatures T_{S_1}, T_{S_0}, and $T_{S_{-1}}$:

$$R_{1M} / R_{00} = \exp(-h\nu_0 / kT_{S_M}), \tag{1}$$

where $h\nu_0$ is the hyperfine splitting of $1S_{1/2}$ [1]. Since $T_* \equiv h\nu_0/k = 0.0681°K \ll T_{S_M}$, we have

$$T_{S_M} \approx T_* \frac{R_{00}}{R_{00} - R_{1M}}. \tag{2}$$

The equilibrium values of the populations R_{FM} are determined by the system of balance equations

$$R_{FM} \sum_{F'M'} W_{FM\to F'M'} = \sum_{F'M'} R_{F'M'} W_{F'M'\to FM}. \tag{3}$$

The coefficients $W_{FM\to F'M'}$ represent the transition probabilities.

The general solution of these equations is the basis for the consideration of all cases of HI orientation, and is of the form

$$\frac{R_{1\pm1}}{R_{00}} = \frac{1}{N}\left\{\left(W_{00\to1\pm1} + \frac{W_{00\to10}W_{10\to1\pm1}}{W_{10}}\right)\right.$$
$$\times\left(\frac{W_{1\mp1\to10}W_{10\to1\mp1}}{W_{10}} - W_{1\mp1}\right)$$
$$-\left(W_{00\to1\mp1} + \frac{W_{00\to10}W_{10\to1\mp1}}{W_{10}}\right)\left(\frac{W_{1\mp1\to10}W_{10\to1\pm1}}{W_{10}}\right.$$
$$\left.\left. + W_{1\mp1\to1\pm1}\right)\right\},$$

$$\frac{R_{10}}{R_{00}} = \frac{1}{N}\left\{\frac{W_{00\to10}}{W_{10}}(W_{11\to1-1}W_{1-1\to11} - W_{11}W_{1-1})\right.$$
$$-\left(W_{00\to11} + \frac{W_{00\to1-1}W_{1-1\to11}}{W_{1-1}}\right)\frac{W_{1-1}}{W_{10}} W_{11\to10}$$
$$\left.-\left(W_{00\to1-1} + \frac{W_{00\to11}W_{11\to1-1}}{W_{11}}\right)\frac{W_{11}}{W_{10}} W_{1-1\to10}\right\}, \tag{4}$$

where $W_{FM} \equiv \sum_{F'M'} W_{FM\to F'M'}$ is the total probability of knocking out the atoms from a given state, and the normalization factor is

$$N = \left(W_{11\to1-1} + \frac{W_{11\to10}W_{10\to1-1}}{W_{10}}\right)$$
$$\times\left(\frac{W_{1-1\to10}W_{10\to11}}{W_{10}} + W_{1-1\to11}\right)$$

$$-\left(W_{11}-\frac{W_{11\to10}W_{10\to11}}{W_{10}}\right)$$
$$\times\left(\frac{W_{1-1\to10}W_{10\to1-1}}{W_{10}}-W_{1-1}\right) \quad (5)$$

The transition probabilities $W_{FM\to F'M'}$ are determined in the general case by three processes (R, L, C).

R. Spontaneous and induced transitions F = 1 ⟷ F = 0($1S_{1/2}$), connected with the 21-cm radio emission.

L. Transitions between levels of the hyperfine structure of $1S_{1/2}$, due to resonant scattering of ultraviolet radiation with $\lambda \sim 1216$–912 Å (Lyman-series transitions).

C. Transitions between levels of the hyperfine multiplet of $1S_{1/2}$, due to collisions of HI atoms with one another or with electrons.

2. ORIENTATION OF HI ATOMS UNDER THE INFLUENCE OF 21-cm RADIO EMISSION

The intensity, spectrum, angular distribution, and polarization of an arbitrary radiation field are described by a density matrix

$$\langle\vartheta\varphi\sigma|\rho(\nu)|\vartheta'\varphi'\sigma'\rangle, \quad (6)$$

where ν is the frequency, ϑ and φ the polar and azimuthal angles, and σ the index of cyclic polarization. The radiation density matrix is so normalized that

$$\langle\vartheta\varphi\sigma|\rho(\nu)|\vartheta\varphi\sigma\rangle=\lambda^3 I_\sigma(\nu\vartheta\varphi)/hc, \quad (7)$$

where $I_\sigma(\nu\vartheta\varphi)$ (erg/cm^{2}sec-sr-Hz) is the emission intensity, i.e., the energy of the radiation of frequency $\nu = c/\lambda$ and polarization σ passing in the $\vartheta_1\varphi$ direction in a unit solid angle through an area of 1 cm^2 per second.

In the representation characterized by a definite photon angular momentum j (relative to the atom), a projection of this momentum m, and a parity $\pi = (-)^{T+j+1}$, the radiation density matrix takes the form

$$\langle jm\tau|\rho(\nu)|j'm'\tau'\rangle$$
$$=\langle jm\tau|\vartheta\varphi\sigma\rangle\langle\vartheta\varphi\sigma|\rho(\nu)|\vartheta'\varphi'\sigma'\rangle\langle\vartheta'\varphi'\sigma'|j'm'\tau'\rangle, \quad (8)$$

where $\langle jm\tau|\vartheta\varphi\sigma\rangle \equiv [\mathbf{Y}_{jm}(\tau)(\vartheta\varphi)]_\sigma$ are the σ-components of the "spherical vector"[2,3].

For isotropic unpolarized radiation we have

$$\langle jm\tau|\rho(\nu)|j'm'\tau'\rangle$$
$$=n(\nu)\delta_{jj'}\delta_{mm'}\delta_{\tau\tau'}[\delta_{mj}+\delta_{mj-1}+\ldots+\delta_{m-j}], \quad (9)$$

where $n(\nu)\equiv\langle\vartheta\varphi\sigma|\rho(\nu)|\vartheta\varphi\sigma\rangle$ is the number of photons per cell of phase space; $n(\nu)$ can be characterized by the temperature $T_R(\nu)$ corresponding to the thermal radiation of an absolutely black body:

$$n(\nu)=\frac{1}{\exp(h\nu/kT_R)-1}\approx\frac{kT_R}{h\nu}. \quad (10)$$

For a directed radiation flux of arbitrary polarization

$$\langle jm\tau|\rho(\nu)|j'm'\tau'\rangle$$
$$=n(\nu)\frac{\sqrt{(2j+1)(2j'+1)}}{8\pi}\,\Omega\sum_{\sigma\sigma'}D_{m\sigma}{}^{j}(\varphi\vartheta0)$$
$$\times[(1+\sigma\eta)\delta_{\sigma\sigma'}+\xi(-)^{\tau}\delta_{\sigma-\sigma'}e^{-i(\sigma-\sigma')\alpha}]D^{j'+}_{\sigma'm'}(0\vartheta\varphi), \quad (11)$$

where $D_{m\sigma}{}^{j}(\varphi\vartheta_0)$ are elements of the rotation matrix $\varphi\vartheta$ in the $jm\sigma$ representation[3,4], and the angles ϑ and φ indicate the direction of the flux. The quantities η and ξ specify the degrees of circular and linear polarization. The angle α determines the direction of the oscillations of the vector **E** in the case of linear polarization. The number of photons per phase cell, $n(\nu)$, determines the strength of the radiation source; as above, $n(\nu)$ can be characterized by a temperature T_R corresponding to absolute black-body radiation; Ω is the solid angle at which the radiation source is seen ($\Omega/4\pi \ll 1$).

Thus, we consider the case when the main process determining the level populations of HI is their interaction with the radio emission, i.e.,

$$W^{(R)}_{FM\to F'M'}\gg W^{(L+C)}_{FM\to F'M'}. \quad (12)$$

The amplitude of the probability of a spontaneous transition with emission of a quantum of radio emission with frequency ν is

$$\langle FM|\hat{S}(\nu)|F'M'jm\rangle=\frac{-(\gamma_0/2\pi)}{(\nu-\nu_0)+i(\gamma_0/4\pi)}C^{FM}_{F'M'jm}, \quad (13)$$

$\gamma_0 = 2.85\times10^{-15}$ sec^{-1} is the total probability of the spontaneous $FM\to F'M'$ transition corresponding to a resonant frequency $\nu_0 = 1420.4$ MHz; $C^{FM}_{F'M'jm}$ is a Clebsch-Gordan coefficient[4]. The transitions between the levels of the hyperfine structure of the $1S_{1/2}$ state, connected with the 21-cm radio emission, are of the M1 type. Their probabilities are determined by the diagonal matrix elements $\rho_m(\nu) = \langle jm\tau|\rho(\nu)|jm\tau\rangle$ with $j = j' = 1$ and $\tau = \tau' = 0$:

$$W^{(R)}_{FM\to F'M'}=\gamma_0(C^{FM}_{F'M'jm})^2\rho_m(\nu_0). \quad (14)$$

Thus, the transition probabilities and populations R_{FM} of interest to us do not depend on the details of the angular distribution and polarization of the radiation, and are determined completely by three parameters $\rho_m(\nu_0)$. The same $\rho_m(\nu_0)$ can correspond to essentially different angular distributions. Thus, in the case of unpolarized radiation with uniform angular distribution in a solid angle 2π, or with an angular distribution $1+\epsilon\cos\vartheta$, the parameters ρ_m will be the same as in the case of isotropic unpolarized radiation, i.e., $\rho_1=\rho_0=\rho_{-1}$.

In the case of a directed flux of polarized radiation, the parameters $\rho_m(\nu)$ take the form

$$\rho_{\pm1}(\nu)=n(\nu)\frac{3}{2}\frac{\Omega}{4\pi}\left[1\pm\eta\cos\vartheta-\frac{\sin^2\vartheta}{2}(1+\xi\cos 2\alpha)\right],$$
$$\rho_0(\nu)=n(\nu)\frac{3}{2}\frac{\Omega}{4\pi}\sin^2\vartheta(1+\xi\cos 2\alpha). \tag{15}$$

In accord with (14), the transition probabilities of an HI atom situated in the field of 21-cm radiation are equal to

$$W^{(R)}_{00\to1M}=\gamma_0\rho_M,\ W^{(R)}_{1M\to00}=\gamma_0(1+\rho_M),\ W^{(R)}_{1M\to1M'}\approx 0. \tag{16}$$

The system (3) leads here to the following relations for the populations:

$$\frac{R_{1\pm1}}{R_{00}}=\frac{W^{(R)}_{00\to1\pm1}}{W^{(R)}_{1\pm1\to00}},\quad \frac{R_{10}}{R_{00}}=\frac{W^{(R)}_{00\to10}}{W^{(R)}_{10\to00}}. \tag{17}$$

These relations give the complete solution of the problem for cases when the HI atoms are oriented by radio emission, and enable us, in accord with (1), to determine the spin temperatures. In the particular case of isotropic unpolarized radio emission, all three spin temperatures are equal and have the same value as the radiation, $T_{S_{+1}}=T_{S_0}=T_{S_{-1}}=T_R$.

In the case of directed unpolarized radio emission, the spin temperatures are

$$T_{S_{+1}}=T_{S_{-1}}=\frac{3}{2}\frac{\Omega}{4\pi}T_R\left(\frac{1+\cos^2\vartheta}{2}\right),$$
$$T_{S_0}=\frac{3}{2}\frac{\Omega}{4\pi}T_R\sin^2\vartheta. \tag{18}$$

In the case of directed radiation which is completely circularly polarized,

$$T_{S_{+1}}=\frac{3}{2}\frac{\Omega}{4\pi}T_R\frac{(1+\cos\vartheta)^2}{2},$$
$$T_{S_0}=\frac{3}{2}\frac{\Omega}{4\pi}T_R\sin^2\vartheta,$$
$$T_{S_{-1}}=\frac{3}{2}\frac{\Omega}{4\pi}T_R\frac{(1-\cos\vartheta)^2}{2}. \tag{19}$$

In the case of completely linearly polarized radiation

$$T_{S_{+1}}=T_{S_{-1}}=\frac{3}{2}\frac{\Omega}{4\pi}T_R(\cos^2\vartheta+\sin^2\vartheta\cos^2\alpha),$$
$$T_{S_0}=\frac{3}{2}\frac{\Omega}{4\pi}T_R 2\sin^2\vartheta\sin^2\alpha. \tag{20}$$

For astrophysical applications, as well as for laboratory practice, greatest interest is attached to the case when the directed radiation, of arbitrary polarization, characterized by a temperature T_{R_1}, is accompanied by a background of isotropic unpolarized radiation characterized by a temperature T_{R_0}. In this case the spin temperatures are

$$T_{S_{\pm1}}=T_{R_0}+T_{R_1}\frac{3}{2}\frac{\Omega}{4\pi}\left[\left(\frac{1+\cos^2\vartheta}{2}\right)\pm\eta\cos\vartheta+\xi\frac{\sin^2\vartheta}{2}\cos 2\alpha\right],$$
$$T_{S_0}=T_{R_0}+T_{R_1}\frac{3}{2}\frac{\Omega}{4\pi}\sin^2\vartheta(1-\xi\cos 2\alpha). \tag{21}$$

3. ORIENTATION OF HI ATOMS UNDER THE INFLUENCE OF ULTRAVIOLET RADIATION

We consider the case when the radiation with $\lambda\sim 1216-912$ Å has sufficiently high intensity, so that

$$W^{(L)}_{FM\to F'M'}\gg W^{(R+C)}_{FM\to F'M} \tag{22}$$

and the populations R_{FM} are determined only by the interaction of the HI atoms with the light.

The amplitude of the resonance scattering of the light, with account taken of the hyperfine splitting, takes the form

$$\langle F'M'j'm'|\hat{S}(\nu)|FMjm\rangle=\sum_{F''}\frac{U(F''j'IJ_a;F'J_b)\,U(F''jIJ_a;FJ_b)}{(\nu-\nu_{F''F})+i(\gamma_{F''}/4\pi)}\times(-)^{F-F'}\left(\frac{\gamma_{F''}}{2\pi}\right)C^{F''M''}_{F'M'j'm'}C^{F''M''}_{FMjm}, \tag{23}$$

where $h\nu_{F''F}$ is the resonance energy, $\hbar\gamma_{F''}$ the energy width of the resonance level, $U(F''jIJ_a:FJ_b)$ a Racah coefficient [4], J_a and J_b the angular momentum of the electron in the ground and excited states, and I the angular momentum of the nucleus.

In resonance scattering of light by HI atoms, the main contribution is made by the Lyman-series transitions $1S_{1/2}\to nP_{1/2}\to 1S_{1/2}$ and $1S_{1/2}\to nP_{3/2}\to 1S_{1/2}$ [1], i.e., E1 transitions corresponding to $j=j'=1$ and $\tau=\tau'=1$. Therefore the entire dependence (8) of $W^{(L)}_{FM\to F'M'}$ on the angular distribution and polarization of light is determined, just as in the case of interaction with radio emission, only by the three parameters

$$\rho_m(\nu)=\langle j=1\,m\,\tau=1|\rho(\nu)|j=1\,m\,\tau=1\rangle.$$

The probability of the transition $FM\to F'M'$ between the levels of the hyperfine structure of

Partial probabilities $W^{(L)}_{FM\rightarrow F'M'}$ of transitions induced by ultraviolet radiation

$FM \rightarrow F'M'$	$\frac{9}{\gamma_{F''}} W^{(L)}_{FM\rightarrow F'M'}$
$00 \rightarrow 00$	$[\rho_1(\bar{\nu}_1) + \rho_0(\bar{\nu}_1) + \rho_{-1}(\bar{\nu}_1)] + 4[\rho_1(\bar{\nu}_2) + \rho_0(\bar{\nu}_2) + \rho_{-1}(\bar{\nu}_2)]$
$00 \rightarrow 10$	$[\rho_1(\bar{\nu}_1) + \rho_{-1}(\bar{\nu}_1)] + [\rho_1(\bar{\nu}_2) + \rho_{-1}(\bar{\nu}_2)]$
$00 \rightarrow 1\pm 1$	$[\rho_0(\bar{\nu}_1) + \rho_{\pm 1}(\bar{\nu}_1)] + [\rho_0(\bar{\nu}_2) + \rho_{\pm 1}(\bar{\nu}_2)]$
$10 \rightarrow 00$	$[\rho_1(\nu_1) + \rho_{-1}(\nu_1)] + [\rho_1(\nu_2) + \rho_{-1}(\nu_2)]$
$10 \rightarrow 10$	$[\rho_1(\nu_1) + \rho_0(\nu_1) + \rho_{-1}(\nu_1)] + [4\rho_0(\nu_2) + {}^1/_2(\rho_1(\nu_2) + \rho_{-1}(\nu_2))(5 + 3\Delta_2)]$
$10 \rightarrow 1\pm 1$	$[\rho_0(\nu_1) + \rho_{\pm 1}(\nu_1)] + [\rho_0(\nu_2) + {}^1/_2\rho_{\pm 1}(\nu_2)(5 - 3\Delta_2)]$
$1\pm 1 \rightarrow 00$	$[\rho_0(\nu_1) + \rho_{\pm 1}(\nu_1)] + [\rho_0(\nu_2) + \rho_{\mp 1}(\nu_2)]$
$1\pm 1 \rightarrow 10$	$[\rho_0(\nu_1) + \rho_{\mp 1}(\nu_1)] + [{}^1/_2\rho_0(\nu_2)(5 - 3\Delta_2) + \rho_{\mp 1}(\nu_2)]$
$1\pm 1 \rightarrow 1\mp 1$	$[2\rho_{\mp 1}(\nu_1)(1 - \Delta_1)] + [{}^1/_2\rho_{\mp 1}(\nu_2)(1 - \Delta_2)]$
$1\pm 1 \rightarrow 1\pm 1$	$[\rho_0(\nu_1) + 2\rho_{\mp 1}(\nu_1)(1 + \Delta_1)] +$ $+ [9\rho_{\pm 1}(\nu_2) + {}^1/_2\rho_0(\nu_2)(5 + 3\Delta_2) + {}^1/_2\rho_{\mp 1}(\nu_2)(1 + \Delta_2)]$

the ground state of HI, due to resonance scattering of light, is

$$W^{(L)}_{FM\rightarrow F'M'} = \sum_{mm'} \int |\langle F'M'j'm'|\hat{S}(\nu)|FMjm\rangle|^2 \rho_m(\nu)\,d\nu. \quad (24)$$

We substitute the amplitude (23) in (24), using the numerical values of all the Racah and Clebsch-Gordan coefficients. We integrate with respect to frequency, assuming that the spectrum of the radiation incident on the HI atoms varies sufficiently smoothly, so that the radiation intensity can be regarded as constant within the limits of the natural energy width of the level. As a result we obtain explicit expressions for the partial transition probabilities (see the table). The resonant frequencies ν_i are defined in the figure. The index 1 pertains to the resonance level $P_{1/2}$, and the index 2 to the level $P_{3/2}$; $\bar{\nu}_1 - \nu_1 = \bar{\nu}_2 - \nu_2 = \nu_0$. In calculating $W^{(L)}_{FM\rightarrow F'M'}$ we took account of the fact that $\nu_1' \approx \nu_1$ and $\nu_2' \approx \nu_2$.

Unlike the ground state $1S_{1/2}$, the magnitude of the hyperfine splitting of the resonance levels $nP_{1/2}$ and $nP_{3/2}$ is smaller than their natural energy width, i.e., the components of the hyperfine doublets of the resonantly excited states overlap partly and interfere. The parameters Δ characterize the degree of overlap of the hyperfine components:

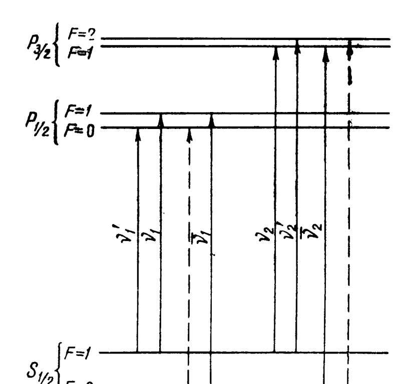
$$\Delta_1 \equiv \frac{(\gamma_{F''}/2\pi)^2}{(\nu_1' - \nu_1)^2 + (\gamma_{F''}/2\pi)^2} \quad \text{for } nP_{1/2},$$
$$\Delta_2 \equiv \frac{(\gamma_{F''}/2\pi)^2}{(\nu_2' - \nu_2)^2 + (\gamma_{F''}/2\pi)^2} \quad \text{for } nP_{3/2}. \quad (25)$$

In astrophysics, greatest interest attaches to scattering, by HI atoms, of the L_α radiation corresponding to the levels $2P_{1/2}$ and $2P_{3/2}$.

Elastic resonance scattering via other higher resonance levels, $nP_{1/2}$ and $nP_{3/2}$, leads to precisely the same orientation, since the angular characteristics and the degree of overlap of the components of the hyperfine structure of these levels are practically the same as for $2P_{1/2}$ and $2P_{3/2}$, namely $\Delta_1 = 0.75$ and $\Delta_2 = 0.95$. The fraction of cascade transitions is small; it is smaller than 2% even at a constant spectrum. Each term of the expressions for $W^{(L)}_{FM\rightarrow F'M'}$ listed in the table should be regarded as the sum of terms of this type for all resonance states with n = 2, 3, 4 etc.

Thus, the expressions listed in the table make it possible to calculate the probabilities $W^{(L)}_{FM\rightarrow F'M'}$ at any intensity or spectrum, and at any angular distribution or polarization of the incident radiation. It is merely necessary to substitute the corresponding values of the parameters $\rho_m(\nu)$.

If the HI atoms are in an isotropic field $n(\nu)$ of ultraviolet radiation with $\lambda = 1216$ Å, and if in addition there is an isotropic background n_R of



21-cm radio emission, then the level populations, in accord with (4), (16), and the table, will be as follows:

$$\frac{R_{11}}{R_{00}}=\frac{R_{10}}{R_{00}}=\frac{R_{1-1}}{R_{00}}=\frac{\gamma_0 n_R+{}^1/_9\gamma_{F''}[n(\bar{\nu}_1)+n(\bar{\nu}_2)]}{\gamma_0(1+n_R)+{}^1/_9\gamma_{F''}[n(\nu_1)+n(\nu_2)]}, \quad (26)$$

corresponding to a spin temperature

$$T_S=T_*\frac{\gamma_0 n_R+{}^1/_9\gamma_{F''}[n(\bar{\nu}_1)+n(\bar{\nu}_2)]}{\gamma_0+{}^1/_9\gamma_{F''}[n(\nu_1)-n(\bar{\nu}_1)+n(\bar{\nu}_2)-n(\nu_2)]}. \quad (27)$$

At a constant spectrum, $n(\nu_1) = n(\bar{\nu}_1)$ and $n(\nu_2) = n(\bar{\nu}_2)$, the spin temperature is determined by the ratio of the probability of the transition induced under the influence of the ultraviolet and radio emission to the probability of the spontaneous transition $F = 1 \to F = 0$. The higher the radiation intensity, the higher the spin temperature. In the case of a dropping spectrum ($dn(\nu)/d\nu < 0$), the spin temperature of the HI atoms will be lower, and in the case of a slowly rising spectrum ($0 < dn(\nu)/d\nu < {}^9/_2(\gamma_0/\gamma_{F''})\nu_0^{-1}$) will be larger than the value T_S corresponding to the constant spectrum.

In the case of a strongly growing spectrum, when

$$\frac{dn(\nu)}{d\nu}>\frac{9}{2}\frac{\gamma_0}{\gamma_{F''}}\frac{1}{\nu_0}, \quad (28)$$

the spin temperature will be negative, i.e., population inversion sets in. Such a situation is possible not only under laboratory conditions, but also under astrophysical conditions, for example when a cloud of atomic hydrogen moves head-on towards a source of L_α radiation at a velocity such that the HI atoms are excited by the red wing of the L_α spectral line.

In the case of directed radiation, the parameters ρ_m for E1 transitions differ from the parameters ρ_m for M1 transitions from (15) only in the sign of ξ. In the particular case of a directed flux of unpolarized radiation propagating along the quantization axis we have

$$\rho_1(\nu)=\rho_{-1}(\nu)=n(\nu)\frac{3}{2}\frac{\Omega}{4\pi}, \qquad \rho_0(\nu)=0. \quad (29)$$

If this flux is sufficiently intense:

$$n(\nu)\frac{\Omega}{4\pi}\gg\frac{100}{1-\Delta_2}\frac{\gamma_0}{\gamma_{F''}}, \quad (30)$$

then the spin state of the HI atom will be characterized by the following populations:

$$\frac{R_{10}}{R_{00}}=\frac{[n(\bar{\nu}_1)+n(\bar{\nu}_2)]}{[n(\nu_1)+n(\nu_2)]+{}^1/_2 n(\nu_2)(1-\Delta_2)},$$

$$\frac{R_{1\pm1}}{R_{00}}=\frac{[n(\bar{\nu}_1)+n(\bar{\nu}_2)][1+n(\nu_2)(1-\Delta_2)/(n(\nu_1)+n(\nu_2))]}{[n(\nu_1)+n(\nu_2)]+{}^1/_2 n(\nu_2)(1-\Delta_2)}, \quad (31)$$

corresponding to the spin temperatures

$$T_{S_0}=T_*\frac{[n(\nu_1)+n(\nu_2)]+{}^1/_2 n(\nu_2)(1-\Delta_2)}{[n(\nu_1)-n(\bar{\nu}_1)]+[n(\nu_2)-n(\bar{\nu}_2)]+{}^1/_2 n(\nu_2)(1-\Delta_2)},$$

$$T_{S_{\pm1}}=T_*\{[n(\nu_1)+n(\nu_2)]+{}^1/_2 n(\nu_2)(1-\Delta_2)\}\cdot\{[n(\nu_1)-n(\bar{\nu}_1)]+[n(\nu_2)-n(\bar{\nu}_2)]+{}^1/_2 n(\nu_2)(1-\Delta_2)[1-2(n(\bar{\nu}_1)+n(\bar{\nu}_2))/(n(\nu_1)+n(\nu_2))]\}^{-1}. \quad (32)$$

It is seen from (31) that the spin of an HI atom in the state $F = 1$ becomes aligned in the direction of the flux $R_{11} = R_{1-1} > R_{10}$. The degree of alignment is determined by the ratio

$$\frac{R_{11}}{R_{10}}=1+\frac{n(\nu_2)}{n(\nu_1)+n(\nu_2)}(1-\Delta_2). \quad (33)$$

The resonance scattering corresponding to the levels $nP_{1/2}$ leads to the equalization of the populations R_{FM}, i.e., it does not result in orientation. Orientation of the spin $F = 1$ is brought about only by scattering via the levels $nP_{3/2}$. There would be no orientation if the components of the hyperfine doublet $nP_{3/2}$ were to overlap completely ($\Delta_2 = 1$). The reason is that the light interacts directly only with the electron spin, but the electron spin in the ground state of the HI atom is $J_a = 1/2$, and such a spin cannot become aligned. Therefore orientation of the total angular momentum $\mathbf{F} = \mathbf{J} + \mathbf{I}$ takes place only as a result of spin interaction between the electron and the proton. Were there no hyperfine splitting, there would likewise be no orientation of F.

If the spectrum of the incident radiation is constant, $n(\nu_1) = n(\bar{\nu}_1) = n(\nu_2) = n(\bar{\nu}_2)$, then

$$\frac{R_{1\pm1}}{R_{00}}=1+\frac{1-\Delta_2}{5-\Delta_2}=1{,}013,$$

$$\frac{R_{10}}{R_{00}}=1-\frac{1-\Delta_2}{5-\Delta_2}=0{,}987. \quad (34)$$

This corresponds to spin temperatures

$$T_{S_{+1}}=T_{S_{-1}}=-T_*\frac{5-\Delta_2}{1-\Delta_2}=-5.2^\circ K,$$

$$T_{S_0}=T_*\frac{5-\Delta_2}{1-\Delta_2}=5.2^\circ K. \quad (35)$$

Population inversion of the sublevels $F = 1$, $M = \pm1$ results not only from the redistribution of the populations among the magnetic sublevels of the $F = 1$ level, but also from the transition of part of the HI atoms from the state $F = 0$ to the state $F = 1$. If the HI atoms are oriented by an intense beam of circularly polarized light ($\eta \neq 0$) directed along the quantization axis, then the ratios R_{10}/R_{00} and $(R_{11} + R_{1-1})/2R_{00}$ will be the same as in the case of the orientation by directed unpolarized radiation (30). However, the spin F = 1 of the hydrogen atom will now be not only aligned but also polarized, i.e., $R_{11} \neq R_{1-1}$:

$$\frac{R_{11}}{R_{1-1}} = \frac{(1+\eta)}{(1-\eta)} \frac{(1+\eta)+2A}{(1-\eta)+2A}, \tag{36}$$

where

$$A \equiv \frac{\eta(\nu_1)(1-\Delta_1) + {}^1/{}_4 n(\nu_2)(1-\Delta_2)}{n(\nu_1)+n(\nu_2)}.$$

If the HI atoms are in an intense directed flux of linearly polarized ($\xi \neq 0$) of ultraviolet radiation propagating in the ϑ, φ direction, then the spins F = 1 of the HI atoms become aligned, so that

$$\frac{R_{10}}{R_{00}} = \frac{1}{N}\left\{[n(\nu_1)+n(\nu_2)](1+\beta)\left(1-\frac{\beta}{3}\right) + 2n(\nu_2)\beta(1-\Delta_2)\right\},$$

$$\frac{R_{1\pm1}}{R_{00}} = \frac{1}{N}\left\{[n(\nu_1)+n(\nu_2)](1+\beta)\left(1-\frac{\beta}{3}\right) - n(\nu_2)(1-\beta)(1-\Delta_2)\right\}, \tag{37}$$

where

$$\beta = {}^1/{}_2(1+\xi\cos 2\alpha)\sin^2\vartheta,$$

and the normalization factor is

$$N \equiv \frac{n(\nu_1)+n(\nu_2)}{n(\bar{\nu}_1)+n(\bar{\nu}_2)}\left\{[n(\nu_1)+n(\nu_2)](1+\beta)\left(1-\frac{\beta}{3}\right) + n(\nu_2)(1-\beta)(1+3\beta)(1-\Delta_2)\right\}.$$

When $\beta = 0$ relations (37) go over into relations (31). It is seen that when the orienting radiation propagates along the quantization axis, then the level populations do not depend on the degree of linear polarization.

4. EFFECT OF COLLISIONS ON THE ORIENTATION OF THE HI ATOMS

An exact account of the collisions in the absence of thermodynamic equilibrium is a problem in itself. In this paper we confine ourselves to qualitative results.

In neutral hydrogen it is necessary to take into account collisions of the HI atoms with one another, not with electrons, and only those collisions that result in transitions between sublevels of the hyperfine structure of the ground state of HI. The most effective in this respect are exchange collisions ($\sigma_{exch} \approx 4 \times 10^{-15}$ cm^2). In principle the collisions tend to equalize the populations of the magnetic sublevels and to make the spin temperature T_S equal to their kinetic temperature T_K. However, the influence of the collisions on the orientation of the HI atoms depends on whether the orienting radiation has circular polarization or not.

If this radiation has circular polarization, then the HI atoms will be partly oriented even when $W^{(C)}_{FM\to F'M'} \gg W^{(R+L)}_{FM\to F'M'}$. This is due to the conservation of the projection of the angular momentum. On the other hand, if the orienting radiation has no circular polarization, then the atoms will be practically non-oriented when $W^{(C)}_{FM\to F'M'} \gg W^{(R+L)}_{FM\to F'M'}$.

Thus, for example, the spin temperatures of HI atoms oriented by 21-cm radio emission characterized by values $\rho_0(\nu_0)$ and $\rho_1(\nu_0) = \rho_{-1}(\nu_0)$ are equal to, in the presence of exchange collisions with $kT_K/h\nu_0$,

$$T_{S_{\pm1}} = T_* \frac{[2c+\gamma_0(\rho_1+1)][3c+\gamma_0(\rho_0+1)]-2c^2}{[4c+\gamma_0(\rho_1+1)]\gamma_0},$$
$$T_{S_0} = T_* \frac{[2c+\gamma_0(\rho_1+1)][3c+\gamma_0(\rho_0+1)]-2c^2}{[4c+\gamma_0(\rho_0+1)]\gamma_0}, \tag{38}$$

where $c \equiv (1/8)\sigma_{exch} v_{at} n_{at}$ is the probability of transition as a result of exchange collisions. When $c \to 0$ formulas (38) go over into (18) or (20), depending on the explicit form of ρ_m.

5. "TRANSPARENCY" OF ORIENTED HI MEDIUM TO 21-cm RADIO EMISSION

A medium consisting of oriented HI atoms has anisotropic properties. The principal quantity characterizing the properties of such a medium is τ, the attenuation (or amplification) coefficient of the transmitted radiation, which depends in our case on the observation direction $\vartheta_{obs}\varphi_{obs}$ and on the type of the investigated polarization κ_{obs}. For 21-cm radio emission

$$\tau = {}^3/{}_8\pi^{-1}\gamma_0 S(\nu)N \times\left[\sum_M D^{1+}_{\sigma M}(0\vartheta_{obs}\varphi_{obs})R_{1M}D^1_{M\sigma'}(\varphi_{obs}\vartheta_{obs}0)\langle\varkappa_{obs}|\sigma\rangle(\sigma\sigma') \times \langle\sigma'|\varkappa_{obs}\rangle - R_{00}\right], \tag{39}$$

where

$$S(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\exp\left[-\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]$$

is the Doppler contour of the spectral line, $\Delta\nu_D = \nu_0\sqrt{kT_K/Mc^2}$, $N \equiv \lambda^2 \int n_{at}(r)\,dr$ is the number of hydrogen atoms in a column with cross section

λ^2 cm^2 along the line of sight; $\langle \sigma | \kappa \rangle$ are the coefficients of transition from the polarization κ to the polarization σ. The populations are normalized so that $R_{11} + R_{10} + R_{1-1} + R_{00} = 1$.

Let us consider three cases, in which the detector recording the radio emission is sensitive to circular polarization, sensitive to linear polarization, and not sensitive to polarization at all.

The attenuation (or amplification) coefficient of circularly polarized radiation with $\lambda = 21$ cm has the following form (the + and – signs pertain to right- and left-hand polarization, respectively):

$$\tau_\pm = {}^3/_8\pi^{-1}\gamma_0 S(\nu) N \left[\left(\frac{R_{11} + R_{1-1}}{4} \right) (1 + \cos^2 \vartheta_{obs}) \pm \cos \vartheta_{obs} \left(\frac{R_{11} - R_{1-1}}{2} \right) + R_{10} \frac{\sin^2 \vartheta_{obs}}{2} - R_{00} \right], \quad (40)$$

or in terms of the spin temperatures

$$\tau_\pm = -\frac{3}{8\pi} \gamma_0 S(\nu) (N R_{00}) \frac{T_*}{T_{S\pm}} \left[1 + \frac{\sin^2 \vartheta_{obs}}{2} \left(\frac{T_{S\pm} - T_{S_0}}{T_{S_0}} \right) \pm \cos \vartheta_{obs} \left(\frac{T_{S_1} - T_{S_{-1}}}{T_{S_1} + T_{S_{-1}}} \right) \right], \quad (41)$$

where $T_{S\pm} \equiv (T_{S_1} + T_{S_{-1}})/2T_{S_1}T_{S_{-1}}$ is harmonic mean of the temperatures $T_{S_{+1}}$ and $T_{S_{-1}}$.

The attenuation (or amplification) coefficient of linearly polarized 21-cm radiation has the form (the angle α_{obs} determines the plane of polarization):

$$\tau_\alpha = {}^3/_8\pi^{-1}\gamma_0 S(\nu) N \left[\left(\frac{R_{11} + R_{1-1}}{4} \right) \times (1 + \cos^2 \vartheta_{obs} + \sin^2 \vartheta_{obs} \cos 2\alpha_{obs}) + \frac{R_{10}}{2} \sin^2 \vartheta_{obs} (1 - \cos 2\alpha_{obs}) - R_{00} \right], \quad (42)$$

or in terms of the spin temperatures:

$$\tau_\alpha = -\frac{3}{8\pi} \gamma_0 S(\nu) (N R_{00}) \times \frac{T_*}{T_{S\pm}} \left[1 + \frac{\sin^2 \vartheta_{obs}}{2} \left(\frac{T_{S\pm} - T_{S_0}}{T_{S_0}} \right) (1 + \cos 2\alpha_{obs}) \right]. \quad (43)$$

The attenuation (or amplification) coefficient of unpolarized radiation is

$$\tau = \frac{3}{8\pi} \gamma_0 S(\nu) N \times \left[\left(\frac{R_{11} + R_{1-1}}{4} \right) (1 + \cos^2 \vartheta_{obs}) + \frac{R_{10}}{2} \sin^2 \vartheta_{obs} - R_{00} \right], \quad (44)$$

or in terms of the spin temperatures

$$\tau = -\frac{3}{8\pi} \gamma_0 S(\nu) (N R_{00}) \frac{T_*}{T_{S\pm}} \left[1 + \frac{\sin^2 \vartheta_{obs}}{2} \left(\frac{T_{S\pm} - T_{S_0}}{T_{S_0}} \right) \right]. \quad (45)$$

A study of the passage of 21-cm radio emission through a medium containing atomic hydrogen is the most convenient method of determining the character and degree of orientation of the HI atoms.

6. COHERENT AMPLIFICATION OF 21-cm RADIO EMISSION

An oriented medium will amplify or attenuate the transmitted radiation ($\tau > 0$ or $\tau < 0$), depending on whether population inversion will set in or not, i.e., $T_{S_M} < 0$ or $T_{S_M} > 0$. The interaction of HI atoms with ultraviolet radiation, unlike the interaction with radio emission, can lead to inversion of the populations of the hyperfine-structure levels of the ground state $1S_{1/2}$. In this case the medium consisting of atomic hydrogen will coherently amplify the 21-cm radio emission.

There are two essentially different causes of population inversion: the unique character of the spectrum, and the anisotropy of the radiation field. This can be seen from the following.

On the one hand, ultraviolet radiation with a rapidly growing spectrum in the region $\lambda \sim 1216$ Å ($Dn/d\nu > {}^9/_2\gamma_0/\gamma_F''\nu_0$) leads, according to (27), to $T_{S_1} = T_{S_2} = T_{S_{-1}} < 0$ even in the case of isotropic angular distribution and in the absence of polarization. The total gain of the radio emission will not depend here on either the direction or the polarization:

$$\tau = \frac{3}{8\pi} \gamma_0 S(\nu) (N R_{00}) \left[\frac{1}{n(\nu)} \frac{dn(\nu)}{d\nu} \nu_0 \right]. \quad (46)$$

On the other hand, a directed beam of ultraviolet radiation, even with a constant spectrum and in the absence of polarization, leads, according to (35), to $T_{S_0} = 5.2$°K and $T_{S_{+1}} = T_{S_{-1}} - 5.2$°K. In this case the total gain of the radio emission will depend on the observation direction ϑ_{obs} and on the position angle α_{obs} of the linear polarization:

$$\tau_\alpha = \frac{3}{8\pi} \gamma_0 S(\nu) (N R_{00}) \frac{T_*}{T_{S_0}} (\cos^2 \vartheta_{obs} + \sin^2 \vartheta_{obs} \cos 2\alpha_{obs}). \quad (47)$$

The foregoing conditions, which are necessary for coherent amplification of 21-cm radio emission, can be produced in laboratory installations. Particular interest attaches, however, to the possibility of coherent amplification of the 21-cm radiation in astrophysics. Tremendous regions of space are filled with atomic hydrogen. The density of outer space is negligibly small, so that the collisions have little effect, but the radiation fluxes, and indeed the anisotropic fluxes, are in many cases quite large. Finally, the gigantic dimensions of the amplifying system and the practical absence of losses make it possible, in principle, to attain considerable amplification of the radiation even at low density of the medium and small degree of population inversion. All this gives grounds for assuming that outer space can have the same properties as a quantum amplifier or generator.

In conclusion, I am deeply grateful to Professor A. Z. Dolginov for a useful discussion of this work.

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Translated by J. G. Adashko
30