ON THE POSSIBLE EXISTENCE OF A QUARK STATE OF MATTER IN STARS

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It is shown that a quark state of matter is possible in superdense stars and that if quarks are parafermions they may constitute the dominant particle concentration.

IT is very important to know the properties of extremely dense matter in connection with various cosmological problems. Many authors have discussed this subject; a bibliography is attached to the doctoral dissertation of G. S. Saakyan.[1] It appears that such objects as stars, after expending their internal energy supply and cooling down, should be converted into superdense bodies consisting of a degenerate gas of elementary particles; this state of matter merits special attention. Investigations have heretofore been devoted mainly to the densities of matter consisting of a gas of ordinary elementary particles (to 10^{41} particles/cm^3).

If, as is now frequently being suggested, strongly interacting particles consist of quarks, the following question arises: Can a state of matter exist (obviously, at extremely high densities such that the elementary particles can break down into their constituent quarks) in which the mass consists mainly of quarks having a concentration greater than that of all other components? The present paper contains some calculations relating to this question.

As is customary, we shall consider matter at absolute zero temperature, consisting of a mixture of quarks and hadrons. For simplicity it will be assumed that quarks can perform a transition only to the neutron state:

$$n \rightleftarrows 3q. \qquad (1)$$

Neutrons, like all other baryons, are fermions; for a degenerate Fermi gas we therefore have

$$N_n / V = p_0{}^3 / 3\pi^2\hbar^3, \qquad (2)$$

where p_0 is the maximum Fermi momentum and N_n/V is the neutron density.

We cannot state in advance by what form of statistics the quarks are governed. Moreover, the usual construction of all baryons out of quarks encounters a difficulty associated with wave-function symmetry. Some authors (as in[2], for example) postulate that in place of the conventional Fermi statistics the quarks obey a form of parastatistics,[3] although other explanations are also possible.[4,5] To permit writing different thermodynamic relations for particles with this form of statistics we must calculate the partition function Sp $e^{-\hat{H}/\Theta}$. However, since we regard the quarks as an ideal gas at absolute zero, it is not actually necessary to calculate this sum. Instead, a density relationship is derived in the same form as for ordinary fermions from a calculation of the ground state. This does not depend on the maximum occupation number in the parastatistics that we are using. Therefore we have

$$N_k / V = gp_k{}^3 / 6\pi^2\hbar^3, \quad k = 1, 2, 3. \qquad (3)$$

Here N_k/V is the density of the k-th quark and g is the maximum number of quarks permitted in a single quantum state.

Since we do not know the true statistics of quarks, g is a theoretical parameter. The foregoing argument means that for parastatistics with any value of g the distribution functions approach the usual step-function form as the temperature approaches zero. It can be shown analogously that both the nonrelativistic and relativistic pressure formulas have the same form at absolute zero as that for ordinary fermions when the factor g is included.

Since spin 1/2 is assumed for quarks they can only be parafermions, as has been proved in[3]. If we do not use the commutation relations that define the parastatistics,[3] but assume that quarks are paraoscillators[6] and that the creation and destruction operators for different states either all commute or all anticommute, then we can write the same formula (3) for the density. The distribution function is then easily calculated from the thermodynamic potential

$$\Omega_i = -T \ln \frac{1 - \exp[(g+1)(\mu - \varepsilon_i)/T]}{1 - \exp[(\mu - \varepsilon_i)/T]}, \qquad (4)$$

with the following mean number of particles in the i-th state:

$$\bar{n}_i = \left[\exp\frac{\varepsilon_i - \mu}{T} - 1\right]^{-1} - (g+1)\left[\exp\frac{\varepsilon_i - \mu}{T}(g+1) - 1\right]^{-1}. \quad (5)$$

For negative chemical potentials and large values of g the second term in $\bar{n}_i$ practically vanishes and we are left with a Bose distribution.

The ratio of the quark and neutron concentrations is

$$q = {}^3/_2 g (p_k / p_0)^3. \quad (6)$$

The lower limit of quark mass is $m_k \geq 8$ BeV; some authors (as in[7]) use $m_k \sim 10$ BeV. One additional possibility suggested in[8] has been considered by Just.[9] The universal constants ħ, c, and κ can be used to construct a quantity having the dimension of a mass with the numerical value $m_0 \sim 10^{19}\, m_n$. Particles having this mass, called maximons, can perhaps be identified with quarks. For either of these masses the condition that the quark state of matter will be allowed energetically is

$$3E_k \leqslant E_n, \quad (7)$$

where E_k and E_n are the Fermi levels for quarks and neutrons.

The ideal gas model does not take into account the finite sizes of particles but this must be done when studying atoms (such as He^3) at high pressures. However, the meaning of the size of a neutron is altogether unclear. It is not safe to make the usual analogy with atoms at high pressures, because atoms have structures that can be destroyed at high pressures and especially when compressed into very low volumes. Neutrons can never be decomposed into pions and other particles by analogy with the decomposition of a helium atom into a nucleus and electrons. Moreover, the concept of proton or neutron size refers only to the range of nuclear forces, which depends on the mass of the quanta in the nuclear field. In this sense an electron is infinitely large. In modern theory the "real" size of both electrons and neutrons is given by the assumption that they are point particles. Consequently their self-energies, calculated according to canonical rules, diverge and require renormalization. Only in the primitive nonrelativistic quark model of a nucleon can we refer to the size of the latter as that of a system of particles. This size is entirely arbitrary at the present time and must not be confused with the range of nuclear forces, which may be identical for both quarks and neutrons. Also, the true situation may differ greatly from the primitive model of the neutron as a simple dynamical system of quarks.[10] It cannot be demonstrated at present that neutron size must be taken into account at high densities. In other words, the treatment of neutron size in the present problem is a somewhat open question, requiring special discussion, although it will be permissible, with this reservation, to use the ideal gas model.[1)]

We shall first consider the case of nonrelativistic quarks in conjunction with relativistic neutrons. The condition (7) becomes

$$3(m_k c^2 + p_k^2 / 2m_k) \leqslant c p_0. \quad (8)$$

For the case of equality, from (8) and (6) we have

$$q = \frac{1}{18} g (p_k / m_k c)^3. \quad (9)$$

We have $q \gg 1$ only when

$$g \geqslant 18 (m_k c / p_k)^3 q. \quad (10)$$

The conditions of the nonrelativistic approximation for quarks are

$$p_k / m_k c = 1/\alpha \ll 1. \quad (11)$$

Taking numerical values of the order 10^2 for q and a, we obtain $g \geq 2 \times 10^9$. It follows that the quark concentration is negligibly small in conventional Fermi statistics (g = 2). With $g = 2 \times 10^9$ and quark mass ~ 10 BeV the particle density is $\sim 10^{46}/cm^3$ and the pressure is $\sim 10^{34}$ atmospheres. It should be noted that this density $\sim 10^{46}/cm^3$ is reasonable because, obviously, when we go to a quark state from a hyperon gas, which prevails at densities $\sim 10^{41}/cm^3$, this gas must be highly compressed within definite limits. If we take $10^{19}\, m_n$ for the quark mass along with the same values of the parameters α and q the density increases by a factor of 10^{54} and the pressure by a factor of 10^{72}.

In the case of relativistic quarks (7) becomes

$$3p_k \leqslant p_0. \quad (12)$$

For the limiting case of equality we have

$$q = g / 18. \quad (13)$$

It is sufficient to take $g = 10^2 — 10^3$ in order to obtain a high concentration of quarks in this case.

The relativistic condition imposes the lower

[1)]The authors are indebted to Ya. B. Zel'dovich, who pointed out the importance of considering the neutron size in the present problem and brought to our attention the article of Ivanenko and Kurdgelaidze, [11] which discusses the posibility of a quark state of matter in stars.

limit $cp_k \geq 10^2$ BeV on the quark momentum for quark mass ~ 10 BeV, and $cp_k \geq 10^{20}$ BeV for quark mass 10^{19} m_n. For the limiting conditions we obtain $\sim 10^{48}/\text{cm}^3$ as the particle density and $\sim 10^{26}$ g/cm^3 as the mass density for quarks of mass ~ 10 BeV. It should be noted that with such statistics ($g \sim 10^2$—10^3) the quark concentration cannot be dominant at nonrelativistic energies. In both the relativistic and nonrelativistic cases very dense matter enters the quark state if quarks are parafermions, but not if they are fermions. In the extreme relativistic case the major amount of mass can be concentrated in quarks even if their concentration is small compared with the concentration of other components. However, this case is impossible for $m_k \sim 10$ BeV. With the given parameters the quark gas pressure is $\sim 10^{40}$—10^{41} atm. For quark mass $\sim 10^{19}$ m_n the density is increased by a factor of 10^{54} and the pressure by a factor of 10^{72}, as in the nonrelativistic case.

The preceding discussion shows that the quark state of matter is possible in superdense stars, and that if the quarks are parafermions their concentration can be dominant, with the other components becoming negligible in all investigations.

The possibility that has been examined here could be important for investigations of the early stage in the evolution of the Universe.

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