ELECTROMAGNETIC EXCITATION OF MAGNETOELASTIC WAVES IN FERROMAGNETS

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Excitation of a magnetoelastic wave in a ferromagnet with an inhomogeneous internal stationary magnetic field by a monochromatic electromagnetic wave is considered. It is shown that the transformation coefficient should experience gigantic oscillations as a function of the stationary external magnetic field strength and of the frequency of the incident electromagnetic wave. The problem is solved for weak coupling between the two magnetoelastic wave branches (strong coupling between the spin and elastic waves[3]) when transition from one wave to another occurs at the boundary of the sample.

1. INTRODUCTION

As shown by Schlomann and his co-workers[1-2], a homogeneous microwave field can excite in a ferromagnet spin waves with wave vector **k** different from zero, if the constant magnetic H(z) inside the sample is inhomogeneous. Figure 1 shows schematically the distribution of the field H(z) along the sample, and the corresponding values of the wave vector of the spin wave (at fixed frequency ω equal to the frequency of the microwave field)

$$k_m^2(z) = \frac{1}{D}\left[\frac{\omega}{\gamma} - H(z)\right], \qquad (1.1)$$

where D is a volume constant and γ the gyromagnetic ratio.

If $H_{min} < \omega/\gamma < H_{max}$, then $k^2(z)$ vanishes at a certain point z_0 where $H(z_0) = \omega/\gamma$, so that the point z_0 (the turning point) divides the sample into two regions: to the left the wave can propagate and to the right ($k^2(z) < 0$) it cannot. In the vicinity of this point, according to Schlomann[1], a spin wave with $\mathbf{k} = 0$ is excited and propagates (to the left) in the inhomogeneous field and goes over into a spin wave with $\mathbf{k} \neq 0$.

If we take into account the magneto-elastic interaction, then the spectrum will not have the form (1.1), and there are two branches of magnetoelastic waves (Fig. 2), the distance between which has a minimum at the point z_1 (intersection point), where the curves of the spectrum of the transverse elastic waves ($k_p^2 = (\omega/s)^2$, s—velocity of transverse sound) would intersect the spin waves (1.1) (if magnetoelastic interaction were to be disregarded). Near the crossing point, the spin wave excited by

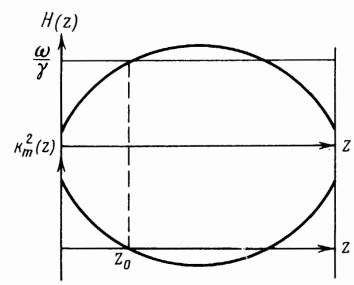

FIG. 1.

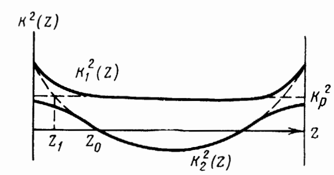

FIG. 2.

FIG. 1. Variation of internal field H(z) and of the squared spin-wave vector $k_m^2(z)$ (1.1) with sample length (z)

($\mathbf{k} \parallel 2$, $\mathbf{H} \parallel z$, $\nabla\mathbf{H} \parallel z$)

FIG. 2. Variation of squared magnetoelastic-wave vector with sample length (z). The dashed line shows the squared wave vector of the noninteracting spin and elastic waves (k_p^2).

the microwave field at the turning point will interact with the elastic wave, and this will excite sound.

Qualitatively, the interaction of two branches at the point of intersection is determined, as shown in[3], by the value of the parameter $a^2[k'(z_1)]^{-1}$, where a is the minimum difference in the wave vectors $k_2(z_1) - k_1(z_1)$ of the two branches. (The prime here and throughout denotes differentiation with respect to the variable z.) Physically this parameter represents a change in the phase difference (δ) of the interacting waves over the interaction length (l), $\delta \approx (k_2 - k_1)l \approx al$. Since the interaction occurs in the entire region where $k_2 - k_1 \approx a$, we have $l \approx a/k'$ and $\delta \approx a^2/k'$. If $\delta \gg 1$, then the interaction of the branches along the length l reverses sign many times, so that the wave does not go over from branch to branch. This is the case of weak coupling between branches. It corresponds to a strong coupling between the magnetic and elastic waves, since the wave moving along the same branch is transformed, on passing through the point z_1, from magnetic (elastic) into elastic (magnetic) (if the interaction is small far from the crossing point, so that the oscillations are separated there into pure spin and pure elastic). In the opposite case ($\delta \ll 1$) the interaction between the branches is strong, so that the wave goes over practically entirely from one branch to the other, i.e., it remains a spin wave.

The quantitative analysis made by Schlomann and Joseph[1-3] was based on the assumption that the decoupling parameter $a \ll k_p$, and that the problem of the excitation of sound waves by the microwave field was solved stage by stage. The excitation of the spin wave by the microwave field was considered separately[1,2], independently of the transition from the spin wave into the elastic wave[3]. We have carried out the corresponding calculation without assuming ak_p^{-1} to be small, and considered the interaction of the microwave field directly with the magnetoelastic waves.

We investigated the case of weak coupling between the branches ($\delta \gg 1$), when the wave excited by the microwave field propagates along the branch k_2 without going over to the branch k_1. It was assumed that the field H(z) changes sufficiently slowly ($k'k_p^{-2} \ll 1$), making it possible to employ the WKB method. In this approximation, as shown by calculation, the value of the electromagnetic energy absorbed by an infinite sample does not depend on the decoupling parameter a, if the wave k_2 approaches asymptotically that of sound ($k_2(-\infty) = k_p$). In a number of experiments[4-6], the wave excited at the point z_0 was registered near the right end face of the sample. The situation there corresponded precisely to the case of weak coupling between branches, when there is no transition inside the sample from branch k_2 to branch k_1. On the other hand, the wave can reach the right end face of the sample only if it goes over to the branch k_1. We have considered the transition from the branch k_2 to the branch k_1 at the boundary of the sample.

It has been shown that the power of the excited magnetoelastic wave depends periodically on the distance from the turning point to the end face of the sample, with a period equal to half the wavelength on the boundary, $\lambda_2/2 = \pi/k_{2b} \approx \pi/k_p$ ($k_{2b} \approx k_p$ is the value of k_2 on the boundary). Since the position of the turning point is determined by the value of the magnetic field ($H(z_0) = \omega/\gamma$), the transformation coefficient η depends periodically on the value of the external magnetic field and on the frequency, with corresponding periods

$$\Delta H = H'(z_0)\pi / k_p, \quad \Delta\omega = \gamma\Delta H. \tag{1.2}$$

Strong oscillatory relationships between the amplitude of the received signal and the magnetic field were observed experimentally in later work by Schlomann[4,5]; in the same paper, the idea is advanced that these oscillations are connected in some manner with the presence of a resonator, whose role is assumed for the k_2 wave by the region between the end face and the turning point[1)]. Our calculations show that the oscillations of the transformation coefficient η should be very large:

$$\eta_{max} / \eta_{min} \cong (k_{1b} / k_p)^6 (k_{1b} / a)^4; \tag{1.3}$$

(1.3) was obtained under the assumption that $k_{1b}k_p^{-1} \gg 1$ (k_{1b} is the value of k_1 on the boundary).

2. MAGNETOELASTIC BRANCHES IN AN INHOMOGENEOUS MAGNETIC FIELD

The equation of motion for magnetization and elastic displacements in a monochromatic microwave field with circular polarization are[3]

$$u'' + k_m^2 u - av' = -h(M_s D^{-1})^{1/2}, \quad v'' + k_v^2 v + au' = 0. \tag{2.1}$$

The notation is the same as in [3].

The system (2.1) reduces to a single fourth-order equation

$$u^{IV} + [k_m^2(z) + k_p^2 + a^2]u'' + 2(k_m^2)'u' + [(k_m^2)'' + k_m^2 k_p^2]u = q, \tag{2.2}$$

$$q = -hk_p^2(M_s D^{-1})^{1/2}. \tag{2.2'}$$

Since usually, in the appropriate experiments, the exciting field acts only in one half of the sample (the left half in Fig. 2), the right-hand turning point will be disregarded. We therefore consider a spectrum of the type shown in Fig. 3. We study first the free oscillations of such a system, i.e., the solutions of Eq. (2.2) with q = 0.

1. Far from the turning point (z = 0), in the WKB approximation ($k'k^{-2} \ll 1$) the fourth linearly independent solutions are of the form

$$u = \exp\left\{\pm i \int^z k_{1,2}\, d\xi + f_{1,2}\right\}. \tag{2.3}$$

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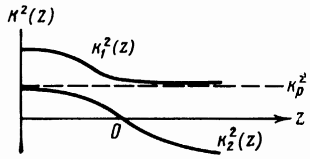
1) Our expression (1,2) for ΔH differs from that given in [4,5] (from intuitive considerations concerning the motion of the crossing point) in that (1.2) contains the gradient of the internal magnetic field at the turning point (and not the crossing point), so that the cause of the oscillations lies precisely in the motion of the turning point (and not the crossing point).



where $k_{1,2}^2$ ($k_1^2 > k_2^2$) are two solutions of the equation

$$k^4 - k^2(k_m^2 + k_p^2 + a^2) + k_m^2 k_p^2 = 0, \tag{2.4}$$

and $f' \sim k'$ are determined in the next approximation in the parameter $k'k^{-2}$:

$$f_1' = -\frac{(k_1^2)' - (k_m^2)'}{k_1^2 - k_2^2} - \frac{k_1'}{2k_1}\quad f_2' = \frac{(k_2^2)' - (k_m^2)'}{k_1^2 - k_2^2} - \frac{k_2'}{2k_2}. \tag{2.5}$$

From (2.3)—(2.5) we obtain[2)]

$$u_{1,2} = Gk_1^{-1/2}\varphi(z)\exp\left\{\pm i\int_{-d}^{z} k_1\,d\xi\right\}, \tag{2.6a}$$

$$u_{3,4} = Fk_2^{-1/2}\psi(z)\cos\left(\int_0^z k_2\,d\xi + \frac{\pi}{3} \mp \frac{\pi}{12}\right); \tag{2.6b}$$

$$\begin{aligned}\varphi(z) &= (k_1^2 - k_p^2)^{1/2}(k_1^2 - k_2^2)^{-1/2},\\ \psi(z) &= [(k_1^2 - k_p^2)(k_1^2 - k_2^2)]^{-1/2},\\ G &= \omega^{-1/2},\ F = 2\omega^{-1/2}ak_p.\end{aligned} \tag{2.6c}$$

$u_{3,4}$ were chosen such as to ensure the joining together with the corresponding solutions at $z \to 0$ (see (2.8) and (2.9)). The normalization factor G is such that the energy flux density in the waves $u_{1,2}$ is[3]

$$S = \omega\,\mathrm{Im}\,[u^*u' + v^*(v' + au)] = \mp 1.$$

The point (−d) will be left undetermined for a while. The factor F is determined from the following considerations. Let there be a wave $e_- = C_3u_3 + C_4u_4$ propagating in the direction of negative z (i.e., $C_3 = -C_4\exp(-i\pi/6)$, see (3.4)). At the chosen normalization we have

$$|S(u_-)| = +|C_4|^2.$$

Unlike the WKB solution of a second-order equation[7], the amplitudes $\varphi(z)$ and $\psi(z)$ in (2.6a) and (2.6b) depend on the coordinate. Physically their spatial variation reflects the transition of the same wave from magnetic to elastic in different points of the sample. Therefore, for example, as can be readily verified, if $k_1(\infty) \to k_p$, then $\varphi(\infty)\varphi^{-1}(-\infty) \ll 1$, since the wave $u_{1,2}$ goes over on moving from left to right from magnetic to elastic.

2. The condition for the applicability of the WKB method for the branch k_2 is violated near the turning point. Since the wavelength at this point becomes infinite for the corresponding solutions, these solutions themselves will be noticeably altered, as will be shown later (see (2.8)—(2.10)), at large distances $L \approx [(k_2^2)']_{z=0}^{-1/3}$, so that $Lk_2 \gg 1$. Therefore the equation for two slowly varying solutions near z = 0, discarding small terms and expanding $k_m^2 = (k_m^2)'z$, takes the form

$$u'' - \alpha^2 zu = 0, \tag{2.7}$$

$$\alpha^2 = -(k_m^2)'_{z=0}[1 + (a/k_p)^2]^{-1} = -(k_2^2)'_{z=0} > 0. \tag{2.7'}$$

In the case in question $(a^2(k')^{-1} \gg 1$, as shown in[3], there is no transition from branch k_2 to branch k_1. Therefore suitably chosen linearly-independent solutions (2.7) and the solutions $u_{3,4}$ (2.6) constitute two linearly independent solutions of (2.2), given for different values of z. Two other linearly independent solutions are $u_{1,2}$ (2.6) in the entire region of variation of z.

The solutions of (2.7), which go over at large negative z into $u_{3,4}$ (2.6), are[8]

$$u_3 = \frac{\pi^{1/2}}{3}\left(\frac{2}{3}\alpha\right)^{-1/3}F\psi(0)t^{1/3}[J_{1/3}(t) + J_{-1/3}(t)] \quad (z < 0),$$

$$u_3 = \frac{1}{(3\pi)^{1/2}}\left(\frac{2}{3}\alpha\right)^{-1/3}F\psi(0)t^{1/3}K_{1/3}(t) \quad (z > 0), \tag{2.8}$$

$$u_4 = \left(\frac{\pi}{3}\right)^{1/2}\left(\frac{2}{3}\alpha\right)^{-1/3}F\psi(0)t^{1/3}J_{1/3}(t) \quad (z < 0), \tag{2.9}$$

$$u_4 = -\left(\frac{\pi}{3}\right)^{1/2}\left(\frac{2}{3}\alpha\right)^{-1/3}F\psi(0)t^{1/3}I_{1/3}(t) \quad (z > 0),$$

$$t = {}^2/_3\alpha|z|^{3/2}. \tag{2.10}$$

Here $J_\nu(t)$, $I_\nu(t)$ and $K_\nu(t)$ are cylindrical functions. The asymptotic form of (2.8) and (2.9) at large values of z < 0 coincides with (2.6). For large z > 0, u_3 decreases exponentially and u_4 increases exponentially.

3. EXCITATION OF WAVES BY A MICROWAVE FIELD

Equation (2.6b) together with (2.8) and (2.9) describes two waves belonging to the branch k_2. It is precisely these waves, inasmuch as they have a turning point ($k_2 = 0$), which can be excited in the vicinity of this point by a homogeneous microwave field. On the other hand the branch k_1 (2.6a) does

2)The expression given here for $u_{3,4}$ is valid far from the turning point from the left (when z < 0). However, the form of $u_{3,4}$ from the right for large z > 0 is not needed here.

not interact directly with a homogeneous microwave field. In order to obtain an expression for the excited spin waves, it is necessary to solve the inhomogeneous equation (2.2). Since the solutions of the homogeneous equation u_{1-4} are known, we can use the Lagrange method

$$u = \sum_{i=1}^{4} C_i(z) u_i(z). \qquad (3.1)$$

Then $C_i' = qW_iW^{-1}$, with

$$W_{1,2} = \mp \begin{vmatrix} u_{2,1} & u_3 & u_4 \\ u_{2,1}' & u_3' & u_4' \\ u_{2,1}'' & u_3'' & u_4'' \end{vmatrix}, \quad W_{3,4} = \mp \begin{vmatrix} u_1 & u_2 & u_{4,3} \\ u_1' & u_2' & u_{4,3}' \\ u_1'' & u_2'' & u_{4,3}'' \end{vmatrix}, \qquad (3.2)$$

W = const is the Wronskian of (2.2).

We consider two cases: an infinite medium (1), and a semi-infinite medium bounded from the left (2). In both cases, the boundary conditions at $z = 0$ reduce to the requirement that the solution be finite and that there be no reflected waves (from the point $z = \infty$). This yields

$$C_4(+\infty) = C_1(+\infty) = 0. \qquad (3.3)$$

The boundary conditions for $C_{2,3}$ are different for the two situations analyzed below.

1. If the medium is infinite, then there should be no waves reflected from the point $z = -\infty$, i.e.,

$$C_2(-\infty) = 0, \quad C_3(-\infty) + C_4(-\infty)\exp\left(-i\frac{\pi}{6}\right) = 0. \qquad (3.4)$$

From (3.2) we see that

$$C_{1,2}' \sim u_{2,1} \sim \exp\left(\mp i \int^{z} k_1\, d\xi\right)$$

is a rapidly oscillating function, and therefore

$$C_1(z) \approx C_1(\infty) = 0, \quad C_2(z) \approx C_2(-\infty) = 0. \qquad (3.5)$$

Thus,

$$u = C_3(z) u_3(z) + C_4(z) u_4(z). \qquad (3.6)$$

At the chosen normalization of the functions $u_{3,4}$ (2.6b) and (2.6′) and of the condition (3.4), the energy flux carried away by the excited wave is $|C_4(-\infty)|^2$, where

$$C_4(-\infty) = qW^{-1}\int_{\infty}^{-\infty} W_4\, dz,$$

and a contribution is made to this integral, obviously, only near the region of small z, for when z is large the integrand in (3.2) oscillates strongly ($z < 0$) or decreases exponentially ($z > 0$). Therefore we can use in the calculations the expressions for $u_{3,4}$ (2.8) and (2.9). The appropriate calculations yield

$$C_4(-\infty) = q\alpha^{-1}\frac{2\pi^{1/2}}{F\psi(0)k_1^2(0)}\cdot\frac{1}{3}\int_0^{\infty} dt \times \left[J_{1/3}(t) + J_{-1/3}(t) + \frac{\sqrt{3}}{\pi}K_{1/3}(t)\right]. \qquad (3.7)$$

Using (2.2), (2.6), and (2.7) and recognizing that $k_1^2(0) = k_p^2 + a^2$ (2.4), we obtain the density of the energy flux carried away by the excited wave per unit time

$$S = 4\pi\frac{M_s}{D}h^2\frac{k_p{}^2 a^2}{|(k_m{}^2)'|_{z=0}}F^{-2}. \qquad (3.8)$$

In the limiting cases when $a \ll k_p$ or $k_p^2 \ll k_m^2(-\infty) + a^2 = k_1^2(-\infty)$, the wave outgoing along the k_2 branch is elastic, and the energy flux density in it is independent of the coupling parameter a[3]:

$$S = \pi M_s \omega h^2 (H')^{-1}_{z=0}, \qquad (3.9)$$

in accord with the result of[3], which, however, was obtained under the assumption that $a \ll k_p$ in an interaction between an electromagnetic microwave field and a wave which is pure spin (at the turning point).

2. We now consider a semi-infinite medium with a boundary situated at the point $z = -d$[4]. We are interested in a wave that goes over to the region $z = \infty$. Since the wave cannot propagate from the point $z = 0$ along the branch k_2 to the right, and the wave k_1 does not interact directly with the homogeneous electromagnetic field, it follows that only the wave k_1, excited by the wave k_2 from the surface ($z = -d$), will pass into the region $z = \infty$. Since the wave k_2 is excited at the point $z = 0$, it is clear that the coefficient of wave transition from the branch k_2 to k_1 should depend periodically on the distance from the turning point to the boundary, with a period equal to half the wavelength $\lambda_2 = 2\pi k_{2b}^{-1}$ (the index b will henceforth denote quantities on the boundary at $z = -d$).

Going over to concrete calculations, we note that the boundary conditions (3.3) remain in force also in this case, as does the first relation of (3.5). The second relation of (3.5) takes the form

$$C_2(z)\,|_{z\to-\infty} \equiv C_2(-d). \qquad (3.10)$$

If the boundary is far enough from the turning point (beyond the region of interaction of the k_2 waves with the microwave field), so that $d \gg |(k_2^2)'|^{-1/3}$, then $C_4(-d) \approx C_4(-\infty)$ and is determined as before

[3]To avoid misunderstanding, we recall that we cannot assume that $a = 0$, since we have assumed that $a^2(k')^{-1} \gg 1$.

[4]We simultaneously choose, by the same token, a definite phase factor in the waves $u_{1,2}$ (2.6).

by formula (3.7). At the chosen normalization of the k_2 wave, the energy flux density carried away is $|C_2|^2$. For the boundary conditions on the end face $z = -d$ we choose the vanishing of the normal component of the field intensities and of the spatial magnetization at these points, i.e.,[5]

$$v' + au = u'' + (k_m^2 + a^2)u = 0 \Big| \quad u' = 0 \Big|_{z=-d}.$$

From this, using (2.6b) and (2.6′), we obtain

$$C_2 = iC_4\beta^{1/2}(\cos\varphi + i\beta\sin\varphi)^{-1}, \qquad (3.11)$$

$$\beta = \frac{k_{2b}}{k_{1b}}\frac{k_p^2 - k_{2b}^2}{k_{1b}^2 - k_p^2} \quad (1 > \beta > 0), \qquad \varphi = \int_{-d}^{0} k_1\,d\xi - \frac{\pi}{4}. \qquad (3.11')$$

Thus, the energy flux density carried away by the wave is

$$S = S_\infty\beta[\cos^2\varphi(d) + \beta^2\sin^2\varphi(d)]^{-1}, \qquad (3.12)$$

S_∞ is the energy flux density carried away by the wave in an infinite sample (3.9).

As expected, the transformation coefficient η of the sample oscillates as a function of the distance from the turning point to the end face of the sample, with

$$\eta_{max}\eta_{min}^{-1} = \beta^{-2}. \qquad (3.13)$$

If on the end face we have $k_{2b} \approx k_p \ll k_{1b}$ and $k_{1b} \approx (k_m^2 + a^2)^{1/2}$, then

$$\eta_{max}/\eta_{min} = (k_{1b}/k_p)^6(k_{1b}/a)^4 \gg 1, \qquad (3.14)$$

so that (3.14) is a giant quantity in this rather typical case. Thus, we have interference between a sequence of waves, in which each preceding wave traverses a path which is shorter by 2d than the succeeding wave. Understandably, the change in the distance 2d by λ_{2b} does not change the relative phase shift of two successfully waves, as a result of which S (Eq. (3.12)) is indeed a periodic function of d with a period $\lambda_{2b}/2$[6]. Corresponding to this is also the periodicity of the magnetic field and of the frequency with periods (1.2).

The oscillatory dependence of (3.12) has also another aspect. The poles ω_j of the denominator of (3.11)

$$\cos\varphi(d,\omega) + i\beta\sin\varphi(d,\omega) = 0 \qquad (3.15)$$

obviously give the spectrum of the natural frequencies of the resonator made up of the region between the turning point and the boundary (for the k_2 wave).

These are complex frequencies, since the interaction on the surface with the wave k_1 makes this resonator open. Neglecting this interaction ($\beta = 0$) makes these frequencies real and they can be determined from the condition

$$\int_{-d}^{0} k_2\,d\xi = \pi\left(n + \frac{3}{4}\right) \qquad (n = 0, 1, \ldots). \qquad (3.16)$$

Thus, resonant absorption of the microwave field by the aforementioned resonator takes place, with the parameter β playing the role of friction, so that when the resonance condition (3.16) is satisfied the power absorbed by the resonator is inversely proportional to β.

The authors thank A. G. Gurevich who called their attention to this topic and E. K. Kudinov and B. D. Laĭkhtman for a useful discussion.

Note added in proof (2 August 1966). We have recently learned that the transition from the k_2 branch to the k_1 branch on the boundary was considered in [9]. However, the value τ_{21} calculated in [9] (called by the authors the coefficient of transition from branch 2 to branch 1), which is equal to the ratio of the sound amplitudes at waves $\exp(ik_1z)$ and $\exp(-ik_2z)$, is not amplitude transition coefficient in the customary sense, since the square of this quantity is not equal to the ratio of the energy fluxes in these waves. Under the boundary conditions employed by us, the root of the ratio of the energy fluxes is $\tau_{21} = 2\beta^{1/2}(1+\beta)^{-1} \approx 2\beta^{1/2}$. The reasons are as follows: 1) since the wave vectors are different in the branches k_2 and k_1, it is necessary to calculate the amplitude ratio at the waves $k_{1,2}^{-1/2}\exp(ik_{1,2}z)$ and not $\exp(ik_{1,2}z)$; 2) since the k_1 wave on the boundary is almost pure spin, and K_2 is almost elastic, the transition coefficient is determined by the ratio of the amplitude of the magnetic component in the wave $k_1(u_1)$ to the amplitude of the elastic component in the wave $k_2(v_2)$ (on the boundary). Therefore

$$\tilde{\tau}_{21} = \tau_{21}\left(\frac{k_1}{k_2}\right)^{1/2}\frac{u}{v_1} \approx 2\beta^{1/2} \gg \tau_{21}.$$

[5]We shall assume for simplicity that the field h at the boundary is zero ($q(-d) = 0$).

[6]We note that the value of S averaged over the position of the turning point relative to the boundary at distances exceeding λ_{2b} equals, as it should S_∞. This averaging can be carried out, for example, if the constant field inside the sample is inhomogeneous in the transverse cross section, and its variation in this cross section exceeds ΔH (1.2) or if the wave is nonmonochromatic and the scatter of the frequencies in it exceeds $\Delta\omega$ (1.2).

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