QUASILINEAR THEORY OF CERENKOV AND CYCLOTRON DAMPING OF ELECTROMAGNETIC WAVES IN A PLASMA

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Cerenkov absorption of Alfven and fast magnetic-sound waves in a plasma with a high gas-kinetic pressure ($4\pi n_0 T/H_0^2 \gtrsim 1$) is investigated in the quasilinear approximation. Cyclotron absorption of the fast magnetic-sound wave by the ions and of the ordinary and extraordinary waves by the electrons are investigated in the same approximation in a plasma with a low gas-kinetic pressure ($4\pi n_0 T/H_0^2 \ll 1$) (n_0 and T are respectively the plasma density and temperature and H_0 is the external magnetic field). It is shown that the different components of the Alfven waves as well as the fast magnetic-sound waves which propagate almost parallel to the external magnetic field have different damping rates. The difference in the damping rates is of the order of the reciprocal wave-diffusion time for the ions in velocity space. It is also shown that the formation of a plateau on the ion distribution function can lead to a strong increase (compared with the linear theory) of the Cerenkov absorption of the Alfven waves. (In a plasma with a low gas-kinetic pressure the formation of a plateau always leads to a reduction of the damping rate). The damping rates for the cyclotron damping of the fast magnetic-sound wave and of the ordinary and extraordinary waves can also increase with wave amplitude as a consequence of the plateau formation.

1. INTRODUCTION

Cerenkov and cyclotron absorption of the energy of different branches of the electromagnetic waves in a uniform magnetoactive plasma have been investigated in the quasilinear approximation in a paper by Rowlands and the present authors.[1] However, the expressions obtained in [1] for the damping rates of the Alfven and fast magnetic-sound waves apply only for a low-pressure plasma, in which case the Cerenkov absorption of these waves by the ions is exponentially small.

In the present work we determine the damping rate for the Alfven wave and also for the fast magnetic-sound wave propagating almost parallel to a magnetic field, this absorption being a consequence of the Cerenkov absorption of these waves by the ions in a plasma with a high gas kinetic pressure for the case in which the Alfven velocity is smaller than or of the order of the ion thermal velocity. It is shown that the different components of the electric field in these waves have different nonlinear damping rates. The difference between the damping rates for the different components is of the order of the reciprocal time for wave diffusion of the ions in velocity space. In contrast with the cases considered earlier, the formation of a plateau on the ion distribution function can lead to a strong enhancement (compared with the linear theory) of the damping as a consequence of the Cerenkov absorption of the Alfven waves.

We also determine the damping rate for the fast magnetic-sound wave (under conditions of ion cyclotron resonance) and the ordinary and extraordinary waves (under conditions of electron cyclotron resonance) in a plasma with a low gas kinetic pressure. The formation of a plateau can also increase the damping rate for these waves.

2. BASIC EQUATIONS

The background distribution function for the resonance particles is found from the equation[1,2]

$$\frac{\partial f^\alpha}{\partial t} = \pi\left(\frac{e_\alpha}{m_\alpha}\right)^2 \sum_{\mathbf{k}} \sum_{n=-\infty}^{\infty} \frac{1}{v_\perp} R\Big\{ v_\perp (Rf^\alpha)\cdot \left| E_1 \frac{n}{a} J_n(a) - i\eta_\alpha E_2 J_n'(a) + \frac{v_\parallel}{v_\perp} J_n(a) E_3 \right|^2 \delta(\omega_\mathbf{k} - n\omega_\alpha - k_\parallel v_\parallel) \Big\} + \frac{\partial}{\partial v_\parallel}\left[D_c^{(\alpha)} \frac{\partial (f^\alpha - f_M^{(\alpha)})}{\partial v_\parallel} \right], \qquad (2.1)$$

where

$$R = \frac{\omega_{\mathbf{k}} - k_{\|} v_{\|}}{\omega_{\mathbf{k}}} \frac{\partial}{\partial v_{\perp}} + \frac{k_{\|} v_{\perp}}{\omega_{\mathbf{k}}} \frac{\partial}{\partial v_{\|}},$$

$$a = \frac{k_{\perp} v_{\perp}}{\omega_{\alpha}}, \quad \omega_{\alpha} = \frac{|e_{\alpha}| H_0}{m_{\alpha} c}, \quad \eta_{\alpha} = \frac{e_{\alpha}}{|e_{\alpha}|},$$

f_M^{α} is the Maxwellian distribution function corresponding to temperature T_{α} for particles of species α, E_i is the projection of the electric field along the axis $\mathbf{e}_i$ ($\mathbf{e}_1 \parallel \mathbf{k}_{\perp}$, $\mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_1$, $\mathbf{e}_3 \parallel \mathbf{H}_0$; $\mathbf{k}_{\perp}$ is the component of the wave vector $\mathbf{k}$ perpendicular to the magnetic field $\mathbf{H}_0$ and $D_c^{(\alpha)}$ is the collisional diffusion coefficient for particles of species α. An expression for $D_c^{(\alpha)}$ is given in [1].

In the general case, the fields E_i will exhibit strong spatial dispersion and, consequently, different damping rates $\gamma = -\mathrm{Im}\,\gamma_i$ where

$$\gamma_i(t) = \partial \ln E_i(t) / \partial t, \tag{2.2}$$

which are determined from the following equations:[3]

$$\Lambda_{ij} E_j = 0, \quad \Lambda_{ij} = \varepsilon_{ij} + k^2 c^2 (k_i k_j / k^2 - \delta_{ij}) / \omega_i \omega_j,$$
$$\varepsilon_{ij} = \delta_{ij} + 4\pi i \sigma_{ij}(\omega_j) / \omega_i, \quad \omega_j = \omega_{\mathbf{k}} + i\gamma_j(t), \tag{2.3}$$

where $\omega_{\mathbf{k}}$ is the frequency for a wave characterized by wave vector $\mathbf{k}$ and $\sigma_{ij}(\omega)$ is the conductivity tensor for the plasma which is given by formulas derived from the linear theory.

We note that Eq. (2.1) can be used only when the following condition is satisfied (cf. [3]):

$$\frac{\Delta k}{k} \gg \frac{\gamma}{k_{\|} |v_{\|} - \partial \omega_{\mathbf{k}} / \partial k_{\|}|}, \tag{2.4}$$

where Δk is the width of the wave packet and $v_{\|}$ is the projection of the velocity of the resonance particles in the direction of the magnetic field. The condition in (2.4) is not satisfied for waves that exhibit a linear dispersion relation $\omega_{\mathbf{k}} \approx kv$ (in particular the Alfven wave and the fast magnetic-sound wave that propagates almost parallel to the magnetic field $\mathbf{H}_0$). The equation for the background distribution function for these waves has a nonlocal time character. However, Eq. (2.1) can be used for these waves in the quasistationary state ($\partial f^{\alpha}/\partial t \approx 0$) for driven oscillations when the wave diffusion of the particles is balanced by collisional diffusion. Eqs. (2.2) and (2.3) also apply for waves with linear dispersion.

3. CERENKOV DAMPING OF MAGNETOHYDRODYNAMIC WAVES

We consider the damping of low-frequency ($\omega \ll \omega_i$) long-wave ($kv_i \ll \omega_i$) oscillations of a plasma which correspond to the magnetohydrodynamic waves of ordinary magnetohydrodynamics. We consider magnetohydrodynamic waves in a plasma with a high ion pressure $\kappa_i = 4\pi n_0 T_i / H_0^2 \gtrsim 1$. The damping of these waves in a plasma with a low pressure ($\kappa_i \ll 1$) has been treated in the quasilinear approximation in [1].) The dispersion equation for these waves is of the following form ($n_j = kc/\omega_{\mathbf{k}} + i\gamma_j$):

$$(\varepsilon_{11} - n_1^2 \cos^2 \vartheta)[\varepsilon_{22} - n_2^2 - \varepsilon_{32}\varepsilon_{23} / (\varepsilon_{33} - n_3^2 \sin^2 \vartheta)]$$
$$= -\varepsilon_{12}\varepsilon_{21} - \{\varepsilon_{32}\varepsilon_{21}(\varepsilon_{13} + n_1 n_3 \sin\vartheta \cos\vartheta)$$
$$+ (\varepsilon_{31} + n_1 n_3 \sin\vartheta\cos\vartheta)[\varepsilon_{12}\varepsilon_{23} - (\varepsilon_{22} - n_2^2)$$
$$\times (\varepsilon_{13} + n_1 n_3 \sin\vartheta\cos\vartheta)]\}(\varepsilon_{33} - n_3^2 \sin^2\vartheta)^{-1}. \tag{3.1}$$

It is known from a linear theory that the right side of Eq. (3.1) is small so that in the zeroth approximation this equation splits into two equations:

$$n_1^2 \cos^2 \vartheta - \varepsilon_{11} = 0, \tag{3.2}$$

$$n_2^2 - \varepsilon_{22} + \varepsilon_{32}\varepsilon_{23} / \varepsilon_{33} = 0. \tag{3.3}$$

The first of these equations determines the frequency of the Alfven wave $\omega = k_{\|} v_A$ while the second determines the frequency of the fast and slow magnetic-sound waves. When $\kappa_i \gtrsim 1$ the magnetic-sound waves are highly damped if the angle ϑ is not close to zero ($\gamma \sim \omega$). Hence, below in the case $\vartheta \sim 1$ we shall only consider the damping of the Alfven wave.

For the Alfven wave we can write $\gamma_i = 0$ in the expressions for the tensor ϵ_{ij} in Eq. (3.1). Under these conditions the tensor ϵ_{ij} can be written

$$\varepsilon_{11} = n_A^2 = \frac{\Omega_i^2}{\omega_i^2} = \frac{c^2}{v_A^2}$$
$$\varepsilon_{12} = i n_A^2 \frac{\omega}{\omega_i} \left[1 + \varkappa_i \left(1 - \frac{3}{2} \mathrm{tg}^2 \vartheta\right)\right] \equiv i n_A^2 \frac{\omega}{\omega_i} q,$$
$$\varepsilon_{13} = -2\varkappa_i n_A^2 \,\mathrm{tg}\,\vartheta \left(\frac{\omega}{\omega_i}\right)^2, \quad \varepsilon_{22} = n_A^2 + 2\varkappa_i n_A^2 \,\mathrm{tg}^2 \vartheta\, f_2,$$
$$\varepsilon_{23} = -i n_A^2 \,\mathrm{tg}\,\vartheta \frac{\omega_i}{\omega} f_1, \quad \varepsilon_{33} = \frac{\omega_i^2 n_A^2 f_0}{\omega^2 \varkappa_i}, \tag{3.4*}$$

where

$$f_{1,2} = 2z_i e^{-z_i^2} \int_0^{z_i} e^{t^2} dt - i\sqrt{\pi}\, z_i e^{-z_i^2} \left(\varphi_{1,2}^{(i)} - \frac{i}{\pi} \psi_{1,2}^{(i)}\right),$$

$$f_0 = 1 + \frac{T_i}{T_e} - 2z_i e^{-z_i^2} \int_0^{z_i} e^{t^2} dt + i\sqrt{\pi}\, z_i e^{-z_i^2} \left(\varphi_0^{(i)} - \frac{i}{\pi} \psi_0^{(i)}\right),$$

*tg ≡ tan.

$$\varphi_n^{(i)} = \left(\int_0^\infty \frac{\partial f^i}{\partial v_\parallel} v_\perp^{2n} dv_\perp^2 \Big/ \int_0^\infty \frac{\partial f_M^i}{\partial v_\parallel} v_\perp^{2n} dv_\perp^2 \right)_{v_\parallel = \omega_k / k_\parallel}$$

$$\psi_n^{(i)} = \int_0^\infty \int_{-\infty}^\infty \frac{\partial (f^i - f_M^i)}{\partial v_\parallel} P \frac{1}{v_\parallel - \omega_k / k_\parallel} v_\perp^{2n} dv_\perp^2 dv_\parallel$$

$$\cdot \left(\int_0^\infty \int_{-\infty}^\infty \frac{\partial f_M^i}{\partial v_\parallel} \delta \left(v_\parallel - \frac{\omega_k}{k_\parallel} \right) v_\perp^{2n} dv_\perp^2 dv_\parallel \right)^{-1}. \quad (3.5)$$

In the dispersion equation (3.1) we can write $\omega_j = k_\parallel v_A$ everywhere except in the expression for n_1 ($n_1 = kc/\omega_1 = kc/\omega_k + i\gamma_1$). Carrying out this procedure we find

$$\frac{\gamma_1}{\omega} = \frac{\omega^2}{\omega_i^2} (q_1 + q_2), \quad (3.6)^\dagger$$

where

$$q_1 = \frac{i}{2} q \frac{\operatorname{ctg}^2 \vartheta \, q f_0 - 2\varkappa_i f_2}{f_0 + \varkappa_i (f_1^2 + 2 f_2 f_0)},$$

$$q_2 = -\frac{i}{2} \varkappa_i \operatorname{tg}^2 \vartheta \frac{1 + 2\varkappa_i f_2}{f_0 + \varkappa_i (f_1^2 + 2 f_2 f_0)}$$

Now, using Eq. (2.3) to find the ratios E_3/E_1 and E_2/E_1 and introducing Eq. (3.4) for ϵ_{ij} we find the difference in the damping rates

$$\gamma_3 - \gamma_1 = \frac{\partial}{\partial t} \ln \frac{E_3}{E_1} = \frac{\partial}{\partial t} \ln \frac{q f_1 + \operatorname{tg}^2 \vartheta (1 + 2\varkappa_i f_2)}{f_0 + \varkappa_i (f_1^2 + 2 f_0 f_2)}$$

$$\gamma_2 - \gamma_1 = \frac{\partial}{\partial t} \ln \frac{E_2}{E_1} = \frac{\partial}{\partial t} \ln \frac{q f_0 - \varkappa_i \operatorname{tg}^2 \vartheta f_1}{f_0 + \varkappa_i (f_1^2 + 2 f_0 f_2)} \quad (3.7)$$

It is evident that when $\vartheta \sim 1$ the quantities $\gamma_3 - \gamma_1$ and $\gamma_2 - \gamma_1$ are of the order of the reciprocal relaxation time for the background distribution function due to diffusion on the waves. It is evident that for sufficiently strong electric fields the difference in the damping rates (3.7) can be of the order of the damping rate.

In the stationary case, taking account of collisions of resonance ions with nonresonance ions we find that the derivative of the distribution function with respect to $v_\parallel$, which determines the damping of the magnetohydrodynamic waves, is

$$\partial f^i / \partial v_\parallel = (\partial f_M^i / \partial v_\parallel)(1 + D / D_c^{(i)})^{-1}, \quad (3.8)$$

where the diffusion coefficient D is determined by the expression

$$D = \frac{\pi e^2}{2 m_i^2} \sum_k \delta(\omega_k - k_\parallel v_\parallel) \frac{\varepsilon_k}{n_A^2} \varkappa_i^2 \operatorname{tg}^2 \vartheta \left(\frac{\omega}{\omega_i} \right)^4$$

$$\times \left| \frac{\operatorname{ctg}^2 \vartheta \, q f_0 - \varkappa_i f_1}{f_0 + \varkappa_i (f_1^2 + 2 f_0 f_2)} \right|^2 \left| \frac{1 + \operatorname{ctg}^2 \vartheta \, q f_1 + 2\varkappa_i f_2}{\operatorname{ctg}^2 \vartheta \, q f_0 - \varkappa_i f_1} + \frac{v_\perp^2}{2 v_i^2} \right|^2 \quad (3.9)$$

Here, $\epsilon_k = 2|H_2|^2 = |H_2|^2 + (c/v_A)^2 |E_1|^2$ is the spectral density of the energy in the Alfven wave. The quantities $\varphi_n^{(i)}$, which determine the damping rate $\gamma_1 = \gamma_2 = \gamma_3 = \gamma$ in (3.6), are found to be

$$\varphi_n^{(i)} = \frac{1}{n!} \int_0^\infty \frac{x^n e^{-x} dx}{1 + D / D_c^{(i)}} \quad (3.10)$$

We now consider the damping of the Alfven waves in a plasma with a high gas kinetic pressure $\kappa_i \gg 1$ in the case in which the Alfven velocity v_A is appreciably smaller than the ion thermal velocity. For weak high-frequency fields $D/D_c^{(i)} \ll 1$ the quantities $\varphi_n^{(i)} \approx 1$ and (3.6) yields the expressions that are obtained from the linear theory:[4] $\gamma = \gamma_M$ where

$$\frac{\gamma_M}{\omega} = -\frac{\omega^2}{\omega_i^2} \frac{\varkappa_i^{3/2}}{\sqrt{8\pi}} \left(\operatorname{ctg} \vartheta - \frac{3}{2} \operatorname{tg} \vartheta \right)^2. \quad (3.11)$$

In strong fields $D/D_c^{(i)} \gg 1$ the diffusion coefficient can be written in the form

$$D(x) = A + B(x - x_0)^2,$$

where

$$A = \frac{\pi^3 e^2 \varkappa_i^2}{8 m_i^2} \sum_k \delta(\omega_k - k_\parallel v_\parallel) \frac{\varepsilon_k}{n_A^2}$$

$$\times \operatorname{tg}^2 \vartheta \left(\frac{\omega}{\omega_i} \right)^4 \frac{[\varphi_1 (\operatorname{ctg}^2 \vartheta - 3/2) + 2\varphi_2]^2}{\psi^2 + \pi^2 \varphi_2^2}$$

$$B = \frac{\pi^2 e^2 \varkappa_i^3}{4 m_i^2} \sum_k \delta(\omega_k - k_\parallel v_\parallel) \frac{\varepsilon_k}{n_A^2} \operatorname{tg}^2 \vartheta \left(\frac{\omega}{\omega_i} \right)^4 \frac{(\operatorname{ctg}^2 \vartheta - 3/2)^2}{\psi^2 + \pi^2 \varphi_2^2}$$

$$x_0 = \frac{1 + 2 \operatorname{ctg}^2 \vartheta}{\sqrt{2\pi \varkappa_i} (1 + T_i / T_e)(2 \operatorname{ctg}^2 \vartheta - 3)},$$

$$\psi \equiv \psi_{1,2} \cong \ln \frac{\omega - k_\parallel v_1}{k_\parallel v_2 - \omega} + \frac{k_\parallel (v_2 - v_1)}{2(\omega - k_\parallel v_1)} - \frac{k_\parallel (v_2 - v_1)}{2(k_\parallel v_2 - \omega)}.$$

The second and third terms in ψ are obtained by integration over the boundaries of the resonance region $v_\parallel \approx v_{1,2}$. It is evident that

$$A/B \sim \varphi_{1,2}^2 / \varkappa_i \ll 1.$$

We use the notation $y = B/D_c^{(i)}(0)$. Below, for reasons of simplicity, we limit our analysis to the case in which

$$1 \ll y \ll \varkappa_i. \quad (3.12)$$

Inasmuch as $A/D_c^{(i)}(0) \ll 1$ in this case, the quantity in (3.10) can be written in the form

$$\varphi_n^{(i)} = \frac{1}{n!} \int_0^\infty \frac{x^n e^{-x} dx}{1 + y(x - x_0)^2 [D_c^{(i)}(0) / D_c(x)]}.$$

Taking account of the relation in (3.12) we then find that

$$\varphi_0^{(i)} = \pi / 2\sqrt{y}, \quad \varphi_1^{(i)} = \ln y / 2y, \quad \varphi_2^{(i)} = 3(\pi - 2)/4y.$$

† ctg ≡ cot.

Using the expressions that have been obtained for $\varphi_n^{(i)}$ we then obtain the damping rate

$$\frac{\gamma}{\omega} = -\left(\frac{\pi\varkappa_i}{2}\right)^{3/2}\left(\frac{\omega}{\omega_i}\right)^2 \frac{(\mathrm{ctg}\,\vartheta - \mathrm{tg}\,\vartheta\,{}^3/_2)^2}{\psi^2}\varphi_2{}^{(i)}. \quad (3.13)$$

The ratio of the nonlinear damping rate (3.13) to the linear damping rate (3.11) is

$$\frac{\gamma}{\gamma_M} = \frac{\pi^2\varphi_2{}^{(i)}}{\psi^2} = \frac{3\pi^2(\pi-2)}{4\psi^2 y}. \quad (3.14)$$

This ratio can be greater than unity if

$$1/\varkappa_i \ll \psi^2 < 3\pi^2(\pi-2)/4y. \quad (3.15)$$

It is evident that for waves characterized by $\omega/k_\parallel \approx (v_1 + v_2)/2$ the quantity $\psi^2 \ll 1/\kappa_i$ and $\gamma \gg \gamma_M$ if $y \sim \sqrt{\kappa_i} \gg 1$.

Usually, the reduction in the derivative of the distribution function for the resonance particles (with respect to $v_\parallel$) caused by wave diffusion leads to a reduction of the Cerenkov absorption. However, in the case considered here, a plasma with a high gas-kinetic pressure, strong wave diffusion can lead to an increase in the Cerenkov absorption of the Alfven waves by the plasma ions.

The damping rate for the Alfven wave, given by Eq. (3.6), and the coefficient for ion diffusion due to the Alfven waves (3.9), increase as $1/\vartheta^2$ when $\vartheta \to 0$. However, these expressions can only be used when $\vartheta^2 \gg \omega/\omega_i$. The damping of the fast magnetic-sound wave is reduced as ϑ is reduced and when $\vartheta \ll 1$ the fast magnetic-sound wave is weakly damped even when $\kappa_i \gtrsim 1$. At small values of ϑ the phase velocities of these waves are close to the Alfven velocity so that both waves are very similar.

We now consider the absorption of the Alfven and fast magnetic-sound waves excited by external sources with random phase in the case of small ϑ for steady-state oscillations when the diffusion due to the waves is balanced by collisions.

Solving the dispersion equation we find that the complex frequencies for the Alfven wave and the fast magnetic-sound wave are determined by the expressions

$$\omega = kv_A(1 - q_\pm), \quad |q_\pm| \ll 1,$$

where the quantities are $q_\pm$ given by

$$q_\pm = \frac{1}{4}\vartheta^2\left[1 - \varkappa_i\left(\frac{f_1^2}{f_0} + 2f_2\right)\right] \pm \left\{\frac{\vartheta^4}{16}\left[1 + \varkappa_i\left(\frac{f_1^2}{f_0} + 2f_2\right)\right]^2 + \frac{q\omega^2}{4\omega_i^2}\left(q - 2\vartheta^2\varkappa_i\frac{f_1}{f_0}\right)\right\}^{1/2}. \quad (3.16)$$

When $\vartheta^2 \gg \omega/\omega_i$ (3.16) becomes (3.6). The expression in (3.16) has been obtained in [4] for a Maxwellian ion distribution.

When $\vartheta \ll 1$ the coefficient for ion diffusion due to the Alfven wave and the fast magnetic-sound wave assumes the following form:

$$D = \frac{\pi e^2}{2m_i^2}\sum_{\mathbf{k}}\delta(\omega - k_\parallel v_\parallel)\frac{\varepsilon_{\mathbf{k}}}{n_A{}^2}\left(\frac{\omega}{\omega_i}\right)^2\varkappa_i{}^2\vartheta^2 \times \frac{|x + f_1/f_0|^2}{1 + (\omega_i/q\omega)^2|2q_\pm + \varkappa_i\vartheta^2(2f_2 + f_1{}^2/f_0)|^2}. \quad (3.17)$$

We note that the expressions obtained above for the damping and the diffusion coefficient for $\vartheta \ll 1$ apply when the plasma pressure is not too large, in which case $\sigma \equiv (\omega/\omega_i)\kappa_i$ is appreciably smaller than unity. When $\sigma \sim 1$ the phase velocities of the Alfven wave and the fast magnetic-sound wave can be appreciably different from the Alfven velocity. When $\vartheta = 0$ the refractive index for these waves is given by $n = n_{1,2}$ [5] where

$$n_{1,2}^2 = n_A{}^2/(1 \mp \sigma). \quad (3.18)$$

In this case the characteristic frequencies are $\omega = \omega_{1,2}$ where

$$\omega_{1,2} = (k^4v_i{}^4/4\omega_i{}^2 + k^2v_A{}^2)^{1/2} \mp k^2v_i{}^2/2\omega_i. \quad (3.19)$$

The damping for the waves given by (3.19) is ($\beta_i = v_i/c$)

$$\left(\frac{\gamma}{\omega}\right)_{1,2} = \sqrt{\frac{\pi}{2}}\,\frac{1\mp\sigma}{2\mp\sigma}\,\beta_i n_{1,2}\vartheta^2\varphi_2{}^{(i)}e^{-z_i{}^2} \quad (3.20)$$

The coefficient for diffusion on waves with frequencies given by (3.19) is given by

$$D_{1,2} = \frac{\pi e^2}{2m_i^2}\sum_{\mathbf{k}}\delta(\omega - k_\parallel v_\parallel)\frac{\varepsilon_{\mathbf{k}}}{n_A{}^2} \times \vartheta^2\varkappa_i{}^2\left(\frac{kv_A}{\omega_i}\right)^2\left|x + \frac{f_1}{f_0}\frac{n_{1,2}^2}{n_A{}^2}\right|^2. \quad (3.21)$$

In the region $\vartheta^2 \ll \omega/\omega_i$ the expression in (3.16) yields (3.20) for the damping rate if the quantity σ is neglected in the latter. The diffusion coefficients (3.17) and (3.21) are also the same when $\vartheta^2 \ll \omega/\omega_i$ and $\sigma \ll 1$. It follows from (3.20) that at small ϑ ($\vartheta^2 \ll \omega/\omega_i$) the wave diffusion reduces the damping for the Alfven and magnetic-sound waves ($\varphi_2^{(i)} < 1$).

4. CYCLOTRON DAMPING OF ELECTROMAGNETIC WAVES

We now investigate the damping of electromagnetic waves at frequencies close to or equal to the electron or ion cyclotron frequencies; it is assumed that these waves propagate in a plasma with a low gas kinetic pressure $\kappa_i + \kappa_e \ll 1$.

For a plasma with high density ($v_A \ll c$) and $\omega \approx \omega_i$ it is possible that an Alfven wave can propagate but this wave is subject to strong damping when $\omega \to \omega_i$. The damping rate for this wave for the case in which the resonance ions are located in the tail of the Maxwellian distribution has been found in [1] in the quasilinear approximation. The magnetic-sound wave remains weakly damped when $\omega \to \omega_i$. The expressions obtained in [1] cannot be used for the damping of the magnetic-sound wave.

Two waves can propagate in the plasma when $\omega \approx \omega_e$: these are the ordinary wave and the extraordinary wave. At small values of ϑ the extraordinary wave is highly damped when $\omega \to \omega_e$. When $\vartheta \to 1$ the wave is weakly damped. The ordinary wave is weakly damped for any value of ϑ. The damping of the extraordinary wave for small ϑ has been given in the quasilinear approximation in [1] for the case in which the resonance electrons have velocities along the magnetic field which are appreciably greater than the electron thermal velocity.

In this section we shall consider ion cyclotron absorption of the fast magnetic-sound wave and electron cyclotron absorption of the ordinary and extraordinary waves.

We start by considering the fast magnetic-acoustic wave. Assuming that in the present case the quantity ϵ_{33} is appreciably greater than the other components of the tensor ϵ_{ij} and appreciably greater than the square of the refractive index, we find from Eq. (2.3) that the quantity E_3 can be neglected. Under these conditions Eq. (2.3) assumes the form

$$(\varepsilon_1 - n^2\cos^2\vartheta)E_1 + i\varepsilon_2 E_2 = 0,$$
$$-i\varepsilon_2 E_1 + (\varepsilon_1 - n^2)E_2 = 0, \qquad (4.1)$$

where

$$\varepsilon_1 = \varepsilon_2 + n_A{}^2 = i\sqrt{\frac{\pi}{8}}\,\frac{n_A{}^2\omega}{k_\| v_i}e^{-z^2}(\Phi + i\psi) - \frac{1}{4}n_A{}^2,$$

$$z = (\omega - \omega_i)/\sqrt{2}\,k_\| v_i,$$

$$\psi = \frac{2}{\sqrt{\pi}}\int_0^z e^{t^2}dt + \frac{1}{\pi}\int_0^\infty xe^{-x}dx\int_{v_1}^{v_2} P\frac{1}{v_\| - v_{res}}\,\frac{\eta}{1+\eta}dv_\|,$$

$$v_{res} = (\omega - \omega_i)/k_\|.$$

From the dispersion equation

$$(\varepsilon_1 - n^2\cos^2\vartheta)(\varepsilon_1 - n^2) - \varepsilon_2{}^2 = 0$$

we determine the refractive index (or the frequency) and the damping rate for the magnetic-sound wave

$$n^2 = n_A{}^2/(1+\cos^2\vartheta), \qquad (4.2)$$

$$\frac{\gamma}{\omega} = \frac{\sqrt{\varkappa_i}\cos\vartheta\sin^4\vartheta}{\sqrt{8}\pi(1+\cos^2\vartheta)^{5/2}}\,\frac{e^{z^2}\Phi}{\Phi^2+\psi^2} \qquad (4.3)$$

where

$$\Phi = \int_0^\infty\frac{xe^{-x}dx}{1+\eta}, \quad x = \frac{v_\perp{}^2}{2v_i{}^2}, \quad \eta = \frac{D}{D_c{}^{(i)}}. \qquad (4.4)$$

(In the case of a Maxwellian distribution $\Phi = 1$, the expression for the damping of the magnetic-sound wave for $\omega \approx \omega_i$ has been treated in [6].)

The coefficient of ion wave diffusion is

$$D \equiv \tilde{D}x = \frac{\pi e^2\sin^4\vartheta\varkappa_i v_\perp{}^2\varepsilon_k}{8m_i{}^2\omega_i{}^2|v_\|\cos\vartheta - \partial\omega_k/\partial k_\||}\,\frac{e^{2z^2}}{\Phi^2+\psi^2}. \qquad (4.5)$$

In deriving (4.3)–(4.5) it was assured that the wave packet is sufficiently narrow in the magnetic field direction[1], so that the inequality $\Delta k/k \ll k_\| v_i/\omega_i \sim \sqrt{\kappa_i}$ holds.

We note that in the case of strong wave diffusion ($D/D_c^{(i)} \gg 1$) in the region $|\psi| \ll 1$ (more precisely $|\psi| \ll |\Phi| \ll 1$) the damping of the magnetic-sound wave increases proportionately to $1/\Phi$. It should be noted, however, that the amplitude of the electric field cannot be given as a specified quantity which is independent of Φ. The field amplitudes are proportional to the amplitudes of the extraneous current densities j^{ext}:

$$E_i = \rho_{ik}j_k{}^{ext}.$$

When $\Phi \to 0$ and $z = 0$ the coefficients ρ_{ik} are proportional to Φ so that for small Φ and $z = 0$ the diffusion coefficient $D \sim \epsilon_k/\Phi^2$ is independent of Φ and the absolute magnitude of the spectral energy absorbed per unit volume of plasma, $\gamma\epsilon_k$, is independent of the amplitude of the extraneous currents for the case of large amplitudes since $\gamma \sim 1/\Phi$, $\Phi \sim D_c/D \sim 1/(j^{ext})^2$ and $\epsilon_k \sim (j^{ext}\Phi)^2$. Thus, when the oscillations of the exciting currents j^{ext} increase the absolute magnitude of the absorbed energy no longer depends on the current amplitudes (saturation).

We turn now to the electron cyclotron resonance. The dielectric tensor at frequencies close to ω_e for a plasma characterized by a low pressure ($\kappa_e \ll 1$) can be given by

$$\varepsilon_{11} = i\sigma + 1 - v/4, \quad \varepsilon_{22} = i\sigma + 1 - v/4 - 2i\alpha,$$
$$\varepsilon_{12} = \sigma - iv/4 - \alpha,$$
$$\varepsilon_{33} = 1 - v + \varepsilon_{33}', \quad \varepsilon_{23} = i\varepsilon_{13},$$
$$\varepsilon_{13} = {}^1/{}_2 v\,\mathrm{tg}\,\vartheta[1 + i\sqrt{\pi}ze^{-z^2}(\Phi + i\psi)],$$
$$\varepsilon_{33}' = (v\sin^2\vartheta/\sqrt{2}\cos\vartheta)\beta_e nz[1 + i\sqrt{\pi}ze^{-z^2}(\Phi + i\psi)],$$
$$\sigma = \sqrt{\pi/8}ve^{-z^2}(\Phi + i\psi)/\beta_e n\cos\vartheta, \quad \alpha = \sigma\beta_e{}^2n^2\sin^2\vartheta,$$
$$z = (\omega - \omega_e)/\sqrt{2}k_\| v_e, \quad v = (\Omega_e/\omega)^2 < 2, \quad \beta_e = v_e/c. \qquad (4.6)$$

Here, the quantities Φ and ψ are determined by Eqs. (4.1) and (4.4), in which we now write $x = v_\perp^2/2v_e^2$ and $D_c = D_c^{(ee)} + D_c^{(ei)}$. For values of the angle ϑ which are not too close to zero, the diffusion coefficient and the damping rate are given by

$$D = [e^2 v_\perp{}^2 \sin^4\vartheta (2-v)^2 (2 + 2\cos^2\vartheta - 4v + v\sin^2\vartheta)^2 \varepsilon_\mathbf{k}] \times [2\pi\omega^2 | v_\| \cos\vartheta - \partial\omega_\mathbf{k} / \partial k_\| | (1-v)^2 (2\cos^2\vartheta - 2v + v\sin^2\vartheta)^2 (\Phi^2 + \psi^2)]^{-1} e^{2z^2}, \quad (4.7)$$

$$\frac{\gamma}{\omega} = \sqrt{\frac{2}{\pi}} \frac{\beta_e n \cos\vartheta}{v} \frac{P}{Q} \frac{\Phi}{\Phi^2 + \psi^2} e^{z^2}, \quad (4.8)$$

$$P = ({}^3/_4 v \sin^2\vartheta - v\cos^2\vartheta) n^4 + [-(1-v)(1 - {}^1/_4 v)(1 + \cos^2\vartheta) - (1 - {}^1/_2 v)\sin^2\vartheta + v({}^1/_2 v - 1) \times \sin^2\vartheta + {}^1/_4 v^2 \operatorname{tg}^2\vartheta (1 + \cos^2\vartheta)] n^2 + (1-v)(1 - {}^1/_2 v) + {}^1/_4 v^2 (v-2) \operatorname{tg}^2\vartheta,$$

$$Q = 2\sin^2\vartheta n^4 - (2 + \sin^2\vartheta - 2v) n^2 + v n^2 - 3v + 2v^2.$$

The dependence of the frequency on wave vector is determined by the dispersion equation of the zeroth approximation ($v = \Omega_e^2/\omega_e^2$):

$$\sin^2\vartheta n^4 - (2 - 2v + \sin^2\vartheta) n^2 + (1-v)(2-v) = 0.$$

We note that Eq. (4.8) is obtained under the assumption that $\gamma \ll k_\| v_e$. Since $n \sim 1$, when ϑ is not close to zero $\Phi \sim 1$ and $|z| \lesssim 1$ and we find from Eq. (4.8) that $\gamma/\omega \sim \beta_e$, that is to say, $\gamma \sim k_\| v_e$. In this case Eq. (4.8) can only be used to obtain orders of magnitude (strictly speaking, the quasilinear approximation does not apply). For small values of ϑ for the ordinary wave we have $P \sim \sin^4\vartheta \ll 1$ so that $\gamma \sim k_\| v_e \sin^4\vartheta \ll k_\| v_e$. We note, among other things, that for the ordinary wave $P \ll 1$ even when $\vartheta \sim 30°–45°$. In the region of exponentially small damping ($|z| \gg 1$) Eq. (4.8) can be used for any $\vartheta \sim 1$. (For a Maxwellian electron distribution the expressions for the damping of the ordinary and extraordinary waves have been obtained in [7] for $|z| \gg 1$ and in [8] for the general case $|z| \gtrsim 1$.)

In the electron cyclotron resonance the damping for certain values of **k** (as in the case of the ion cyclotron resonance on the magnetic-sound wave) increases as $1/\Phi$ in the region $\Phi \ll 1$. However, the absolute magnitude of the absorbed spectral energy for these values of **k** (at increased amplitudes of the exciting currents) tends to approach a constant value since the amplitude of the high frequency field in the plasma $\sim \Phi$ in this case.

For all other **k**, we find $\psi^2 \gg \Phi^2$ for $\Phi \ll 1$ and the absorbed power $\gamma\epsilon_\mathbf{k}$ is proportional to $(j^{ext})^4$.

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101