NEGATIVE REABSORPTION OF SYNCHROTRON RADIATION

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It is shown that in contrast to vacuum, in which reabsorption of synchrotron radiation of a system of relativistic electrons is always positive, reabsorption in a medium may be negative, provided the relativistic electron spectrum is appropriately chosen. The reabsorption frequency dependence is discussed for the case of monoenergetic and power-law spectra of the emitted electrons.

TWISS[1] has made the statement that a system of relativistic electrons moving in a magnetic field and producing synchrotron radiation can have negative reabsorption. His reasoning, however, turned out to be incorrect because he failed to take into account the statistical weight when trying to find the degree of reabsorption by the Einstein-coefficient method.[1)] Weiss, calling attention to this circumstance, has proved that there is no negative reabsorption of synchrotron radiation for a system of relativistic electrons in vacuum regardless of the choice of the energy spectrum of the electrons (see the review[3]).

We shall show in this article that relativistic electrons ($E \gg mc^2$) moving in a medium (plasma) on which a magnetic field is superimposed have under certain conditions a negative reabsorption and consequently intensify rather than attenuate the radiation incident on them. In particular, a system of relativistic electrons amplifies also its own synchrotron radiation, as a result of which its level exceeds the summary intensity of the spontaneous synchrotron radiation from all the electrons of this system.

It must be emphasized that the finding of the conditions under which negative reabsorption of synronous radiation is realized is of great significance in radiophysics and in radioastronomy. The coherent synchrotron radiation resulting from negative reabsorption may turn out to be quite promising in the interpretation of the part of the radio emission from quasars and a few other cosmic sources. The radioastronomy aspects of coherent synchrotron radiation will be discussed in another article.

1)For the same reason, the conclusion by Slysh[2] that negative reabsorption of synchrotron radiation by relativistic electrons in vacuum is incorrect.

1. As is well known (see [3,4]) the reabsorption coefficient of synchrotron radiation, obtained by the Einstein-coefficient method, is

$$\mu = -\frac{c^2}{4\pi\nu^2}\int_0^\infty \frac{d}{dE}\left[\frac{N(E)}{E^2}\right]E^2Q(\nu, E)\,dE. \qquad (1)$$

It can be represented in a somewhat different form, taking the integral by parts:

$$\mu = \frac{c^2}{4\pi\nu^2}\int_0^\infty \frac{N(E)}{E^2}\frac{d}{dE}[E^2Q(\nu, E)]\,dE. \qquad (2)$$

In (1) and (2) ν is the frequency, c the speed of light, N(E) the energy spectrum of the relativistic electrons, $Q(\nu, E)$ the power of the synchrotron radiation of one electron with energy E in a unit frequency interval in a single normal wave.

We note that under the derivative sign in (1) we have in fact the electron momentum distribution function f(p). This can be readily verified by recognizing that for an isotropic distribution f(p) and N(E) are related by

$$f(p)\cdot 4\pi p^2 dp = N(E)\,dE, \qquad (3)$$

where $E = pc$ in the relativistic case.

Formulas (1) and (2) have been written out under the condition that the system of relativistic electrons is in vacuum (the refractive index is $n = 1$). We can ascertain, however, that the influence of the medium ($n \neq 1$), in the case when $1 - n \ll 1$, does not change the form of the expressions (1) and (2) for μ; it affects only the formula for the spectral power $Q(\nu, E)$ of the synchrotron radiation.

In an isotropic and sufficiently dilute plasma, for which

$$\nu_H / \nu \ll 1, \quad \nu_L^2 / 2\nu^2 \ll 1, \quad n^2 \approx 1 - \nu_L^2/\nu^2$$

(ν_H is the gyrofrequency of the electrons and ν_L

the natural frequency of the plasma) the normal (ordinary and extraordinary) waves are circularly polarized, and the power radiated in one of the waves amounts, accurate to terms of the order of mc^2/E, to one-half of the total power of the synchrotron radiation of the electron: $Q(\nu, E) \approx \frac{1}{2} Q_{tot}(\nu, E)$, where[4]

$$Q_{tot}(\nu, E) = 2\sqrt{3}\pi \frac{e^2}{c} \nu_H \left[1 + \frac{\nu_L^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2\right]^{-1/2} \times \frac{\nu}{\nu_c'} \int_{\nu/\nu_c'}^{\infty} K_{5/3}(\eta)\, d\eta. \quad (4)$$

Here $K_{5/3}(\eta)$ is a MacDonald function

$$\nu_c' = \frac{3}{2} \nu_H \left(\frac{E}{mc^2}\right)^2 \left[1 + \frac{\nu_L^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2\right]^{-3/2}; \quad (5)$$

$$\nu_H = eH/2\pi mc, \qquad \nu_L^2 = e^2 N/\pi m, \quad (6)$$

H is the magnetic-field component orthogonal to the electron velocity, and N is the electron density in the plasma. It follows from (4) and (5) that in the region

$$\frac{\nu_L^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2 \ll 1 \quad \text{(region I)} \quad (7)$$

the influence of the medium on the synchrotron radiation of the electron (and by the same token on the reabsorption coefficient) is insignificant; to the contrary, the influence of the medium becomes decisive in the other limiting case, when

$$\frac{\nu_L^2}{\nu^2}\left(\frac{E}{mc^2}\right)^2 \gg 1 \quad \text{(region II)}. \quad (8)$$

2. Let us determine first the contribution made to the reabsorption coefficient at the frequency ν by the electrons from the energy interval (7) (region I). According to (4) and (5) we have in this interval

$$Q(\nu, E) = \sqrt{3}\pi \frac{e^2}{c} \nu_H z \int_z^{\infty} K_{5/3}(\eta)\, d\eta, \quad (9)$$

where

$$z \equiv \frac{\nu}{\nu_c'} = \frac{2}{3} \frac{\nu}{\nu_H} \left(\frac{mc^2}{E}\right)^2. \quad (10)$$

Therefore

$$\frac{d}{dE}[E^2 Q(\nu, E)] = \frac{8\pi}{3\sqrt{3}} \frac{e^2}{c} (mc^2)^4 \frac{\nu^2}{\nu_H} E^{-3} K_{5/3}(z). \quad (11)$$

Since this derivative is positive, the interval (7) gives a positive contribution to the reabsorption coefficient, equal to

$$\mu^{\mathrm{I}} = \frac{2}{3\sqrt{3}} \frac{e^2 c}{\nu_H} (mc^2)^4 \int_0^{E^2 \ll E^{*2}} \frac{N(E)}{E^5} K_{5/3}(z)\, dE. \quad (12)$$

Here z is connected with E by relation (10) and

$$E^* = \frac{\nu}{\nu_L} mc^2. \quad (13)$$

Thus, the upper limit in the integral is chosen such as to satisfy the condition (7) that the medium have a weak influence.

It is appropriate to note here that in vacuum ($\nu_L = 0$) the inequality (7) is satisfied for all energies, and that in (12) the upper limit can be replaced by infinity. In this case (12) determines the total reabsorption coefficient. In accordance with the conclusion of Weiss,[3] the coefficient is always positive, i.e., there is no intensification of synchrotron radiation in vacuum regardless of the energy spectrum of the relativistic electrons.

3. We turn now to the investigation of the interval (8) (region II), where the medium plays a significant role. According to (4) and (5) we have in this interval

$$Q(\nu, E) = \frac{2\pi}{\sqrt{3}} \frac{e^2}{c} \frac{\nu_L^2}{\nu} \int_z^{\infty} K_{5/3}(\eta)\, d\eta, \quad (14)$$

where

$$z \equiv \frac{\nu}{\nu_c'} \approx \frac{2}{3} \frac{\nu_L^3}{\nu_H \nu^2} \frac{E}{mc^2}. \quad (15)$$

Then

$$\frac{d}{dE}[E^2 Q(\nu, E)] = \frac{2\pi}{\sqrt{3}} \frac{e^2}{c} \frac{\nu_L^2}{\nu} \left[2E \int_z^{\infty} K_{5/3}(\eta)\, d\eta - E^2 K_{5/3}(z) \frac{dz}{dE}\right] = \sqrt{3}\pi \frac{e^2}{c} \frac{\nu_H \nu}{\nu_L} mc^2 \Phi(z). \quad (16)$$

We denote by $\Phi(z)$ the function

$$\Phi(z) = 2z \int_z^{\infty} K_{5/3}(\eta)\, d\eta - z^2 K_{5/3}(z). \quad (17)$$

Taking (14) and (16) into account and using (2), we find that the contribution made by the electrons from interval (8) to the reabsorption coefficient μ is

$$\mu^{\mathrm{II}} = \frac{\sqrt{3}}{4} e^2 c \frac{\nu_H}{\nu \nu_L} mc^2 \int_{E^2 \gg E^{*2}}^{\infty} \frac{N(E)}{E^2} \Phi(z)\, dE. \quad (18)$$

z is connected with E by (15); the lower limit of integration, in which E^* is taken from (13), is chosen to satisfy the condition (8) that the medium play an important role.

The function $\Phi(z)$ is sign-alternating, since

$$\int_0^{\infty} \Phi(z)\, dz = \int_0^{\infty} 2z \int_z^{\infty} K_{5/3}(\eta)\, d\eta\, dz - \int_0^{\infty} z^2 K_{5/3}(z)\, dz = 0.$$

This can be readily verified by evaluating the first integral by parts. From the plot of $\Phi(z)$ in Fig. 1 it is clear that

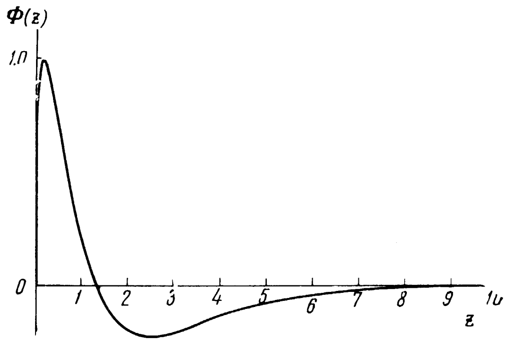

FIG. 1

$$\Phi(z) > 0 \text{ for } 0 < z < \tilde{z};$$
$$\Phi(z) < 0 \text{ for } z > \tilde{z}.$$

The reversal of the sign occurs when

$$z = \tilde{z} \approx 1.35. \tag{19}$$

If $z \ll 1$ we have

$$\Phi(z) \approx \frac{16\pi}{3\sqrt{3}\,\Gamma(^1/_3)}\left(\frac{z}{2}\right)^{1/3}; \tag{20}$$

when $z \gg 1$

$$\Phi(z) \approx -\sqrt{\pi/2}\,z^{3/2}e^{-z}. \tag{21}$$

To obtain the total coefficient of synchrotron-radiation reabsorption in the presence of a medium (plasma) it is obviously necessary to add expressions (12) and (18) and add further to them the appropriate integral in the intermediate interval; we must therefore use here in the calculation of μ the exact formula (4) for $Q(\nu, E)$, and not the approximations (9) and (14). To gain an idea of the magnitude and character of the reabsorption in each concrete case (for a broad energy spectrum of the relativistic electrons), a suitable calculation must be made over the entire energy interval. However, since $\Phi(z)$ is negative when $z > \tilde{z}$, it is clear that when the electron spectrum is specially chosen such that the main contribution to the reabsorption coefficient is made by the energy region $E > \tilde{E}$ corresponding to $z > \tilde{z} \approx 1.35$, namely

$$E > 2mc^2\nu_H\nu^2/\nu_L^3,$$

the reabsorption is assuredly negative and the synchrotron radiation will be intensified in the relativistic-electron system itself.

The negative reabsorption in the medium is the result of a substantial change in the energy dependence of the electron synchrotron radiation from a unit solid angle $E^2Q(\nu, E)$ compared with the corresponding dependence in vacuum. The point is that $E^2Q(\nu, E)$ increases monotonically with increasing E in vacuum, but in a medium its variation is nonmonotonic, first increasing and then decreasing with increase in E (see (9) and (14)).

We shall illustrate the effect of negative reabsorption of synchrotron radiation by means of two concrete examples, chosen because of their significance to radio-astronomical observations.

4. We consider first a system of electrons with a narrow ("monoenergetic") spectrum whose maximum occurs at an energy E_0. The width ΔE of the spectrum is assumed to be sufficiently small to be able to take the functions $E^{-5}K_{5/3}(z)$ and $E^{-2}\Phi(z)$ outside the integral signs in (12) and (18), using their values at the point $E = E_0$. After performing this operation, we obtain

$$\mu = \mu^{\mathrm{I}} = \frac{2}{3\sqrt{3}}\,\frac{e^2c}{\nu_H}\,\frac{(mc^2)^4}{E_0^5}\,N_0K_{5/3}(z_0),$$
$$z_0 = \frac{2}{3}\,\frac{\nu}{\nu_H}\left(\frac{mc^2}{E_0}\right)^2 \tag{22}$$

when $E_0^2 \ll E^{*2}$ and

$$\mu = \mu^{\mathrm{II}} = \frac{\sqrt{3}}{4}e^2c\,\frac{\nu_H}{\nu\nu_L}\,\frac{mc^2}{E_0^2}N_0\Phi(z_0),\quad z_0 = \frac{2}{3}\,\frac{\nu_L^3}{\nu_H\nu^2}\,\frac{E_0}{mc^2} \tag{23}$$

when $E_0^2 \gg E^{*2}$. We recall that $E^* = mc^2\nu/\nu_L$; N_0 in (22) and (23) denotes the concentration of the relativistic electrons.

The reabsorption coefficient depends on the frequency in the following manner: At high frequencies (in region I), $\mu(\nu)$ is described by (22). In this case the reabsorption is positive; the coefficient μ is exponentially small when ν is large, and increases with decreasing frequency. Bypassing the region

$$\nu \sim \nu_L E_0/mc^2, \tag{24}$$

where $\mu(\nu)$ is not described by the simple formulas (22) and (23), we go over to region II, where (23) is valid. An idea of the frequency dependence of the degree of reabsorption, defined by (23), can be gained from Fig. 2, which shows a plot of $\sqrt{z}\;\Phi(z) \propto \mu^{\mathrm{II}}$ as a function of $1/\sqrt{z} \propto \nu$.

In region II, depending on the concrete relations

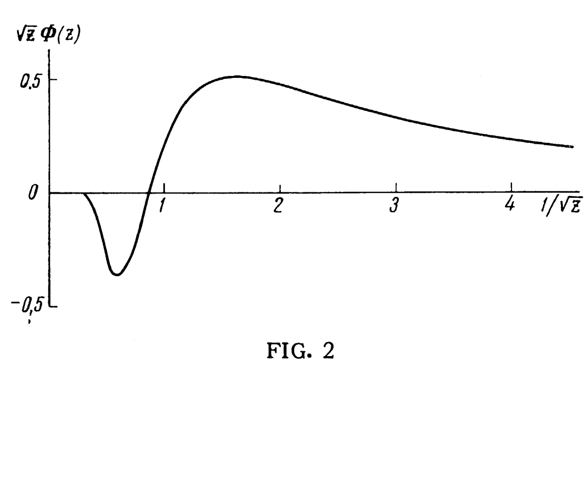

FIG. 2

between the system parameters, there are realized two cases. If the frequency corresponding to $z_0 = \tilde{z} \approx 1.35$, namely

$$\nu \approx \left(0.5 \frac{\nu_L{}^3}{\nu_H} \frac{E_0}{mc^2} \right)^{1/2}, \quad (25)$$

is much lower than the characteristic frequency (24):

$$\frac{\nu_L}{2\nu_H} \ll \frac{E_0}{mc^2}, \quad (26)$$

(i.e., $\Phi(z_0)$ reverses sign in region II), then the reabsorption coefficient in the start of this interval is positive as before, and becomes negative only at the frequency (25). The degree of negative reabsorption $-\mu(\nu)$ reaches a maximum value at the frequency

$$\nu_{max} \approx \left(0.24 \frac{\nu_L{}^3}{\nu_H} \frac{E_0}{mc^2} \right)^{1/2} \quad (z_{max} \approx 2.8), \quad (27)$$

after which it decreases rapidly with frequency like

$$\mu(\nu) \backsim \nu^{-4} e^{-a/\nu^2} \quad (28)$$

(see (21) and (23)).

It is clear from Fig. 2 that the frequency interval occupied by the region of appreciable amplification of the synchrotron radiation is quite narrow and lies within the limits $\pm 0.3\, \nu_{max}$.

In the other case, when the frequency (25) is comparable with or larger than the frequency (24)

$$\nu_L / 2\nu_H \gtrsim E_0 / mc^2, \quad (29)$$

the reabsorption coefficient will be negative in the entire interval $\nu^2 \ll \nu_L^2 (E_0/mc^2)^2$, where the influence of the medium is quite appreciable. The sign reverses in this case in the frequency region (24). We note that when

$$\nu_L / 2\nu_H \gg E_0 / mc^2 \quad (30)$$

$z_0 \gg 1$ in the entire region $E_0^2 \gg E^{*2}$, and the degree of negative reabsorption, will be exponentially small at all frequencies at which it exists (see (21)). This is in fact connected with the rapid decrease of the synchrotron radiation when condition (3) is satisfied (see [5]).

Let us see at which magnetic fields H and at which electron energies E_0 in a plasma with concentration N is the maximum negative reabsorption coefficient realized at a specified frequency ν (compared with other frequencies and with values $-\mu$ at other H and E_0). It is clear that to this end it is necessary above all that ν coincide with ν_{max} (Eq. (27)). When this circumstance is taken into account, it follows from (23) that when ν_L is constant the degree of negative reabsorption increases with decreasing E_0. But the energy E_0 can be reduced only to a certain limit, namely to $E_0 = aE^*$, where E^* is given by (13) and a is a coefficient equal to several units, for with further increase of E_0 the degree of negative reabsorption will decrease. It follows from the conditions $\nu = \nu_{max}$ and $E_0 = aE^*$ that the values of E_0 and ν_H (i.e., H) corresponding to the maximum degree of negative reabsorption are determined (for a monoenergetic electron spectrum) by the relations

$$E_0 = a \frac{\nu}{\nu_L} mc^2, \qquad \nu_H = 0.24 a \frac{\nu_L{}^2}{\nu}. \quad (31)$$

With this, in accord with (23), the reabsorption coefficient reaches the value

$$\mu \approx - \frac{2.3 \cdot 10^{-2}}{a} \frac{e^2}{mc} \frac{\nu_L{}^2}{\nu^4} N_0. \quad (32)$$

In calculating the numerical coefficient in (32), the value of $\Phi(z)$ at the point $z = z_{max}$ was taken from the diagram of Fig. 1.

5. By way of a second example we consider a power-law energy spectrum of the form

$$N(E) = \begin{cases} AE^\gamma & \text{for } E_1 < E < E_2 \\ 0 & \text{for } E < E_1,\ E > E_2 \end{cases}, \quad (33)$$

confining ourselves, for simplicity, to the case when $E_1^2 \gg E^{*2}$, i.e.,

$$\nu^2 \ll \nu_L{}^2 (E_1 / mc^2)^2. \quad (34)$$

Then, obviously, the degree of reabsorption is completely determined by the region where the medium has an appreciable influence, so that

$$\mu = \mu^{\rm II} = \frac{\sqrt{3}}{4} e^2 c \frac{\nu_H}{\nu \nu_L} mc^2 A \int_{E_1}^{E_2} E^{\gamma-2} \Phi(z)\, dE,$$

where z is connected with E by (15). Changing the variable, we obtain

$$\mu = \frac{\sqrt{3}}{4} e^2 c \frac{\nu_H}{\nu \nu_L} \left(\frac{3\nu_H \nu^2}{2\nu_L{}^3} \right)^{\gamma-1} (mc^2)^\gamma A \int_{z_1}^{z_2} z^{\gamma-2} \Phi(z)\, dz; \quad (35)$$

the limits z_1 and z_2 are determined by the values of E_1 and E_2.

We note first that in the frequency interval

$$\nu^2 \gg \frac{2}{3} \frac{\nu_L{}^3}{\nu_H} \frac{E_2 - E_1}{mc^2}, \quad (36)$$

the difference $z_2 - z_1 \ll 1$ and the reabsorption coefficient (35) reduces in fact to formula (23) for the monoenergetic spectrum.

At very low frequencies, for which $z_1 \gg 1$ and $z_2 \gg 1$, i.e., in the interval

$$\nu^2 \ll \frac{2}{3} \frac{\nu_L{}^3}{\nu_H} \frac{E_1}{mc^2}, \quad (37)$$

the degree of reabsorption is negative and is exponentially small. If inequality (36) is already satisfied in this interval, the variation of $\mu(\nu)$ is described in almost the entire range where reabsorption is significant by the formulas obtained for the monoenergetic spectrum.

In the opposite case, i.e., in the frequency interval

$$\nu^2 \gg \frac{2}{3}\frac{\nu_L^3}{\nu_H}\frac{E_1}{mc^2}, \tag{38}$$

the situation is more complicated. The expression (35) for $\mu(\nu)$ simplifies here if it falls with increasing frequency into the region where $z_1 \ll 1$ and $z_2 \gg 1$:

$$\frac{2}{3}\frac{\nu_L^3}{\nu_H}\frac{E_1}{mc^2} \ll \nu^2 \ll \frac{2}{3}\frac{\nu_L^3}{\nu_H}\frac{E_2}{mc^2}. \tag{39}$$

The condition $z_2 \gg 1$ allows us to replace the upper limit in the integral of (35) by infinity, since the integrand decreases rapidly when z is large. At the same time the inequality $z_1 \ll 1$ makes it possible to replace the lower limit by zero, if the integral of $z^{\gamma-2}\Phi(z)$ converges. Since the function $z^{\gamma-2}\Phi(z)$ is proportional to $z^{\gamma-5/2}$ when $z \ll 1$ (see (20)), it is clear that convergence will be assured if $\gamma > 2/3$.

Assuming this condition satisfied and taking the foregoing into account, we obtain

$$\mu(\nu) \approx \frac{\sqrt{3}}{4}e^2c\frac{\nu_H}{\nu\nu_L}\left(\frac{3\nu_H\nu^2}{2\nu_L^3}\right)^{\gamma-1}(mc^2)^\gamma A\int_0^\infty z^{\gamma-2}\Phi(z)\,dz.$$

Substituting here the expression (17) for $\Phi(z)$ and transforming the first of the obtained integral by parts, we obtain for $\mu(\nu)$

$$\mu(\nu) \approx \frac{\sqrt{3}}{4}e^2c\frac{\nu_H}{\nu\nu_L}\left(\frac{3\nu_H\nu^2}{2\nu_L^3}\right)^{\gamma-1}(mc^2)^\gamma$$

$$\times A\left(\frac{2}{\gamma}-1\right)\int_0^\infty z^\gamma K_{5/3}(z)\,dz, \tag{40}$$

where

$$\int_0^\infty z^\gamma K_{5/3}(z)\,dz = 2^{\gamma-1}\Gamma\left(\frac{\gamma}{2}+\frac{4}{3}\right)\Gamma\left(\frac{\gamma}{2}-\frac{1}{3}\right).$$

The integral in (40) is always positive, and therefore the degree of reabsorption will be negative if

$$\gamma > 2. \tag{41}$$

It was noted above that the electron momentum distribution function f(p) is proportional to $E^{-2}N(E)$, i.e., $E^{\gamma-2}$ for the spectrum under consideration. It is clear therefore that amplification of the synchrotron radiation will occur when the distribution function f(p) increases with increasing p in the energy interval $E_1 < E < E_2$.

According to (40), in the interval (39) the degree of negative reabsorption $-\mu(\nu)$ increases with increasing frequency when $\gamma > 2$, and the degree of positive reabsorption $\mu(\nu)$ decreases with increasing ν when $\gamma < 2$.

We note finally that when the frequency is shifted from the interval (39) towards lower values of ν the degree of negative reabsorption first becomes larger than the value given by (40) above, since at frequencies

$$\nu^2 \sim \frac{2}{3}\frac{\nu_L^3}{\nu_H}\frac{E_1}{mc^2}$$

we have $z_1 \sim 1$ and the region of integration over positive values of the function $\Phi(z)$ shrinks. To the contrary, when the frequency increases beyond the interval (39), the degree of negative reabsorption decreases because of the shrinkage of the interval of integration in (35) over the region of negative values of the function $\Phi(z)$. This is connected with decrease of z_2; when $z_2 \lesssim 1$, i.e., at frequencies when

$$\nu^2 \gtrsim \frac{2}{3}\frac{\nu_L^3}{\nu_H}\frac{E_2}{mc^2},$$

the reabsorption coefficient certainly becomes positive.

6. When account is taken of reabsorption in the radiating system, the intensity of the outward radiation is

$$I = (a/\mu)(1 - e^{-\mu L}), \tag{42}$$

where L is the length of the homogeneous system (a and μ are constant) and a is the emissivity, equal to

$$a = \int_0^\infty Q(\nu, E)N(E)E^2\,dE.$$

It is easy to derive concrete expressions for the monoenergetic and power-law particle spectra considered above. We shall not present the corresponding formulas, noting only that in the absence of reabsorption ($\mu = 0$) we have for the intensity $I = aL$. If the reabsorption is positive, then $I < aL$ (absorption), and if it is negative $I > aL$ (amplification). In the latter case the intensity of outward radiation exceeds by a factor $(e^{-\mu L} - 1)(-\mu L)^{-1}$ the level of radiation in the absence of reabsorption. The effect of amplification is negligible if $-\mu L \ll 1$. To the contrary, when $-\mu L \gg 1$, the negative reabsorption causes a sharp increase in the radiation level. At high intensities, however, the reaction of the radiation on the character of the particle energy distribution becomes appreciable. Then formula (42), which has been obtained in a linear approximation, will no longer hold and the intensity must be determined by a nonlinear approach.

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