LASER MODE INTERACTION IN THE COURSE OF Q-SWITCHING

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The mode locking process in a laser in the course of Q-switching is considered. It is shown that allowance for mode pulling by the Doppler line center reveals the existence of a minimum degree of modulation whereby synchronization is possible. The analysis of a two-mode laser can be reduced to the classical problem of synchronizing an oscillator by an external signal, which is solved by a successive simplification method. This method allows one to determine the mode locking range for any number of equidistant modes. Mode capture has been observed experimentally, the mode locking range is determined, and a qualitative agreement of experimental results with the theory under consideration is established.

NONLINEAR laser processes have recently been the subject of considerable attention. The nonlinearity of the active medium under certain conditions causes mutual synchronization of laser modes.[1] The emission may also be synchronized by impressing an external signal upon the laser at oscillation frequency,[2] or by a parametric method which varies either the dielectric constant or resonator losses at the frequency of mode beats. The latter case was discussed by Di Domenico,[3] who showed that mode locking occurs when resonator losses are modulated at a frequency equal to the mode beat frequency. In such a process, mode separation becomes equal to the modulation frequency and all beats have the same constant phase. The locked mode interference makes the laser emission pulsed. The case in which modulation frequency differs from the mode beat frequency has been discussed in [4, 5]. There, as in [3], the nonlinear nature of amplification in the gas mixture was not considered; it was found that a decrease in the detuning between the frequency of the modulated signal and the beat frequency led to an increase in the intensity of light generated by the laser. However, the assumption that gain is linear does not yield a stationary solution with zero detuning and does not permit determination of the mode locking range of the laser. The experiments [5, 6] on the oscillation intensity as a function of the detuning are approximate and need to be refined.

Q-switching at a frequency close to the mode beat frequency has a cumulative effect and constitutes an example of a resonant interaction with a system. Therefore, at small input losses, such a problem can be treated by the standard methods used in the study of oscillating systems subject to weak resonant signals.

The purpose of this paper is an investigation of laser mode locking in the course of Q-switching, taking account of the nonlinear amplification of the gas mixture and mode pulling, as well as the determination of the mode locking range and comparison with experiment.

DERIVATION OF EQUATIONS

The field of each laser mode is represented by a plane wave propagating along the resonator axis; this is valid for a laser generating the fundamental mode. Q-switching is accomplished by varying the reflection coefficient of one of the mirrors (for example, by means of an electro-optical crystal placed at the mirror). The amplitude-modulated light reflection coefficient ρ can be conveniently represented, following Di Domenico, by

$$\rho = \frac{1-\alpha-\beta_m\cos\omega_m t}{1+\alpha+\beta_m\cos\omega_m t}, \quad 1 \gg \alpha > \beta_m. \tag{1}$$

Here, α and β_m are the constant and variable losses and ω_m is the modulation frequency. Let us consider the case were the modulation frequency differs little from the mode beat frequency:

$$\omega_m = \omega_n - \omega_{n-1} + \delta, \quad \delta < \omega_n Q^{-1}, \tag{2}$$

Q is the quality factor of the resonator. The case when $\omega_m \approx 2(\omega_n - \omega_{n-1})$, $\omega_m \approx 3(\omega_n - \omega_{n-1})$, etc. is considered analogously, provided two, three, etc. groups of modes which do not interact with one another appear within the Doppler line. In each group, the modes are separated from one another by a distance $\sim \omega_m$ and satisfy a condition of the type (2). Expanding the field in the laser in terms of the resonator eigenfunction, we obtain an equation for the n-th mode:

$$\frac{d^2x_n}{dt^2} + \omega_n Q^{-1}\frac{dx_n}{dt} + \omega_n^2 x_n = -\varepsilon^{-1}\frac{d^2P}{dt^2} + \frac{2\beta_m\omega_n}{\pi}$$
$$\times \cos\omega_m t\left\{\frac{1}{n-1}\frac{dx_{n-1}}{dt} + \frac{1}{n+1}\frac{dx_{n+1}}{dt}\right\}$$
$$\times \frac{d^2P_n}{dt^2} \approx -\nu_n^2 P_n, \quad \nu_n = \omega_n + \Delta_n. \quad (3)$$

Here P_n is the polarization of the active medium of the laser, ν_n is the oscillation frequency, and ϵ is the dielectric constant.

The above equation is best studied by the method of slowly varying amplitudes. Let us assume that the field of the n-th mode satisfies the expression:

$$x_n = E_n(t)\sin(\nu_n t + \varphi_n(t)), \quad (4)$$

where E_n and φ_n are an amplitude and phase that vary slowly over one period. Polarization of the medium is represented by

$$P_n = P_n^{(1)}(t, \omega_n - \omega)\cos(\nu_n t + \varphi_n)$$
$$+ P_n^{(2)}(t, \omega_n - \omega)\sin(\nu_n t + \varphi_n), \quad (5)$$

ω is the center of the Doppler line, and $P_n^{(1)}$ and $P_n^{(2)}$ are defined after Lamb.[1] The form of these functions is of no further interest; it is important merely that $P_n^{(1)}$ ensure saturation of the gas-mixture gain. In the definition of $P_n^{(2)}$ we confine ourselves to the first Lamb approximation; hence, $P_n^{(2)} = \gamma(\omega_n - \omega)E_n$, where $\gamma(\omega_n - \omega)$ is independent of E_n. Consequently, oscillation-frequency pulling by the gain curve is taken into account, but mode interaction in the absence of modulation is not.

Substituting (4) into (5) and (3) and using the standard procedure, we get

$$\dot{E}_n + (\omega_n Q^{-1} - \sigma(E_n))E_n = \beta[E_{n+1}\cos\Phi_n + E_{n-1}\cos\Phi_{n-1}],$$
$$\dot{\Phi}_n = -\varkappa_n + \delta + \beta\left[\frac{E_{n-1}}{E_n}\sin\Phi_{n-1} - \left(\frac{E_{n+1}}{E_n} + \frac{E_n}{E_{n+1}}\right)\times\sin\Phi_n + \frac{E_{n+2}}{E_{n+1}}\sin\Phi_{n+1}\right],$$
$$\varkappa_n = \nu_n(P_n^{(2)} - P_{n+1}^{(2)})(\varepsilon E_n)^{-1},$$
$$\sigma(E_n) = \nu_n P_n^{(1)}(E_n)(\varepsilon E_n)^{-1}, \quad (6)$$

where $\Phi_n = \varphi_n - \varphi_{n+1} + (\Delta_n - \Delta_{n+1} + \delta)t$ is the relative phase, and $\beta = \omega_m\beta_m/2\pi$. If the laser resonator excites n modes, the system (6) contains n amplitude and n − 1 phase equations. From the amplitude equation (6) it follows that if only the n-th mode is amplified in the laser, then the amplitudes of the remaining modes attenuate like $E_{n\pm\kappa}/E_n \approx (\beta_m/\alpha)^{\kappa}$.

Further analysis of the system (6) will be based on the successive simplification method developed by Khokhlov.[7] The applicability of this method is based on the fact that a weak signal has a small effect on the amplitude of oscillations, so that the main parameters of the process are described by the phase equations. The amplitude and phase of the electric field of the n-th mode is represented as

$$E_n = E_n^0 + \mu e_n(\mu t), \quad \Phi_n = \Phi_n(t), \quad (7)$$

where E_n^0 is the steady-state amplitude for $\beta = 0$, and μ is a parameter characterizing the smallness of e_n:

$$\mu \approx (E_{n+1}^0 + E_{n-1}^0)(E_n^0|\omega_n Q^{-1} - \sigma(E_n)|_{max})^{-1}.$$

Using (7), we can represent (6) in the form

$$\Phi_n = -\varkappa_n + \delta + \beta[K_{n-1}\sin\Phi_{n-1} - (K_n^{-1} + K_n)\times\sin\Phi_n + K_{n+1}^{-1}\sin\Phi_{n+1}], \quad (8)$$

where $K_n = E_n/E_{n+1}$ is not time dependent.

The expression for amplitude correction e_n is obtained from (6):

$$e_n = \frac{\beta}{|d\sigma/dE_n|_{E_n^0}}[K_n^{-1}\cos\Phi_n + K_{n-1}\cos\Phi_{n-1}]. \quad (9)$$

The most complete analysis of (8) and (9) is possible in the two-mode case. The behavior of modes with indices n and n + 1 can be described by (8) and (9) if K_{n-1} and K_{n+1}^{-1} are set equal to zero:

$$e_n = \frac{\beta}{|d\sigma/dE_n|_{E_n^0}}K_n^{-1}\cos\Phi_n,$$
$$e_{n+1} = \frac{\beta}{|d\sigma/dE_{n+1}|_{E_{n+1}^0}}K_n\cos\Phi_n, \quad (10)$$

$$\dot{\Phi}_n = \delta_1 - \beta(K_n^{-1} + K_n)\sin\Phi_n, \quad \delta_1 = \delta - \varkappa_n. \quad (11)$$

The phase equation (11) coincides with the analogous equation obtained under the same assumptions for the case of an oscillator synchronized by an external signal, and can be integrated by quadratures.

Equation (11) is integrated separately in the regions $|\delta_1| < \beta(K_n^{-1} + K_n)$ and $|\delta_1| > \beta(K_n^{-1} + K_n)$ (see [7]. In the first case, (11) describes limiting motion of the phase to the stable state, when

$$\sin\Phi_n = \delta_1[\beta(K_n^{-1} + K_n)]^{-1}. \quad (12)$$

The mode locking band of a two-mode laser is determined by

$$|\delta_1| = \beta(K_n^{-1} + K_n). \quad (13)$$

The stationary phase values lie within the range $-\pi/2 < \Phi_n < \pi/2$. Therefore, according to (10), the mode amplitudes increase when the detuning δ_1 decreases. Within a detuning range such that $|\delta_1| > \beta(K_n^{-1} + K_n)$, solution of (11) yields an oscillating phase of frequency $\sim\delta_1$ when the detuning is considerable. This corresponds to a simultaneous existence of mode beats with frequencies ω_m and $\nu_{n+1} - \nu_n$. It follows from (11) that the amplitudes e_n and e_{n+1} vary in phase and consequently the average intensity of the laser should also vary with a frequency $\sim\delta_1$.

The synchronization of a three-mode laser with frequencies ν_{n-1}, ν_n, and ν_{n+1}, is described by a system of two phase equations obtained from (8):

$$\dot{\Phi}_n = \delta - \varkappa_n + \beta[K_{n-1}\sin\Phi_{n-1} - (K_n^{-1} + K_n)\sin\Phi_n],$$

$$\dot{\Phi}_{n-1} = \delta - \varkappa_{n-1} + \beta[-(K_{n-1}^{-1} + K_{n-1})\sin\Phi_{n-1} + K_n^{-1}\sin\Phi_n], \quad (14)$$

This system cannot be integrated by quadratures, but permits a qualitative analysis in the phase plane. Setting $X_n = \sin\Phi_n$ and $X_{n-1} = \sin\Phi_{n-1}$, and dividing one equation in (14) by the other, we get

$$\frac{dX_{n-1}}{dX_{n-1}} = \frac{\sqrt{1 - X_{n-1}^2}[\delta - \varkappa_n + \beta(K_{n-1}X_{n-1} - (K_n^{-1} + K_n)X_n)]}{\sqrt{1 - X_n^2}[\delta - \varkappa_{n-1} + \beta(-(K_{n-1}^{-1} + K_{n-1})X_{n-1} + K_n^{-1}X_n)]}. \quad (15)$$

The phase diagrams of this equation are shown in Fig. 1 for the case when system (14) has a stationary solution (X_{n-1}, X_n) determined by the equations

$$K_{n-1}X_{n-1} - (K_n^{-1} + K_n)X_n = (\varkappa_n - \delta)\beta^{-1}, \quad (16)$$

$$-(K_{n-1}^{-1} + K_{n-1})X_{n-1} + K_n^{-1}X_n = (\varkappa_{n-1} - \delta)\beta^{-1};$$

$$|X_{n-1}| < 1, \quad |X_n| < 1, \quad (17)$$

$$X_{n,\,n-1} = (D_{n,\,n-1}^{(\varkappa)} - \delta D_{n,\,n-1})/\beta D, \quad (18)$$

where D is the determinant of system (16), and $D_{n,\,n-1}$ are determinants whose columns containing the coefficients of $X_{n,\,n-1}$ have been replaced

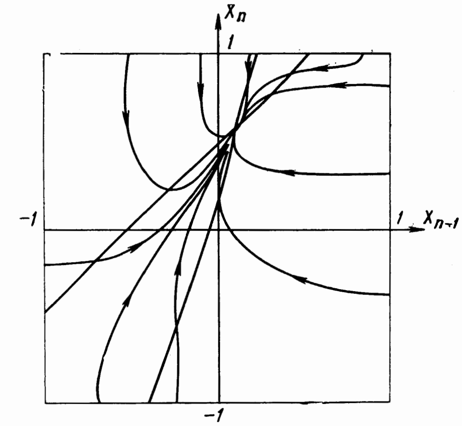

FIG. 1. Relative phase diagram of a three-mode laser for the following parameter values: $K_n = 1$, $K_{n-1} = 2$, $\beta^{-1}(\kappa_n - \delta) = -1$, $\beta^{-1}(\kappa_{n-1} - \delta) = 0.2$.

by unit columns and $D_{n,\,n-1}^{(\kappa)}$ are determinants in which the same columns have been replaced by columns containing the elements $\kappa_{n,\,n-1}$.

The mode locking range of the system is determined by the condition,

$$\left|\frac{D_{n,\,n-1}^{(\varkappa)} - \delta D_{n,\,n-1}}{\beta D}\right| = 1, \quad (19)$$

which is a consequence of (17) and (18). If the center of the Doppler line pulls in the modes so as to render

$$\left|\frac{D_n^{(\varkappa)} - \delta D_n}{\beta D}\right| > 1 \text{ or } \left|\frac{D_{n-1}^{(\varkappa)} - \delta D_{n-1}}{\beta D}\right| > 1 \quad (20)$$

for all values of δ, then laser beats will not be synchronized for the given value of β.

It follows from (18) that within the mode locking range the relative phases begin to differ already in the case of three modes and fail to coincide with the phase of the modulating signal; an exception is the case of $\kappa_n = \kappa_{n-1} = \kappa$, corresponding to a symmetrical arrangement of modes in the Doppler line. The locking range is here determined by the condition

$$|\delta_1| = \beta D / D_m, \quad (21)$$

where D_m is the largest of the determinants $D_{n,\,n-1}$. If the detuning exceeds the locking range and inequalities (17) are not satisfied, the phase diagrams of Eq. (16) are closed curves describing synchronous oscillations of the phases $\Phi_{n,\,n-1}$. We can analyze the interaction of any number of modes in a similar manner.

The expression for the locking range can be considerably simplified if $|D_{n,\,n\pm1,\,n\pm2,\,\ldots}^{(\kappa)}/\beta D| \ll 1$. In this case, (21) is valid, wherein D_m is the largest of all $D_{n,\,n\pm1,\,n\pm2,\,\ldots}$. If, in addition, it is assumed that all mode amplitudes are equal, then for $2n + 1$ modes, for example, we obtain

$$|\delta| = 2\beta / n(n+1). \tag{22}$$

EXPERIMENTAL DETERMINATION OF THE MODE LOCKING RANGE

In the experiment described we used a He-Ne laser ($\lambda = 0.63\mu$, distance between mirrors ~ 163 cm) which generated only longitudinal modes. Q-switching was accomplished by a KDP crystal placed at the flat mirror. The voltage was applied through ring electrodes along the z axis of the crystal (Fig. 2). An FM signal with a frequency of ~ 92 mc and a fixed bias (not shown in Fig. 2) were applied to the signal; the maximum frequency deviation amounted to ~ 100 kc. The same signal was delivered to a photodiode, where it was mixed with a signal at a mode beat frequency. The resulting frequency-difference signal was displayed on the screen of a two-beam oscilloscope; at the same time, the display showed the variation in laser emission intensity, as indicated by a photoelectron multiplier (upper beam).

A typical oscilloscope trace is shown in Fig. 3 (bottom). The absence of beats within a definite frequency interval indicates the locking of the mode beats. The magnitude of the locking range and the amplitudes of the beats located on both sides of the range depend upon the magnitude of the signal impressed on the crystal and upon the bias. Outside the locking range, the laser shows simultaneously mode beats at the frequencies $\sim \omega_n - \omega_{n-1}$ and ω_m. Addition of these signals results in the modulation of light intensity with a frequency $\sim \delta$ (upper trace in Fig. 3).

Without modulation or in the case of a large detuning, it is impossible to observe the beats, except of the cases of self-locking.[5] This is due to the fact that all mode beats have random phases, and their sum at the photodiode has the character of noise. Near the locking range, the beats of all modes are no longer independent, in view of the

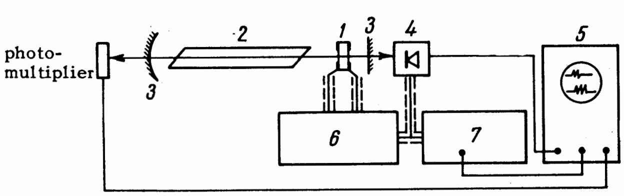

FIG. 2. Diagram of the experimental setup. The oscilloscope is synchronized with voltage used to modulate the generator frequency. 1 – KDP crystal, 2 – gas discharge tube, 3 – mirrors, 4 – photodiode, 5 – two-beam oscilloscope, 6 – amplifier, 7 – FM oscillator.

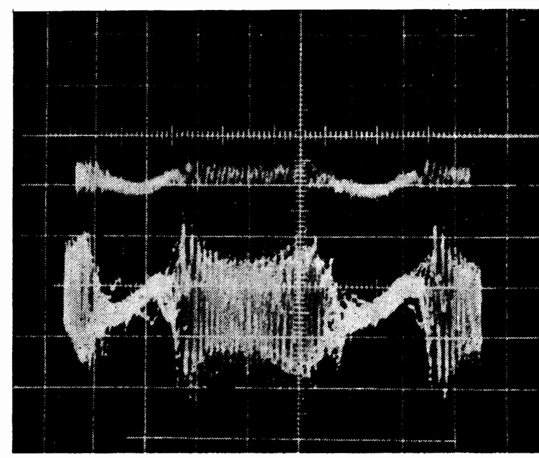

FIG. 3. Oscillographic traces of mode locking. Upper beam shows the emission intensity.

relationship between the phase equations. The case of the three-mode laser indicates that the system is subject to synchronous oscillations of relative phases with a frequency close to that of the detuning. The laser emission intensity is somewhat increased within the locking range, which is in accordance with the amplitude expressions (10) (upper trace in Fig. 3; intensity increases downward)

Figure 4 shows the locking range as a function of the amplitude of modulated losses. In accordance with Eq. (21), it is linear when β_m is large.

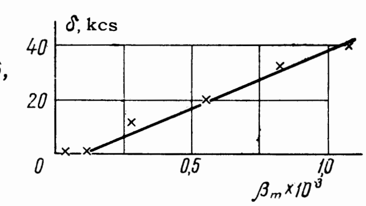

FIG. 4. Locking range, δ, as a function of the degree of modulation characterized by β_m.

The minimum value of β_m at which locking can occur amounts to 10^{-4} for $\alpha \approx 2 \times 10^{-3}$. The slope of the straight line in Fig. 4 corresponds to four or five laser modes with close amplitudes, as it follows from (22). The mode beats can be readily made to lock by adjustment of mirrors. In that case, the oscilloscope screen will show clear beats in the absence of modulation. Nevertheless, when laser synchronization was investigated by means of Q-switching, the self-locking effect was absent.

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