THERMAL CONDUCTIVITY OF THIN DIELECTRIC AND FERRODIELECTRIC FILMS AND FILAMENTS

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The thermal conductivity coefficient is calculated for thin dielectric and ferrodielectric films and filaments at low temperatures, when the mean free path of the phonons and of the spin waves is much larger than the characteristic dimensions of the specimen. It is shown, that for thin films the main contribution to the thermal conductivity is made by gliding phonons and spin waves. In the case of thin filaments the gliding phonons and spin waves give only a correction. Different mechanisms of phonon and spin wave scattering are considered.

1. We consider first the thermal conductivity of a dielectric film. We choose coordinates such that the z axis is normal to the boundaries of the film and the plane $z = 0$ lies on the lower boundary of the film. The deviation of the phonon distribution function χ from the equilibrium Bose function n_0 should be obtained from the following kinetic equation:

$$v_z \frac{\partial \chi}{\partial z} + \frac{\chi}{\tau} = -(\mathbf{v}\nabla T)\frac{\partial n_0}{\partial T}, \qquad (1)$$

where $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the phonon group velocity and τ is the smallest of the relaxation times, due to N-processes, scattering by impurities, etc.

The collision integral in (1) is expressed in terms of the relaxation time τ, the introduction of which is always valid for thin films, since the nonequilibrium addition χ to the distribution function has a sharp maximum for phonons that travel along the surface of the film.

Solving Eq. (1) under boundary conditions $\chi(z = d, v_z < 0; z = 0, v_z > 0) = 0$, we can readily calculate the thermal conductivity coefficient

$$\varkappa_{ik} = -\frac{2}{(2\pi\hbar)^3}\int_0^{2\pi} d\varepsilon\, \varepsilon \frac{\partial n_0}{\partial T}$$

$$\times \int_0^{\pi} \frac{\sin\vartheta\, d\vartheta}{K(\vartheta,\varphi)} n_i n_k \frac{vl}{d}[l - l\cos\vartheta(1 - e^{-d/l\cos\vartheta})], \qquad (2)$$

where $l = v\tau$ is the phonon mean free path, $v_z = |\mathbf{v}|\cos\vartheta$, $\mathbf{n}$ is the group-velocity unit vector, $K(\vartheta, \varphi)$ is the Gaussian curvature of the phonon equal-energy surface, and ϑ and φ are angles determining the position of $\mathbf{n}$ relative to the z axis.

In the case $d \ll l$ considered here, the main contribution to the integral (3) is made by the region of angles for which $d/l \ll \cos\vartheta \ll 1$. Expanding the exponential in (3) in powers of the small parameter $d/l\cos\vartheta$ and assuming that l, n_i, and $K(\vartheta, \varphi)$ are independent of ϑ, we obtain

$$\varkappa_{ik} = \frac{2d}{(2\pi\hbar)^3}\int d\varepsilon\, \varepsilon \frac{\partial n_0}{\partial T}\int_0^{2\pi} v \frac{d\varphi}{K(\varphi, \pi/2)} n_i n_k \ln\frac{l}{d}. \qquad (3)$$

In the case of an isotropic dispersion law ($\epsilon(\mathbf{p}) = vp$ for phonons and $\epsilon(\mathbf{p}) = \alpha p^2$ for spin waves), the effective mean free path is $l_{\text{eff}} \sim d\ln(l/d)$, in analogy with the result obtained by Fuchs for the effective mean free path of electrons in thin metallic films[1].

Thus, the main contribution to the thermal conductivity of the film is made by phonons traveling along the surface of the film. At very low temperatures, the smallest mean free path is the path l^N connected with the normal collisions. Estimates yield for it[2] $l^N \sim a(Mv^2/\Theta_D)(\Theta_D/T)^2$, where a is the lattice constant, M the mass of the atom, v the speed of sound, and Θ_D the Debye temperature. Consequently, in the limit of very low temperatures, so long as $l^N \gg d$, the thermal conductivity of the film is

$$\varkappa \sim \frac{d}{a^3}\left(\frac{T}{\Theta_D}\right)^3 v \ln\left\{\frac{aMv^2}{d\Theta_D}\left(\frac{\Theta_D}{T}\right)^5\right\}.$$

Further, when $l^N \lesssim d$, the diffusion mechanism considered by Gurzhi[2], for which $l_{\text{eff}} \sim d^2/l^N$, comes into play. In samples that are not very pure, scattering by impurities may become important, corresponding to a mean free path $l_i \sim c^{-1}a(\Theta_D/T)^4$ where c is the concentration of the impurity atoms.

For this case we have

$$\varkappa \sim \frac{d}{a^3}\left(\frac{T}{\Theta_D}\right)^3 v \ln\left\{\frac{a}{dc}\left(\frac{\Theta_D}{T}\right)^4\right\}.$$

In ferrodielectrics at low temperatures, when $T \ll \Theta_D^2/\Theta_C$ (Θ_C is the Curie temperature), the thermal conductivity is determined by the spin waves. Normal collisions between spin waves are due to both relativistic and exchange interaction. The mean free paths for the corresponding processes have orders of magnitude[2,3]

$$l_{ss}^{(r)} \sim a\left(\frac{\Theta_C}{\mu M_0}\right)^2\left[1+\exp\left(\beta\frac{\mu M_0}{T}\right)\right],$$

$$l_{ss}^{(e)} \sim a\left(\frac{\Theta_C}{T}\right)^{7/2},$$

where μ is the Bohr magneton, M_0 the nominal magnetization, and β the anisotropy constant.

In thin films of thickness $d < a(\Theta_C/\mu M_0)^2 \sim 10^{-3}$–$10^{-4}$ cm, the main contribution to the thermal conductivity, up to a temperature T_1 determined from the condition $l_{ss}^{(e)}(T_1) \sim d$, will be made by gliding spin waves. Taking this circumstance into account, it is easy to estimate the thermal-conductivity coefficient:

$$\varkappa \approx \frac{\Theta_c d}{\hbar a^2}\left(\frac{T}{\Theta_c}\right)^2 \times \ln\begin{cases} \dfrac{a}{d}\left(\dfrac{\Theta_c}{\mu M_0}\right)^2 \exp\dfrac{\beta\mu M_0}{T}, & T \ll \mu M_0, \\ \dfrac{a}{d}\left(\dfrac{\Theta_c}{\mu M_0}\right)^2, & \mu M_0 \ll T \ll \Theta_c\left(\dfrac{\mu M_0}{\Theta_c}\right)^{4/7}, \\ \dfrac{a}{d}\left(\dfrac{\Theta_c}{T}\right)^{7/2} & \Theta_c\left(\dfrac{\mu M_0}{\Theta_c}\right)^{4/7} \ll T \ll T_1. \end{cases}$$

If the thickness of the film is $d \gtrsim a(\Theta_C/\mu M_0)^2$, then gliding spin waves can be significant up to a temperature T_2 determined from the condition $l_{ss}^{(r)}(T_2) \sim d$. The coefficient of thermal conductivity will be determined in this case by the first two expressions of the preceding formula.

2. The kinetic equation for the thermal conductivity of a dielectric filament is

$$v_x\frac{\partial\chi}{\partial x} + v_y\frac{\partial\chi}{\partial y} + \frac{\chi}{\tau} = -v_z\frac{\partial T}{\partial z}\frac{\partial n_0}{\partial T}. \tag{4}$$

We choose a cylindrical coordinate system (r, ϑ, z) such that the z axis is directed along the filament axis. Solving Eq. (4) by the usual method (see, for example, [4]) with the boundary condition $\chi(r = a,\ v_r \le 0) = 0$, we obtain the following expression for the thermal conductivity of the filament:

$$\varkappa = \frac{1}{\pi a^2(2\pi\hbar)^3}\int d\varepsilon\varepsilon\frac{\partial n_0}{\partial T}\int_0^{2\pi} d\varphi \times \int_0^1 \frac{dt\, t\sqrt{1-t^2}}{\tilde{K}(t,\varphi)}\, l \int_{-a}^{a} dy\,\alpha\left(1-\frac{lt}{\alpha}e^{-\alpha/lt}-\frac{lt}{\alpha}\right), \tag{5}$$

where ψ and φ are the angles that determine the position of the velocity vector relative to the coordinate system, $\alpha = 2(a^2 - y^2)^{1/2}$, $t = \sin\psi$, $y = r\sin(\varphi - \vartheta)$, and

$$\frac{1}{\tilde{K}(t,\varphi)} = \frac{1}{K(\arcsin t, \varphi)} + \frac{1}{K(\pi - \arcsin t, \varphi)}.$$

Expanding the exponential under the integral sign in powers of the small parameter a/lt and putting $\tilde{K}(t, \varphi) = \tilde{K}(0, \varphi)$ and $l = l(0)$, we obtain after integrating with respect to y

$$\varkappa = \frac{2}{(2\pi\hbar)^3}\int d\varepsilon\varepsilon\frac{\partial n_0}{\partial T}v \times \int_0^{\pi}\frac{d\varphi}{\tilde{K}(0,\varphi)}\,a\left[\frac{4}{3\pi}+\frac{a}{4l}\ln\frac{a}{l}-\frac{29}{48}\frac{a}{l}\right]. \tag{6}$$

The last two terms in (6) are due to phonons traveling along the boundary. Unlike the preceding case, the gliding phonons do not make the main contribution to thermal conductivity, only a correction. This is due to the fact that in the case of the filament an entire degree of freedom drops out and the number of phonons traveling along the surface is small compared with the number of phonons traveling in arbitrary fashion. In the limit as $l \to \infty$ we obtain the well known result of Casimir[5].

The results obtained above are essentially connected with two assumptions: a) the scattering of phonons on the boundaries is diffuse, b) the mean free path of the phonons is much larger than the characteristic dimensions of the sample ($l \gg d$). Inasmuch as real samples are always rough, the assumption that the scattering is diffuse is always justified. The only possible exception is the region of extremely low temperatures, when the phonon wavelength λ is large compared with the characteristic dimensions of the roughness of the surface ($\lambda \sim a\Theta_D/T$ and consequently $\lambda \sim 10^{-6}$ cm only when $T/\Theta_D \sim 10^{-2}$). Under these conditions the reflection is specular. The second condition ($l \gg d$) is always attained for sufficiently thin films.

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