THE ROLE OF LIGHT ABSORPTION BY FREE CARRIERS IN A SEMICONDUCTOR LASER

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It is shown that the kinetic equations for the semiconductor-radiation system in the stationary case, with account taken of photon absorption by free carriers, can have three solutions: a stable laser solution (a) corresponding to a weak absorption by free carriers, an unstable laser solution (b) corresponding to a strong absorption by free carriers, and a non-laser solution (c). The lowest pump energy value which can yield additional solutions has been computed as an example for the case of an intrinsic semiconductor with direct band-to-band radiative transitions.

THE absorption of light by free carriers in a semiconductor laser was first considered in[1]. It was found that in a number of real cases such an absorption can practically preclude the realization of laser action. Nevertheless, the analysis carried out in[1] is based upon arbitrarily selected values of carrier concentration, corresponding to the stationary state of the semiconductor—radiation system. The basic defect of such an approach is not the quantitative inaccuracy (which, incidentally, is quite high) inherent in the criterion for laser action, but rather the following problem. As will be shown below, free-carrier absorption, given specified values of semiconductor and resonator parameters, temperature, and pump energy, may lead to three distinct stationary states of the system under consideration, corresponding to substantially different carrier concentrations. Only one such state can yield a stable laser solution. Thus, identical conditions may lead to different stationary states of the system, depending upon the manner in which the conditions were imposed upon the system. Such a conclusion could not have been reached in the course of the elementary analysis performed in[1].

The present work contains the result of an orderly investigation of the semiconductor—radiation system, including the determination of the quasi-level of the chemical potential for electrons and holes. It was such an approach that led to the establishment of the possibility that the above three stationary states could exist.

It is convenient to separate the cases which differ in terms of energy levels involved in radiative transitions in a semiconductor. Let us consider in some detail the simplest case of direct band-to-band transitions in an intrinsic semiconductor. (The characteristic parameters of a stationary laser state without free-carrier absorption have been determined for such a system in[2].)

Let us neglect the binding of carriers to form excitons, avoiding for a while any evaluation of the validity of such an approximation. The usual assumptions[2], allowing for free-carrier absorption, permit the following formulation of the kinetic equations for the stationary state:

$$\frac{dq^j}{dt}=B_e{}^j(q^j+1)-B_a{}^jq^j-(\alpha^j+2\zeta^j\mathfrak{N})q^j=0, \qquad (1)$$

$$\frac{d\mathfrak{N}}{dt}=\Phi-\sum_j B_e{}^j(q^j+1)+\sum_j B_a{}^jq^j=0, \qquad (2)$$

where B_e^j and B_a^j are the probabilities of emission and absorption of a photon of the j-th mode:

$$B_e{}^j=A\sqrt{\varepsilon^j}(\tau^j)^2, \qquad B_a{}^j=B_e{}^j+A\sqrt{\varepsilon^j}(1-2\tau^j), \qquad (3)$$

$A = e^2\sqrt{m}fv/\pi n^2\hbar^2V$, f is the oscillator strength, m is the effective mass of the electron (hole), n is the index of refraction of the crystal, v is the volume of the crystal, V is the volume of the resonator, $\epsilon^j = \hbar\omega^j - \epsilon_g$, ω^j is mode frequency, and ϵ_g is the width of the forbidden band.

$$\tau^j=[\exp(x^j/2-\eta)+1]^{-1}, \qquad (4)$$

$x^j = \epsilon^j/\Theta$; $\eta = \mu/\Theta$, μ is the chemical potential level of electrons (holes), Θ is the absolute temperature in terms of energy units, $\mathfrak{N}$ is the number of free electrons (holes),

$$\zeta^j=\frac{e^3}{n^2\pi u m^2(\omega^j/2\pi)^2 v} \qquad (5)$$

is the probability of photon capture by a free carrier per unit of time[3], u is mobility, Φ is pump energy, and α^j are losses of the j-th mode due to the resonator.

The following relationships may be conveniently used to replace expressions (1) and (2):

$$q^j = B_e{}^j / F^j, \qquad F^j = \alpha^j + 2\zeta^j \mathfrak{N} + B_a{}^j - B_e{}^j, \tag{6}$$

$$\Phi = \sum_j (\alpha^j + 2\zeta^j \mathfrak{N}) q^j. \tag{7}$$

A laser solution of Eqs. (6) and (7) was considered in[2], with

$$0 < \eta \ll 1 \tag{8}$$

(let us from now on call it solution a). The criterion for the existence of solution a, connected with the free-carrier absorption, has the form,

$$2\zeta^0 C F_{1/2}(\eta) \ll \alpha^0, \qquad \alpha^0 = (\alpha^j)_{min}, \tag{9}$$

where the Fermi integral, $F_{1/2}(\eta) \sim 1$, and C is the density of states in the band, approximately equal to $5 \times 10^{15}\ vT^{3/2}$. Since in real cases, ζ amounts to $(10^{-7} - 10^{-9})v^{-1}$, and $\alpha^0 \sim 10^9 - 10^{11}\ sec^{-1}$, condition (9) can be fulfilled, at least at sufficiently low temperatures.

We shall now show that, in addition to the above-mentioned solution (a) of (6) and (7), which lies close to small values of η (see (8) above), two other solutions of this system may also exist.

For simplicity, we shall look for these additional solutions when $\eta \gg 1$. Here,

$$\tau^j = \begin{cases} 1, & x^j < 2\eta \\ 0, & x^j > 2\eta \end{cases}, \tag{10}$$

$$\Phi = \sum_j^{(2\eta)} \frac{\alpha^{*j} A \sqrt{\Theta x^j}}{\alpha^{*j} - A\sqrt{\Theta x^j}}, \tag{11}$$

$$\alpha^{*j} = \alpha^j + \frac{8C\zeta^j \eta^{3/2}}{3\sqrt{\pi}},$$

and summation is carried out from x^j to $x^j_{max} = 2\eta$.

It is apparent from (11) that there are two types of solutions of this equation. One of these (let us call it b) corresponds to the case where the numerators of certain modes tend to zero. Singular modes are those with $x^j_{max} = 2\eta$, since only in this case will each of the remaining modes have $q^j > 0$. This is a laser-type solution. However, singular modes are distinguished here not by virtue of minimal losses in the resonator, but actually by virtue of the highest number of states associated with transitions having the singular-mode frequency. In such a case, stimulated emission has no directivity.

Another type (type c) of solution of (11) corresponds to the case where $\alpha^{*j} > A\sqrt{2\Theta\eta}$. Such a solution is possible when η is sufficiently large, because α^* increases as $\eta^{3/2}$, i.e., faster than $A\sqrt{2\Theta\eta}$. In this case, there are no singular modes (non-laser solution), $\Phi \sim \eta^{3/2}$.

The solutions of kinetic equations for a laser based on direct band-to-band transitions are diagrammatically illustrated in the figure.

In accordance with the assumption made above, we shall limit the discussion to the case of

$$8C\zeta^j\eta^{3/2} / 3\sqrt{\pi} \gg \alpha^j$$

for all j. Proceeding from summation to integration in (11), and computing the integral, we obtain

$$\Phi = 2GA\sqrt{\Theta}\beta^{-3/2} f(\eta), \tag{12}$$

where

$$\beta = \frac{8C\bar{\zeta}}{3\sqrt{\pi}A\sqrt{\Theta}} = \frac{4\sqrt{2}m_0 k n^2 \bar{\zeta} T V}{3\hbar e^2 f v},$$

$$G = \frac{8\pi V \varepsilon_g^2 \Theta}{(2\pi\hbar\tilde{c})^3},$$

$\tilde{c}$ is the velocity of light in the medium, m_0 is the mass of a free electron, the superscript line means averaging over the modes, and k is the Boltzmann constant;

$$f(\eta) = F(t) = -t^{5/2}\left[t^2 \ln\left(1 - \frac{\sqrt{2}}{t}\right) + \sqrt{2}t + 1\right], \quad t = \beta\eta. \tag{13}$$

When $\Phi \to \infty$, the right-hand side of (12) can, as is apparent from (13), tend to infinity, as $t \to \sqrt{2} + 0$ (b), or as $t \to \infty$ (c), which corresponds to two new solutions of (6) and (7). Expressions (12) and (13) describe the curve b − c in the figure. Condition $\partial\Phi/\partial\eta = 0$ allows us to find the coordinates of the point on the curve b − c at which Φ is a minimum:

$$t_{min} = 1.98, \quad F(t_{min}) = 6.1, \quad \Phi_{min} = 12.2A\sqrt{\Theta}\beta^{-3/2}G. \tag{14}$$

The threshold pump value for the laser solution (a), in which free-carrier absorption is negligible ($\beta < \lambda$), equals, according to[4],

$$\Phi_{\text{thr}} = G\alpha^0 S,$$

$$S = \xi \begin{cases} 0{,}4 / \lambda\xi, & \xi \gg 1, \quad \lambda\xi \gtrsim 2 \\ \dfrac{2}{3}\ln\dfrac{e}{4\lambda\xi}, & \xi \gg 1, \quad \lambda\xi \ll 1 \\ \dfrac{\pi\sqrt{2/3}}{\sqrt{\xi - 1}} + \dfrac{2}{3}\ln\dfrac{1}{2\lambda}; & \xi \simeq 1 \end{cases}, \tag{15}$$

where $\xi = \bar{\alpha}/\alpha^0$, and $\lambda = \alpha^0/A\sqrt{\Theta} < 1$.

The ratio $\kappa = \Phi_{min}/\Phi_{thr}$, according to (14) and (15), equals

$$\varkappa = \frac{12{,}2}{\beta^{3/2}\lambda S}. \tag{16}$$

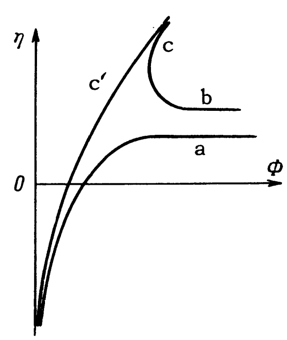
Assuming, as an estimate, that T = 100°K, f = 5 $\times 10^{-2}$, $n^2 = 3$, $\zeta = 10^{-7}$ sec^{-1}, $\lambda \cong 1$, $\xi \gg 1$, v = V, and computing $\beta = 0.5$, we obtain

$$\varkappa \cong 70. \tag{17}$$

It can be seen that, with the selected values of parameters, an experimental realization of the additional solutions will require a substantial increase in pump energy. However, this is entirely feasible when, for example, fast electrons[5] are used to excite the semiconductor.

It follows from (15) and (16) that increasing λ, given $\xi \gg 1$, or the value of $\xi \cong 1$, may cause a considerable decrease in κ. In particular, the case may actually occur when β is close to λ; the losses of singular and non-singular modes will then differ to a slight extent only.

Equation (2) indicates that solutions a and c are stable, while solution b is unstable. When ζ is sufficiently large, solutions of the a and b types do not exist (curve c′ in the figure). This represents the case, discussed in[1], when laser action is not possible at all.



Solutions of kinetic equations for the stationary state of the semiconductor–radiation system. a – Laser solution corresponding to a weak absorption by free carriers; b – laser solution with strong absorption by free carriers; c – non-laser solution. Curve c′ (left) corresponds to the case where there are no laser solutions (a) and (b).

As a rule, the above assumption, neglecting the role of carrier binding into excitons, is not valid in actual cases. A quasi-equilibrium is usually established between the excitons and free carriers, preventing laser action in direct band-to-band transitions in such cases[6].

It must be noted that solutions (b) and (c) correspond to a high carrier concentration, for which the screening effect of the electron-hole gas may prove to be substantial. The losses due to the exciton formation from electron and hole binding may be considerably lower in such a case than in the usual laser state (a). This circumstance may thus promote the feasibility of solutions (b) and (c).

In fact, as it is seen from[7], if the free-carrier concentration

$$\mathfrak{N} / v \gg 1 / 3\pi^2 r_{ex}^3, \tag{18}$$

where r_{ex} is the exciton radius, and degeneracy is present, the screening radius will be

$$r_0 = \tfrac{1}{2}\hbar n e^{-1} m^{-1/2} (\mathfrak{N} / v)^{-1/6},$$

and, according to (18),

$$r_0 / r_{ex} \ll (a_B' / r_{ex})^{1/2} \lesssim 1, \tag{19}$$

where $a_B' = a_B n^2 m_0/m$ and a_B is the Bohr radius. Even if the exciton radius is shorter or the carrier concentration lower, satisfying a condition converse to that of (18), the screening radius of the form[7]

$$r_0 = 2^{-1/2} (\hbar^2 n^2 v / \pi \mathfrak{N} m e^2)^{1/4},$$

can be much shorter than r_{ex}, if

$$\frac{1}{3\pi^2 r_{ex}^3} \gg \frac{\mathfrak{N}}{v} \gg \frac{1}{3\pi^2 r_{ex}^3} \frac{a_B'}{r_{ex}}. \tag{20}$$

For example, if $r_{ex} \approx 3 \times 10^{-7}$ cm, $n^2 = 3$, $m/m_0 \approx 1$, and $\mathfrak{N}/v \sim 10^{17}$ cm^{-3}, condition (20) is met, while, if $\mathfrak{N}/v > 10^{18}$ cm^{-3}, conditions (18) and (19) are met.

Satisfaction of the inequality $r_0 \ll r_{ex}$ may result in the absence of exciton states, or in a considerable increase in the time of electron and hole binding into excitons. The latter event is particularly significant if the binding proceeds via excited exciton states ($r'_{ex} \gg 10^{-7}$ cm).

The conclusion concerning the existence of several stationary solutions of kinetic equations for optical transitions in a semiconductor may also be drawn for the case of other than the band-to-band transitions.

For example, it would be interesting to consider a laser using the radiative impurity-to-band transitions in a semiconductor[8]. Here, one could satisfy the condition that $\eta \gg 1$ with a small number of excitons. This would require a high concentration of recombination levels of the acceptor type (for the band-impurity-donor transitions) and a high electron population of the impurity levels.

Consequently, an excessively high pump energy, in an experimental attempt to obtain laser action in a semiconductor, incurs the danger of running into a branch of the curve b – c in the diagram, with the attendant poor-laser or non-laser emission. Review[9] fails to report any experimental realization of different stationary states under the same conditions of stimulated emission in a semiconductor. The above analysis shows that such a situation is capable of realization.

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