OSCILLATIONS OF THE ATTENUATION AND VELOCITY OF SOUND WAVES IN SEMICONDUCTORS AND METALS IN THE PRESENCE OF A STRONG MAGNETIC FIELD

V. I. PUSTOVOĬT and I. A. POLUÉKTOV

Institute of Physico-technical and Radio-engineering Measurements;
P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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It is shown that an important factor in the establishment of the amplitude of the attenuation oscillations of ultrasound waves in degenerate semiconductors and metals is the formation of a space charge by the wave and of a Coulomb field connected with it. This circumstance is taken into account in setting up a dispersion equation for propagation of ultrasound waves in semiconductors and metals. It can be seen from the equation that in order to investigate the nature of the oscillations a knowledge of the complex longitudinal conductivity of the medium is required. In order to find the complex permittivity of the medium under conditions of Landau quantization, the equation for the density matrix operator is considered. It is shown that for the calculation of the longitudinal permittivity of the medium the equation for the density matrix in which collisions are taken into account can be reduced to the "kinetic equation" for a single matrix element. The possibility of the oscillation of the imaginary and real part of the conductivity is considered. It is shown that the imaginary part of the conductivity oscillates only under very stringent conditions. Attenuation and sound velocity oscillations connected with oscillations of the real part of the conductivity are considered. The results are compared with experiment.

GUREVICH et al.[1] first showed, in a strong magnetic field the attenuation of ultrasound waves in degenerate semiconductors and metals exhibits oscillations with changing magnetic field. The amplitude of these oscillations can attain considerable values. The nature of the attenuation oscillations of sound waves consists in the fact that in the region of the diffuseness of the Fermi surface for $\hbar\Omega \gg \kappa T$ (Ω—cyclotron frequency) only narrow intervals of values of k_z (the quasimomentum along the magnetic field) turn out to be allowed. The position of these intervals changes with the magnetic field and the electrons which move in phase with the sound wave turn out to be either in the region of the diffuseness of the Fermi surface or outside it. In considering the absorption of ultrasound in semiconductors and metals, the authors of [1] and also those of the subsequent papers on this subject[2-4] calculated the energy of the ultrasound wave absorbed by the conduction electrons. In doing this they took no account of the fact that a violation of the local equilibrium of the electrons is possible in the sound wave, i.e., there appears a space charge, which leads in turn to the appearance of a corresponding Coulomb field which must be taken into account.

In order to take into account the effect of the Coulomb field of the space charge, one must solve the self-consistent problem of the equations of the theory of elasticity, the equations of electrostatics, and the equations which define the dielectric permittivity tensor of the medium under the Landau quantization conditions.

As will be shown below, with the appearance of a space charge in the sound wave considerable changes occur in the oscillations of the absorption as a function of the magnetic field, viz., the ratio of the minimum attenuation to the maximum attenuation is no longer proportional to the oscillations of the real part of the conductivity, but is given by a more complex expression whose oscillation amplitude is as a rule considerably less than the corresponding oscillation amplitude of the real part of the conductivity. If a space charge is produced in the sound wave, oscillations can occur in the change of the velocity of the sound waves, due to the oscillations of the real part of the conductivity.

We shall show below that when the real part of the conductivity of the electron gas oscillates there are, generally speaking, no oscillations of the imaginary part of the conductivity, i.e., of the polarizability. Nonetheless, when more stringent conditions are satisfied [lower temperatures and higher magnetic fields; see Eqs. (36a) and (37)], oscillations of the imaginary part of the conductivity are also possible, i.e., the velocity oscillations and the attenuation of sound waves can undergo further changes.

In this paper we consider the simplest case when the equal-energy surface of the electrons (holes) is a sphere located at the center of the Brillouin zone. Clearly the Fermi surface of real semiconductors and metals is most frequently not isotropic and there are several minima (valleys). Under the conditions of a complex zone structure the propagating sound wave changes the energy of the electron, and at that generally speaking differently in each minimum (valley). This leads to the circumstance that quasi-equilibrium "intervalley" transitions will take place in the wave, which may affect the process of production of space charge. A case is possible, for example, in which the intervalley transitions fully compensate the space charge, and when consequently the results of [1-4] turn out to be applicable.[1)] Nevertheless, for each specific case of a given zone structure one can always indicate a series of directions of wave propagation with a given polarization vector for which the intervalley transitions will not fully compensate the space charge, and for which our discussion remains therefore qualitatively valid. A more rigorous treatment of the entire kinetics of the process with account of intervalley transitions constitutes a rather complicated problem which requires separate investigation.

1. THE DISPERSION EQUATION FOR THE PROPAGATION OF ULTRASOUND IN SEMICONDUCTORS AND METALS

The equations describing the propagation of ultrasound waves in a solid with account of their interaction with the conduction electrons (holes) are of the form

[1)] Full compensation of the space charge occurs when the change of the energy of the electrons in one group of minima is equal and opposite in sign to that of another group. Such a case is realized, for instance, in germanium when a transverse wave propagates in the [100] direction with the polarization along the [010] direction. In this direction the wave interacts most strongly with the carriers which gives, in particular, rise to the acousto-electric effect.

$$\rho\frac{\partial^2 u_i}{\partial t^2}-\lambda_{iklm}\frac{\partial u_{lm}}{\partial x_k}=-\sum_{\alpha=e,\,h}\Lambda_{ik}^{(\alpha)}\frac{\partial}{\partial x_k}n_{\sim}^{(\alpha)}(\mathbf{x},t). \tag{1}$$

Here the indices e and h denote as usual electrons and holes, ρ is the density of the lattice, λ_{iklm} is the modulus of elasticity tensor, $\mathbf{u}$ is the displacement vector, u_{ik} is the deformation tensor, related to the displacement vector by the usual relation, and Λ_{ik} is the tensor of the electron-phonon interaction constants; $n_{\sim}^{(\alpha)}(\mathbf{x}, t)$ is the deviation of the electron (hole) concentration from the equilibrium value caused by the sound wave.

Owing to the deformation of the lattice, the energy of the electron (hole) in the conduction band changes and consequently the deformed lattice acts upon the electron (hole) with a force $\mathbf{F}^{(\alpha)} = -\nabla\Lambda_{ik}u_{ik}$. However in the acoustic wave there occurs in addition the production of space charge $en_{\sim}(\mathbf{x}, t)$ and the electron is therefore acted upon by an additional force $e\mathbf{E}'$ due to the Coulomb field of the space charge $\mathbf{E}'$. As noted already by Pippard[5] (see also [6]), account of this effect may alter considerably the nature of the propagation of elastic waves in solids. In order to take into account the field of the space charge, one must consider the equations of electrostatics:

$$\operatorname{div}\varepsilon_0\mathbf{E}'=4\pi e(n_{\sim}^{(e)}-n_{\sim}^{(h)}),\qquad \operatorname{rot}\mathbf{E}'=0, \tag{2*}$$

where ϵ_0 is the dielectric permittivity of the lattice. Employing further the continuity equations of both components of the electron-hole plasma in conjunction with Eqs. (2), we obtain the Fourier components of the nonequilibrium values of the electron (hole) density:

$$n_{\sim}^{(e,\,h)}(\omega,q)=-\frac{q^3\sigma_{\|}^{(e,\,h)}(\omega,q)}{e^2\omega}\times\frac{(i\varepsilon_0\omega/4\pi)\Lambda^{(e,\,h)}-\sigma_{\|}^{(h,\,e)}(\omega,q)\,(\Lambda^{(e)}+\Lambda^{(h)})}{i\varepsilon_0\omega/4\pi+\sigma_{\|}^{(e)}(\omega,q)+\sigma_{\|}^{(h)}(\omega,q)}u_z, \tag{3}$$

where $\sigma_{\|}^{(\alpha)}(\omega, q) \equiv \sigma_{zz}^{(\alpha)}(\omega, q)$ is the longitudinal conductivity. In obtaining (3) we took into account that all quantities depend on the coordinates and time according to the exp $(i\omega t - iqz)$ law.

The obtained expression for $n_{\sim}^{(\alpha)}(\omega, q)$ and expression (1) allow one to construct the dispersion equation of elastic waves with account of their interaction with the electron-hole carrier plasma. For longitudinal waves the dispersion equation takes on the form

$$\omega^2-q^2v_s^2=\frac{iq^4}{e^2\omega\rho}\times\frac{(i\varepsilon_0\omega/4\pi)[\sigma_{\|}^{(e)}\Lambda^{(e)2}+\sigma_{\|}^{(h)}\Lambda^{(h)2}]+\sigma_{\|}^{(e)}\sigma_{\|}^{(h)}(\Lambda^{(e)}+\Lambda^{(h)})^2}{i\varepsilon_0\omega/4\pi+\sigma_{\|}^{(e)}+\sigma_{\|}^{(h)}} \tag{4}$$

*rot = curl.

(v_s is the velocity of sound).

For semimetals, as well as for intrinsic semiconductors, the concentration of the electrons and holes is approximately equal. Since the inequality $|\sigma_{\|}^{(\alpha)}| \gg \epsilon_0\omega/4\pi$ is generally satisfied, only the last term in the right-hand side of (4) is appreciable, and the dispersion equation has in this case the form

$$\omega^2 - q^2 v_s^2 = \frac{iq^4(\Lambda^{(e)} + \Lambda^{(h)})^2}{e^2\omega\rho} \frac{\sigma_{\|}^{(e)}(\omega, q)\,\sigma_{\|}^{(h)}(\omega, q)}{\sigma_{\|}^{(e)}(\omega, q) + \sigma_{\|}^{(h)}(\omega, q)}. \quad (5)$$

The attenuation of elastic waves is in this case

$$\operatorname{Im} q = -\frac{\omega^2(\Lambda^{(e)} + \Lambda^{(h)})^2}{2e^2\rho v_s^5} \operatorname{Re} \frac{\sigma_{\|}^{(e)}(\omega, q)\,\sigma_{\|}^{(h)}(\omega, q)}{\sigma_{\|}^{(e)}(\omega, q) + \sigma_{\|}^{(h)}(\omega, q)}. \quad (6a)$$

From the dispersion equation (5) one can also obtain the change in the phase velocity of the sound

$$\frac{\Delta v_s}{v_s} = -\frac{\omega(\Lambda^{(e)} + \Lambda^{(h)})^2}{2e^2\rho v_s^4} \frac{\operatorname{Im}\sigma_{\|}^{(e)}|\sigma_{\|}^{(h)}|^2 + \operatorname{Im}\sigma_{\|}^{(h)}|\sigma_{\|}^{(e)}|^2}{|\sigma_{\|}^{(e)} + \sigma_{\|}^{(h)}|}. \quad (6b)$$

If no account is taken of the electric field $\mathbf{E}'$ which appears in the sound wave (i.e., one assumes $\mathbf{E}' = 0$), then the nonequilibrium electron density is determined directly from the continuity equation

$$e\omega n_{\sim}^{(e)}(\omega, q) = q j_{\sim}^{(e)}(\omega, q),$$

where $j_{\sim}^{(e)}(\omega, q) = \sigma_{\|}^{(e)}(\omega, q) e^{-1} q \Lambda^{(e)} u$ is the electron current, Substituting $n_{\sim}^{(e)}(\omega, q)$ in Eq. (1), we find that

$$\operatorname{Im} q \sim \operatorname{Re}\sigma_{\|}^{(e)}(\omega, q), \qquad \Delta v_s / v_s \sim \operatorname{Im}\sigma_{\|}^{(e)}(\omega, q).$$

One sees readily that this case corresponds to [1-4].

It follows from Eqs. (6) that the effect of the holes is very appreciable, even if there is no direct interaction of the holes with the phonons ($\Lambda^{(h)} = 0$). Physically this is fully understandable and related to the fact that when the electron distribution in space is disturbed, the holes are also redistributed in such a way that the condition of quasineutrality is on the whole fulfilled [if $|4\pi\sigma_{\|}^{(\alpha)}/\epsilon_0\omega| \gg 1$].

It is seen from expressions (5), (6a), and (6b) that to investigate the attenuation and change of velocity of sound one must know not only the real but also the imaginary part of the longitudinal permittivity of the system under the Landau quantization conditions.

2. DERIVATION OF THE "KINETIC EQUATION"

As can be seen from the dispersion equation (1), the interaction of the electrons (holes) with phonons is fully determined by the longitudinal component of the dielectric permittivity tensor

$$\varepsilon_{\|}(\omega, q) = \varepsilon_0 + (4\pi / i\omega)\sigma_{\|}(\omega, q).$$

To calculate $\epsilon_{\|}(\omega, q)$, one must consider the following problem. Let the system be acted upon by a plane electric wave

$$\mathbf{E}(z, t) = \mathbf{E}_{\sim} e^{i\omega t - iqz}, \qquad \mathbf{E}_{\sim} \parallel \mathbf{q} \quad (7)$$

and it is required to find the current induced by this wave in the system. We assume that an external magnetic field satisfying the following conditions has been imposed on the system:

$$\varkappa T \ll \hbar\Omega < \mu,$$

where μ is the chemical potential of the electron gas, $\Omega = eH/mc$ is the cyclotron frequency of the carriers in the magnetic field, T is the temperature, and κ is the Boltzmann constant. We shall assume that the magnetic field is along the z axis, i.e., along the direction of the propagation of the wave (7). For a quantum system the current induced by the wave is given by the expression

$$j_z(z, t) = \operatorname{Sp}(\hat{j}_z\hat{\rho}), \quad (8)$$

where $\hat{\rho}$ is the density matrix operator and $\hat{\mathbf{j}}$ the current operator:

$$\hat{\mathbf{j}}(z, t) = \frac{e}{2}(\hat{\mathbf{v}}\delta(z - z') + \delta(z - z')\hat{\mathbf{v}}). \quad (9)$$

Here $\hat{\mathbf{v}}$ is the velocity operator, $\hat{\mathbf{v}} = i\hbar^{-1}[\hat{\mathscr{H}}, \mathbf{r}]$, and $\hat{\mathscr{H}}$ is the Hamiltonian of the system.

In order to determine $\hat{\rho}$, we consider the equation

$$i\hbar\frac{\partial\hat{\rho}}{\partial t} = [\hat{\mathscr{H}}, \hat{\rho}], \quad \hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \hat{V} + \hat{\mathscr{H}}_{\sim},$$
$$\hat{\mathscr{H}}_0 = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2$$

where $\hat{\mathscr{H}}_0$ is the Hamiltonian of the free motion of the electron in the magnetic field; $\mathbf{A}$ is the vector potential which it is convenient to choose in the form $A_x = A_z = 0$, $A_y = xH$; $\hat{V}$ is the scattering potential; $\mathscr{H}_{\sim} = e\Phi_{\sim}(z, t)$ is the Hamiltonian of the interaction with the longitudinal wave (7), and $\Phi_{\sim}(z, t)$ is the scalar potential of the wave with $\mathbf{E}_{\sim} = -\nabla\Phi_{\sim}(z, t)$. The operator $\hat{\rho}$ will be represented in the form

$$\hat{\rho} = \hat{\rho}' + \hat{\rho}_{\sim}, \quad (11)$$

where $\hat{\rho}'$ is the density matrix operator in the absence of a wave, $\hat{\rho}_{\sim}$ is a perturbation due to the wave (7). Substituting (11) in (10) and linearizing (10) in $\hat{\mathscr{H}}_{\sim}$, we obtain respectively

$$i\hbar\partial\hat{\rho}'/\partial t = [\hat{\mathscr{H}}_0 + \hat{V}, \hat{\rho}'], \quad (12a)$$

$$i\hbar\partial\hat{\rho}_{\sim}/\partial t = [\hat{\mathscr{H}}_0 + \hat{V}, \hat{\rho}_{\sim}] + [\hat{\mathscr{H}}_{\sim}, \hat{\rho}'].\tag{12b}$$

The eigenfunctions and eigenvalues of the operator $\mathscr{H}_0$ are, as is well known,[7]

$$|n, k_y, k_z\rangle \equiv |\alpha\rangle = \frac{1}{\sqrt{L_y L_z}} \exp(ik_y y + ik_z z)\Phi_n\left(x - \frac{\hbar k_y}{m\Omega}\right),\tag{13}$$

$$\varepsilon_{n,k_z} = \varepsilon_\alpha = \hbar\Omega(n + 1/2) + \hbar^2 k_z^2/2m.\tag{14}$$

(We assume for simplicity a parabolic dispersion law.) Here Φ_n is a normalized oscillator function; n is the oscillator quantum number, and k_y and k_z are the wave numbers of the electrons. We are interested in the conductivity of the medium over the varying field of the wave (7). Therefore expression (8) for the current will include only the part of the density matrix operator $\hat{\rho}$ due to the wave, i.e., $\hat{\rho}'$. The Fourier component of the average value of the current (8) in the representation (13) is of the form

$$j_z(\omega, q) = \frac{e\hbar}{m}\sum_\alpha\left(k_z + \frac{q}{2}\right)\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle.\tag{15}$$

Consequently, in order to find the average current (15), one must know only those matrix elements $\hat{\rho}_{\sim}$ in which states differ in the wave number k_z, i.e.,

$$\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle \equiv \langle n, k_y, k_z + q|\hat{\rho}_{\sim}|n, k_y, k_z\rangle.\tag{16}$$

Let us consider Eq. (12a) for the operator $\hat{\rho}'$. Considering $\hat{V}$ a perturbation, we obtain from this equation

$$\langle\alpha'|\hat{\rho}'|\alpha\rangle = \begin{cases} f(\varepsilon_\alpha) \equiv f_\alpha & \text{for} \quad \alpha' = \alpha \\ \langle\alpha'|\hat{V}|\alpha\rangle(f_{\alpha'} - f_\alpha)/(\varepsilon_{\alpha'} - \varepsilon_\alpha) & \text{for} \quad \alpha' \neq \alpha \end{cases}.\tag{17}$$

Here $f(\epsilon_\alpha)$ is the electron equilibrium distribution function (which will hereafter be assumed to be a Fermi distribution). Substituting (17) in (12b), we obtain equations for the matrix elements $\langle\alpha'|\rho_{\sim}|\alpha\rangle$:

$$(\varepsilon_{\alpha'} - \varepsilon_\alpha - \hbar\omega + is)\langle\alpha'|\hat{\rho}_{\sim}|\alpha\rangle = \langle\alpha'|[\hat{\mathscr{H}}_{\sim}, \hat{\rho}']|\alpha\rangle + \langle\alpha'|[\hat{V}_0, \hat{\rho}_{\sim}]|\alpha\rangle,\tag{18}$$

where s is a small parameter indicating the rule for circuiting poles. Equation (18) can be split into two respectively for $\alpha' = \alpha + q$ and $\alpha' \neq \alpha + q$:

$$(\varepsilon_{\alpha+q} - \varepsilon_\alpha - \hbar\omega + is)\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle = q^{-1}ieE_{\sim}(f_{\alpha+q} - f_\alpha) + \sum_{\alpha''\neq\alpha+q}\langle\alpha + q|\hat{V}|\alpha''\rangle\langle\alpha''|\hat{\rho}_{\sim}|\alpha\rangle - \sum_{\alpha''\neq\alpha}\langle\alpha + q|\hat{\rho}_{\sim}|\alpha''\rangle\langle\alpha''|\hat{V}|\alpha\rangle,\tag{19a}$$

$$(\varepsilon_{\alpha'} - \varepsilon_\alpha - \hbar\omega + is)\langle\alpha'|\hat{\rho}_{\sim}|\alpha\rangle = \frac{ieE_{\sim}}{q}\langle\alpha'|\hat{V}|\alpha + q\rangle \times\left[\frac{f_{\alpha'-q} - f_\alpha}{\varepsilon_{\alpha'-q} - \varepsilon_\alpha} - \frac{f_{\alpha'} - f_{\alpha+q}}{\varepsilon_{\alpha'} - \varepsilon_{\alpha+q}}\right] + \langle\alpha'|\hat{V}|\alpha + q\rangle \times\{\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle - \langle\alpha'|\hat{\rho}_{\sim}|\alpha' - q\rangle\}\tag{19b}$$

In the last two sums in (19a) we omitted terms containing diagonal matrix elements of the scattering potential $\hat{V}$ which lead to a shift of the energy levels ϵ_α by a quantity $\langle\alpha|\hat{V}|\alpha\rangle$. This quantity does not depend on the state α if $\hat{V}$ does not contain differentiation or integration operators. This shift can be avoided by changing the point from which the energy is reckoned. The same result will, however, be obtained if in all calculations $\langle\alpha|\hat{V}|\alpha\rangle$ is assumed to be zero. The following approximation was employed in Eq. (19b):

$$\langle\alpha'|[\hat{V}, \hat{\rho}_{\sim}]|\alpha\rangle \approx \langle\alpha'|\hat{V}|\alpha + q\rangle\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle - \langle\alpha'|\hat{\rho}_{\sim}|\alpha' - q\rangle\langle\alpha' - q|\hat{V}|\alpha\rangle;\tag{20}$$

the validity of this approximation follows directly from the structure of Eqs. (19). Indeed, the matrix elements $\langle\alpha'|\hat{\rho}_{\sim}|\alpha\rangle$ for $\alpha' \neq \alpha + q$ are small compared with $\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle$, since the former are proportional to $E_{\sim}|V|$, and the latter simply to $E_{\sim}$. This circumstance allows one to retain in the sum (20) only terms containing $\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle$.

Substituting further (19b) in (19a), we obtain[2)] a "kinetic equation" for the matrix elements $\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle$. In the general case this equation will depend on the position of all scattering centers (if the scattering is by phonons, then the result will depend on the phase of the oscillations). Therefore the obtained equation must be averaged over the random distribution of the scattering centers or the phases of the oscillations. Such an averaging will be denoted below by an upper bar. Finally we obtain

$$(\varepsilon_{\alpha+q} - \varepsilon_\alpha - \hbar\omega + is)\langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle = \frac{ieE_{\sim}}{q}(f_{\alpha+q} - f_\alpha) + i^2\frac{\pi eE_{\sim}}{q}\sum_{\alpha''\neq\alpha}\overline{|\langle\alpha''|\hat{V}|\alpha\rangle|^2}\{\delta^- F_1 + \delta^+ F_2\} + \pi i\sum_{\alpha''\neq\alpha}\overline{|\langle\alpha|\hat{V}|\alpha\rangle|^2}\{\langle\alpha'' + q|\hat{\rho}_{\sim}|\alpha''\rangle - \langle\alpha + q|\hat{\rho}_{\sim}|\alpha\rangle\}(\delta^+ + \delta^-),\tag{21}$$

2) We note that this procedure for obtaining the "kinetic equation" for $\langle\alpha + q|\rho_{\sim}|\alpha\rangle$ is in essence close to the method used by Kohn and Luttinger [8] in constructing the quantum theory of transport phenomena in a constant field.

where

$$F_1 = \frac{f_{\alpha+\hbar\omega} - f_{\alpha+q}}{\varepsilon_\alpha - \varepsilon_{\alpha+q} + \hbar\omega} - \frac{f_{\alpha''} - f_{\alpha''+q-\hbar\omega}}{\varepsilon_{\alpha''} - \varepsilon_{\alpha''+q} + \hbar\omega},$$

$$F_2 = \frac{f_\alpha - f_{\alpha+q-\hbar\omega}}{\varepsilon_\alpha - \varepsilon_{\alpha+q} + \hbar\omega} - \frac{f_{\alpha''+\hbar\omega} - f_{\alpha''+q}}{\varepsilon_{\alpha''} - \varepsilon_{\alpha''+q} + \hbar\omega}; \qquad (22)$$

$$\delta^+ = \delta(\varepsilon_{\alpha''} - \varepsilon_{\alpha+q} + \hbar\omega), \quad \delta^- = \delta(\varepsilon_{\alpha''+q} - \varepsilon_\alpha - \hbar\omega). \quad (23)$$

One sees readily that Eq. (21) is in essence a kinetic equation for the matrix element $\langle \alpha + q \mid \hat{\rho}_\sim \mid \alpha \rangle$. However there is also a difference between it and the usual form of the kinetic equation which consists in the fact that an additional term has appeared in (21) which is connected with the combined action of the wave and of the scattering. This term makes a contribution to the expression for the average current only when the current induced by the wave is being considered. In essence a similar situation has already been considered in [9] (see also [8]), where the magnetoresistance in a homogeneous and constant field was calculated. In the presence of a wave when the average value of the current is determined by the nondiagonal matrix elements $\langle \alpha + q \mid \hat{\rho}_\sim \mid \alpha \rangle$, this term must be taken into account; however, in the case considered for $\hbar\omega$, $\hbar/\tau \ll \epsilon$ where ϵ is the characteristic energy of the electrons, one can omit it. Indeed, making use of the fact that for $q = 0$ this term vanishes, we find

$$\sum_{\alpha''} \overline{|\langle\alpha''|\hat{V}|\alpha\rangle|^2}(\delta^- F_1 + \delta^+ F_2)$$

$$= \hbar\omega \cdot \frac{\hbar^2 q k_z}{m} \frac{\partial^2 f_\alpha}{\partial\mu^2} \sum_{\alpha''} \overline{|\langle\alpha|\hat{V}|\alpha''\rangle|^2} \left\{ \delta^{+'} + \frac{k_z''}{k_z} \delta^{-'} \right\}$$

$$+ \frac{\hbar^2 q k_z}{m} \frac{\partial^2 f_\alpha}{\partial\mu^2} \sum_{\alpha''} \overline{|\langle\alpha''|\hat{V}|\alpha\rangle|^2} \left\{ \delta^+ + \frac{k_z''}{k_z} \delta^- \right\}. \qquad (23a)$$

i.e., in the region $\hbar\omega$, $\hbar/\tau \ll \epsilon$ expression (23) gives an unimportant contribution. In deriving (21) we took the electron scattering into account in the first nonvanishing approximation. Account of the following orders in the scattering potential leads to the appearance of the corresponding corrections to the first Born approximation for the cross section for scattering by one center. In the final result this leads to the necessity of using in the "kinetic equation" the scattering cross section correct in the accepted approximation (just as in the case of the calculation of the conductivity in a constant field [3]). In addition, in higher approximations there will appear interference of waves scattered by two, three, etc. scattering centers. Allowance for this effect[8] (see [3]) also leads to correction of the scattering cross section in the corresponding approximation.

3. LONGITUDINAL PERMITTIVITY OF THE ELECTRON-HOLE PLASMA IN A QUANTIZING MAGNETIC FIELD

We shall seek the solution of (21) in the form

$$\langle \alpha + q|\hat{\rho}_\sim|\alpha\rangle = (k_z + q/2)\varphi(\varepsilon_\alpha). \qquad (24)$$

Substituting (21) in (24) and taking into account that the energy of an electron is conserved in a collision (with accuracy up to the energy of a wave quantum $\hbar\omega$), we obtain

$$\langle \alpha + q|\hat{\rho}_\sim|\alpha\rangle = \frac{ieE_\sim}{q}(f_{\alpha+q} - f_\alpha)$$

$$\times \left[\varepsilon_{\alpha+q} - \varepsilon_\alpha - \hbar\omega + 2i\pi \sum_{\alpha''\neq\alpha} \overline{|\langle\alpha|\hat{V}|\alpha''\rangle|^2}\delta(\varepsilon_\alpha - \varepsilon_{\alpha''})\right.$$

$$\left.\times (1 - k_z''/k_z)\right]^{-1}. \qquad (25)$$

From expression (25) it can be seen that a "transport" cross section has appeared in the term responsible for the scattering of the electrons. Formula (25) is correct for $\hbar\omega \ll \epsilon$ where ϵ is the characteristic energy of the electron, which is fulfilled up to very high frequencies ω. Substituting furthermore (25) in (15), we obtain the complex conductivity of the medium over the varying field of the wave:

$$\sigma_\parallel(\omega, q) = \frac{ie^2\hbar}{mq} \sum_\alpha (k_z + q/2)(f_{\alpha+q} - f_\alpha)$$

$$\times \left[\varepsilon_{\alpha+q} - \varepsilon_\alpha - \hbar\omega + 2i\pi \sum_{\alpha''\neq\alpha} \overline{|\langle\alpha|\hat{V}|\alpha''\rangle|^2}\delta(\varepsilon_\alpha - \varepsilon_{\alpha''})\right.$$

$$\left.\times (1 - k_z''/k_z)\right]^{-1}. \qquad (26)$$

We note that in the limit when q and ω tend to zero, expression (26) goes over to

$$\sigma_\parallel(0,0) = \frac{e^2}{m} \sum_\alpha \frac{\hbar^2 k_z^2}{2m} \frac{\partial f_\alpha}{\partial\mu} \left[\frac{\pi}{\hbar} \sum_{\alpha''\neq\alpha} \overline{|\langle\alpha|V|\alpha''\rangle|^2}\right.$$

$$\left.\times \delta(\varepsilon_\alpha - \varepsilon_{\alpha''})(1 - k_z''/k_z)\right]^{-1}, \qquad (27)$$

from which one can, for example, obtain the results of [10, 11] of the calculation of the longitudinal magnetoresistance. In the absence of collision, Eq. (26) goes over naturally into the result of Spector.[12] It also readily shows that for $|V| \to 0$, $q \to 0$ expression (26) gives the known classical result for the longitudinal permittivity of a plasma $\epsilon_\parallel = 1 - \omega_0^2/\omega^2$ where ω_0 is the plasma frequency of the electrons.

Let us further consider the imaginary and real part of the longitudinal conductivity. We introduce a relaxation time defined by the usual relation:

$$\hbar/\tau = 2\pi \sum_{\alpha''\neq\alpha} \overline{|\langle\alpha''|\hat{V}|\alpha\rangle|^2}(1 - k_z''/k_z)\delta(\varepsilon_{\alpha''} - \varepsilon_\alpha). \qquad (28)$$

Since (26) contains the difference of the Fermi functions $f_{\alpha+q} - f_\alpha$ which is close to $\delta(\epsilon_\alpha - \mu)$, one can replace the value of the energy ϵ_α in (28) by μ. Equation (28) undergoes oscillations as a function of the magnetic field; these oscillations indeed determine, as can be seen from (27), the oscillations of the magnetoresistance in the Shubnikov-de Haas effect. Below we shall, however, neglect the Shubnikov-de Haas oscillations compared with the "giant oscillations" of Gurevich et al.[1] The exact condition under which this assertion is correct depends on the concrete scattering mechanism. For example, when the scattering takes place on neutral impurities, the condition for the neglect of the oscillations will, as has been shown by Skobov,[3] be of the form

$$\hbar/\tau \ll \varkappa T, \quad \hbar\Omega \ll \sqrt{\mu\cdot\varkappa T}. \tag{29}$$

From (26) one can obtain the values of the imaginary and real part of the conductivity:

$$\operatorname{Re}\sigma_{\parallel}(\omega, q) = \frac{e^2\hbar^2}{m^2\tau}\sum_{k_y,k_z}\frac{k_z^2}{(\hbar q k_z/m-\omega)^2+\tau^{-2}} \times \sum_{n=0}^{N}\frac{1}{2\varkappa T}\operatorname{ch}^{-2}\left(\frac{\mu-\varepsilon_{n,k_z}}{2\varkappa T}\right), \tag{30a*}$$

$$\operatorname{Im}\sigma_{\parallel}(\omega, q) = \frac{e^2\hbar^2}{m^2}\sum_{k_y,k_z}\frac{k_z^2(\hbar q k_z/m-\omega)}{(\hbar q k_z/m-\omega)^2+\tau^{-2}} \times \sum_{n=0}^{N}\frac{1}{2\varkappa T}\operatorname{ch}^{-2}\left(\frac{\mu-\varepsilon_{n,k_z}}{2\varkappa T}\right), \tag{30b}$$

where the integer N is found from the relation $N = E[(2\mu - \hbar\Omega)/2\hbar\Omega]$. In obtaining (30) use was made of the condition $\hbar\omega \ll mv_{ph}^2$ (v_{ph}—phase velocity).

We introduce now the dimensionless variables

$$y = \frac{\hbar k_z}{mv_\tau}, \quad a = \frac{v_{ph}}{v_{th}}, \quad A_n = \frac{\mu - \hbar\Omega(n+1/2)}{\varkappa T}, \quad b = \frac{1}{qv_{th}\tau}, \quad v_{th} = \left(\frac{2\varkappa T}{m}\right)^{1/2} \tag{31}$$

(v_{th}—thermal velocity of the electrons). In terms of this notation (30) takes on the form

$$\operatorname{Re}\sigma_{\parallel}(\omega, q) = \frac{\Omega}{4\pi^2}\frac{1}{q^2 v_{th}^2\tau a_0}\int_{-\infty}^{+\infty} dy\frac{y^2}{(y-a)^2+b^2} \times \sum_{n=0}^{N}\operatorname{ch}^{-2}\left(\frac{y^2-A_n}{2}\right), \tag{32}$$

$$\operatorname{Im}\sigma_{\parallel}(\omega, q) = \frac{\Omega}{4\pi^2}\frac{\omega}{q^2 v_{th} a_0} \times \int_{-\infty}^{\infty} dy\frac{y^2(y^2-a^2-b^2)}{(y^2-a^2)^2+2b^2(a^2+y^2)+b^4} \times \sum_{n=0}^{N}\operatorname{ch}^{-2}\left(\frac{y^2-A_n}{2}\right), \tag{33}$$

where $a_0 = \hbar^2/me^2$ is the Bohr radius. In going over to (32) and (33) the summation over k_z in (30a) and (30b) was replaced by integration, and in (33) the integrand was symmetrized in y (this is necessary for removing the fictitious divergence of the first integrand in (30b) for $y = \pm\infty$).

The conditions under which the oscillations of the attenuation of ultrasound become observable were considered in [1-4]. Consideration of the oscillations of Re $\sigma_{\parallel}$ given by (32) leads to the same results as those of [1-4]; we therefore only present the result. Oscillations are observed only when the following conditions are satisfied:

$$ql > \left(\frac{\mu}{\hbar\Omega}\right)^{1/2}, \quad ql > \left(\frac{\mu}{\hbar\Omega}\right)^{1/2}\frac{v_F}{v_{ph}}\left(\frac{2\varkappa T}{mv_{ph}^2}\right)^{1/2}, \quad l = v_F\tau. \tag{34}$$

Let us consider now the behavior of the imaginary part of the conductivity as a function of the magnetic field. The integrand in (33) consists of the product of the two functions

$$\varphi_1(y) = \frac{y^2(y^2-a^2-b^2)}{(y^2-a^2)^2+2b^2(y^2+b^2)+b^4}$$

and $$\varphi_2(y) = \sum_{n=0}^{N}\operatorname{ch}^{-2}\left(\frac{y^2-A_n}{2}\right)$$

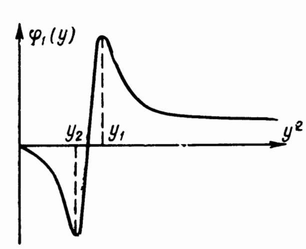
The function $\varphi_2(y)$ consists, as is well known, of a series of peaks of unit height located at the points $y^2 = A_n$ and having a width $\Delta = (1 + A_n)^{1/2} - (A_n)^{1/2}$. The distance between nearest peaks is $(\hbar\Omega/\kappa T + A_n)^{1/2} - (A_n)^{1/2}$. On the other hand, the function $\varphi_1(y)$ whose behavior is shown in the figure, has two extrema at the points

$$y_{1,2}^2 = \frac{a^2+b^2 \pm 2b^2(a^2+b^2)^{1/2}}{a^2-3b^2} \approx a^2\left(1 \pm \frac{2b}{a}\right) \quad (a \gg b), \tag{35}$$

the distance between which is

$$y_1^2 - y_2^2 \approx 4ab = 4v_{ph}^2/\omega\tau v_{th}^2.$$

The ratio of the values of the function φ_1 at the



*ch = cosh.

extrema to its value at infinity turns out to be of the order of $\omega\tau$; therefore, if one is interested in the oscillations of $\mathrm{Im}\,\sigma_{\parallel}(\omega, q)$ it makes sense to consider only the range of frequencies $\omega\tau \gg 1$ where the extremal values of the function φ_1 are relatively large. Clearly, oscillations of the imaginary part of the conductivity as a function of the magnetic field will appear when one of the extrema of the function $\varphi_1(y)$ will coincide with a peak of the function $\varphi_2(y)$. However, in order that these oscillations be noticeable, it is required that in addition to the condition $\omega\tau \gg 1$ the following conditions be also fulfilled: first, the width of the extrema of the first function must be larger than the width of the peaks of the second function, i.e.,

$$\Delta_n < \frac{4}{\omega\tau}\frac{v_{\mathrm{ph}}^2}{v_{\mathrm{th}}^2}, \tag{36}$$

and, secondly, the contribution to the integral in the region $y^2 > a^2 + 2ab$ in which $\varphi_1 \approx 1$ should be smaller than the contribution from the region where the peaks of the first and second function coincide. The second condition yields

$$\omega\tau > \sqrt{\mu / \hbar\Omega} > 1. \tag{37}$$

Taking into account that at the extremum $y^2 = A_n$ should be of the order of a^2, condition (36) can be written in the form

$$\frac{16}{\omega^2\tau^2}\frac{v_{\mathrm{ph}}^2}{v_{\mathrm{th}}^2} + \frac{8}{\omega\tau}\frac{v_{\mathrm{ph}}^3}{v_{\mathrm{th}}^3} > 1. \tag{36a}$$

If the conditions (36) and (37) are not fulfilled, then the imaginary part of the conductivity does not oscillate in a magnetic field, and in this case it is

$$\mathrm{Im}\,\sigma_{\parallel}(\omega, q) \approx \frac{2}{3}\frac{e^2 n_0 \omega}{m v_F^2 q^2} \quad \left(\frac{v_{\mathrm{ph}}}{v_{\mathrm{th}}} \ll \omega\tau < 1\right), \tag{38}$$

which corresponds to the well-known expression for the longitudinal dielectric permittivity at low frequencies[6]

$$\varepsilon_{\parallel} = \varepsilon_0 + 1 / q^2 r_D^2, \quad r_D^2 = (3\mu / 8\pi e^2 n_0)^{1/2},$$

where r_D is the Debye radius.

The phase velocity of the wave is of the order of the sound velocity; for this reason it is very difficult to satisfy condition (38) experimentally. One can, however, attempt to observe oscillations of $\mathrm{Im}\,\sigma_{\parallel}(\omega, q)$ in metals where one has "heavy" holes; in this case one needs stronger magnetic fields to satisfy the condition $\hbar\Omega \gg \kappa T$.

4. OSCILLATIONS IN THE ATTENUATION AND VELOCITY OF SOUND

Above we obtained an expression for the real and imaginary part of the longitudinal conductivity of the medium in order to investigate the nature of the oscillations of the attenuation and velocity of sound. So far a relatively large number of experiments have been carried out to investigate "giant" attenuation oscillations in various substances, viz., in zinc,[13] gallium,[14] lead telluride,[15] and bismuth[16-18]; in all cases the observed oscillations are not as large as predicted in [1]. The chief cause of the decrease in the oscillations of the sound attenuation compared with the oscillations $\mathrm{Re}\,\sigma_{\parallel}(\omega, q)$ is, as has been indicated above, the appearance of a Coulomb field of the space charge produced by the longitudinal wave [see formulas (5) and (6)].

We note, however, that in semiconductors and metals with a complex dispersion law (for instance, with a number of minima of the energy of the electron as a function of its momentum) quasi-equilibrium electron transitions are possible under the influence of the wave between individual minima ("intervalley" transitions; see, for example, [19]). The latter should lead to a decrease in the space charge produced in the longitudinal wave, and should consequently increase the amplitude of the oscillations. We shall not consider this mechanism in this paper; for this reason the results obtained here apply to semiconductors and metals with a Fermi surface which does not differ too much from a spherical surface. For complex Fermi surfaces this kind of mechanism may play an appreciable role in establishing the amplitude of the attenuation and velocity of sound oscillations. Let us consider the attenuation and change of the velocity of sound in bismuth which have been well studied experimentally.[16, 17]

The equal-energy surface of the electrons in bismuth can be represented in the form of three equivalent ellipsoids whose principal axes pass through the centers of the pseudohexagonal planes of the first Brillouin zone;[20] as far as the equal-energy surface of the holes is concerned, it can be approximately regarded to be a sphere.[20] In the experiment in [16] a longitudinal sound wave propagated in a longitudinal magnetic field along the binary and bisector axis. One sees readily that the interaction of such a wave with electron groups belonging to different minima leads to the appearance of a space charge. Since in addition the concentrations of electrons and holes in bismuth are approximately equal, the oscillations of the attenuation and velocity of sound waves are described by expressions (6a) and (6b).

Under conditions when the real part of the conductivity $\mathrm{Re}\,\sigma_{\parallel}(\omega, q)$ oscillates, the imaginary part $\mathrm{Im}\,\sigma_{\parallel}(\omega, q)$ does not oscillate and remains small compared with $\operatorname{Re} \sigma_{\|}(\omega, q)$; therefore the expression for the attenuation (6a) will be of the form

$$\Gamma = \frac{\omega^2 \Lambda^2}{2e^2 \rho v_s^5} \frac{\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q) \operatorname{Re} \sigma_{\|}^{(h)}(\omega, q)}{\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q) + \operatorname{Re} \sigma_{\|}^{(h)}(\omega, q)}, \quad (39)$$

from which it can be seen that the amplitude of the oscillations is determined by the ratio of the real parts of the electron and hole conductivity. Since the effective masses of the holes are usually larger than those of the electrons, $\sigma_0^{(e)} \gg \sigma_0^{(h)}$. This relation, however, may also not be fulfilled for the conductivities $\sigma_{\|}^{(\alpha)}(\omega, q)$ depending on the frequency ω and the wave vector q.

If for holes $ql^{(h)} \ll 1$, $\kappa T \gg \hbar\Omega^{(h)}$, then the hole conductivity can be found from the classical kinetic equation[21, 22]

$$\sigma_{\|}^{(h)}(\omega, q) = \frac{\sigma_0^{(h)}}{1 - iq^2 v_F^{(h)2} \tau^{(h)}/3\omega}. \quad (40)$$

It follows from (40) that $\operatorname{Re} \sigma_{\|}^{(h)}(\omega, q)$ is of the same order of magnitude as the hole conductivity for direct current $\sigma_0^{(h)}$. On the other hand, the limiting value of the electron conductivity when the collisions of the electrons with the lattice are small and there is no magnetic field at zero temperature is, as has been shown in [12],

$$\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q) = \frac{3\pi}{2} \sigma_0^{(e)} \frac{1}{\omega\tau} \left(\frac{v_s}{v_F}\right)^3$$

$$(T = 0,\ H = 0). \quad (41)$$

Since in bismuth $v_S/v_F \sim 10^{-3}$, $\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q) \sim 10^{-9} \sigma_0^{(e)}$. This means that even in the presence of a magnetic field and at a finite temperature $\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q) \ll \sigma_0^{(e)}$; one cannot, therefore, neglect in the denominator of (39) the hole conductivity compared with the electron conductivity. The last condition in fact ensures the appearance of oscillations whose amplitude is determined by the ratio of the minimum value of the real part of the electron conductivity to that of the hole conductivity; if, on the other hand, this ratio is larger than the relative amplitude of the oscillations of $\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q)$ itself, then the amplitude of the oscillations of the attenuation of elastic waves is determined by the amplitude of the oscillations of $\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q)$. Apparently it is precisely the first case which is realized in bismuth,[16] where the relative amplitude of the oscillations of the attenuation of longitudinal waves, observed experimentally, turns out to be of the order of 2–3, while the relative amplitude of $\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q)$ turns out to be considerably larger.[3)]

Let us now consider the change in the oscillations of the velocity of sound waves in bismuth. If no account is taken of the formation of the space charge in the sound wave and the Coulomb field connected with it, then the change in the phase velocity of the waves is determined by the imaginary part of the conductivity $\operatorname{Im} \sigma_{\|}^{(e)}(\omega, q)$. As was explained above, the oscillations of $\operatorname{Im} \sigma_{\|}^{(e)}(\omega, q)$ appear only when conditions (36) and (37) are fulfilled; these conditions have so far not been realized experimentally. Therefore if one does not take into account the formation of a space charge in the wave, no oscillations can occur in the phase velocity. However, oscillations in the change of the phase velocity in bismuth are observed experimentally.[16] Account of the Coulomb field indicates [see (6b)] that the real part of the conductivity $\operatorname{Re} \sigma_{\|}^{(e)}(\omega, q)$ which indeed ensures the appearance of the velocity oscillations really enters in the expression for $\Delta v_S/v_S$. Since the expressions for the attenuation (39) and the change in the sound velocity have opposite signs, their oscillations should be of opposite phase. Precisely this is the nature of the oscillations in the sound velocity observed in bismuth.[16]

Similarly one can explain the nature of the oscillations in the attenuation of longitudinal sound waves in gallium[14] where their amplitude just as in bismuth, is of the order of 2–4.

As regards the oscillations of the attenuation of longitudinal sound in zinc,[13] their nature can be explained if one takes into account that the interaction of the carriers with the sound wave take place not by means of the deformation potential, but by means of the Coulomb field which accompanies this wave in the "charged" lattice. Since the at-

3) In the experiment of Toxen and Tansal [17] the sound wave propagated in bismuth along a binary axis with the polarization vector in the bisector direction. (The magnetic field was perpendicular to the propagation of the wave and was applied along the bisector axis.) It can be shown that in the model of three equivalent ellipsoids the wave should not be accompanied by a space charge, since for two ellipsoids the change in the energy takes place with opposite signs and in the case of the third it vanishes. Therefore all the conditions for the applicability of the discussions [1–4] are fulfilled. Nevertheless, the amplitude of the oscillations is still of the order of 2 – 3 and the effect occurs also in the case of a magnetic field perpendicular to the propagation vector of the wave, whereas it should not occur according to [1]. Possibly this phenomenon is altogether not related to the "giant oscillations," [1] being due to Shubnikov-de Haas type oscillations [11].

tenuation of ultrasound is in this case inversely proportional to the effective conductivity, then with increasing magnetic field when the conductivity decreases the attenuation should increase. This is qualitatively confirmed by the experiment.[13] We note, however, that the above-mentioned mechanism of "intervalley" transitions may play a definite role in the case of zinc. This mechanism apparently plays an appreciable role in determining the character of the oscillations in the impurity semiconductor PbTe where the dispersion law is very complicated.[15]

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