GENERATION DYNAMICS OF A GIANT COHERENT LIGHT PULSE

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The space-time development of a giant light pulse from a Q-switched laser has been investigated. It is shown that the observed giant pulse is due to the superposition of a series of closely spaced pulses generated by neighboring regions in the laser. The effect of an inhomogeneous distribution of population inversion density upon the giant pulse structure has also been investigated.

THE most effective method of generating giant pulses of coherent light, known as the method of Q-switching the laser, was submitted by Hellwarth.[4] The early theoretical investigations of the energy, peak power, and rise and fall rates of the giant pulse[1-5] were based on balance equations for the total energy of the radiation field and for the total number of active particles in the resonator. These equations describe a simple model assuming that the population inversion density and the energy density of the radiation field are uniform within the cavity. Consequently, such a model fails to take into account electromagnetic effects that determine the space-time development of the generated field, as well as the effect of the initial spatial distribution of population inversion and cavity geometry upon the pulse generation dynamics. The delay time, length, and shape of the giant pulse, as computed from the simplified model, may strongly differ (even from the qualitative point of view[6, 7]) from actual values, because, as the following will show, the observed pulse is due to the superposition of a series of closely-spaced pulses generated by neighboring regions of the laser. The necessity to consider the spatial development of the generation region when Q-switching is present was pointed out in [8].

Orderly theoretical analysis of the space-time process of generating a giant pulse of light should take into account the essentially nonlinear, nonstationary interaction of many modes in a resonator with an inhomogeneous population inversion, whose dimensions are much larger than the emitted wavelength. The space-time development of the giant pulse is essentially determined by population inversion inhomogeneities that are transverse to the laser axis. An effective method of investigating nonlinear, nonstationary interaction of many modes in the presence of an inhomogeneous population inversion was developed and used to study passive Q-switching laser processes in solids.[9] The present work makes use of the same method to study Q-switched laser dynamics.

1. DEFINITION OF THE PROBLEM. BASIC EQUATIONS

Let us consider the following model of a Q-switched laser which will completely describe the electrodynamics of a giant pulse development. The active medium is confined by totally reflecting plane-parallel mirrors, spaced at a distance L from one another, along the z-axis (laser axis). The real losses in the laser (absorption and radiation transfer) are considered uniformly distributed within the cavity. Let us assume further that the active medium is characterized by a complex dielectric permittivity, uniform along the y axis, but variable along the x axis which is perpendicular to the laser axis:

$$\varepsilon(x,t)=\begin{cases}\varepsilon_0+i\varepsilon_0''-i\varepsilon''(x,t), & |x|\leqslant a\\ 1, & |x|>a\end{cases}. \tag{1}$$

Here ϵ_0'', designating radiation losses, is related to the photon lifetime, T_0, in the resonator by:

$$\varepsilon_0''=1/T_0\omega_0, \tag{2}$$

while $\epsilon''(x,t)$ designates emission gain and is related to population inversion density, $N(x,t)$, by:

$$\varepsilon''(x,t)=(\lambda/2\pi)\sigma N(x,t), \tag{3}$$

where ω_0 and λ are frequency and wavelength, respectively, of the output emission, and $\sigma=\sigma(\omega_0)$ is the radiative transition cross section.

Q-switched lasers generate many longitudinal (axial) modes and, therefore, the periodic variation in population inversion is quite smooth along the axis. As a result, the conditions governing the development of various longitudinal modes are practically the same. Consequently, an analysis of the significant effects of a transverse development of the giant pulse can be limited to the case of a single longitudinal mode.

The electric field, $\mathscr{E}(x, t)$, in the resonator is defined by the equation:[9]

$$\frac{\partial \mathscr{E}(x,t)}{\partial t} = i\frac{c^2}{2\varepsilon_0\omega_0}\frac{\partial^2 \mathscr{E}(x,t)}{\partial x^2} + \frac{\omega_0}{2\varepsilon_0}[\varepsilon_0'' - \varepsilon''(x,t)]\,\mathscr{E}(x,t), \quad (4)$$

where $\mathscr{E}(x, t) = U(x, t)e^{-i\alpha\, x, t}$; $U(x, t)$ and $\alpha(x, t)$ are the "slowly" varying amplitude and phase of the field at the mirror of the laser, while the field within the cavity has the form,

$$E(x, z, t) = \mathrm{Re}\left\{\mathscr{E}(x,t)\sin\frac{\omega_0 z}{c}e^{-i\omega_0 t}\right\},$$

where $\omega_0 = \pi nc/L$, n is the longitudinal mode index, and c is the velocity of light in the medium.

Variation of $\epsilon''(x, t)$ is defined by the equation:

$$\frac{\partial \varepsilon''(x,t)}{\partial t} = -\frac{2\sigma}{\hbar\omega_0}\varepsilon''(x,t)I(x,t), \quad (5)$$

where $I = c\epsilon_0\overline{E}_2/8\pi$ is beam density (in erg/cm^2 · sec). The following processes, which are slow in comparison to the development time of a giant pulse, have been neglected in (5): spontaneous decay of population inversion and changes in inversion due to pumping.

We solve (4) and (5) with boundary conditions at the lateral surface of the active medium, $\mathscr{E}(\pm a, t) \equiv 0$, corresponding to total reflection from the lateral surface. Using these boundary conditions, we neglect the mirror edge diffraction effect upon the field distribution and losses. Neglect of the contribution of mirror edge diffraction to field distribution is well justified, since the chief role in the spatial development of a giant pulse is played by inhomogeneities (initial and those formed during generation) of the population inversion density. The contribution of diffraction to radiation losses is readily accounted for by introducing an additional diffraction term into ϵ_0'' (which, incidentally, has but a slight effect upon the solution).

When solving the system (4) and (5), it is convenient to express the electromagnetic field inside the laser in the form of a superposition of transverse (corner) modes with time-dependent complex amplitudes:

$$\mathscr{E}(x,t) = \sum_{k=1}^{\infty} a_k(t)U_k(x)e^{-i\Omega_k t}, \quad (6)$$

where $U_k(x)$ and Ω_k are eigenfunctions and eigenvalues of (4) with $\epsilon''(x, t) \equiv 0$, given the above boundary conditions. Substituting (6) in (4) in the usual manner yields a system of equations for amplitudes $a_k(t)$, which can be conveniently represented by

$$a_k(t)e^{i(\Omega_1-\Omega_k)t} = A_k'(t) + iA_k''(t). \quad (6a)$$

A system of an infinite number of equations is obtained as the result:

$$\dot{A}_k'(t) = \frac{\omega_0}{2\varepsilon_0}\sum_{m=1}^{\infty} A_m'(t)\varepsilon_{km}''(t) + (\Omega_k - \Omega_1)A_k''(t),$$

$$\dot{A}_k''(t) = \frac{\omega_0}{2\varepsilon_0}\sum_{m=1}^{\infty} A_m''(t)\varepsilon_{km}''(t) - (\Omega_k - \Omega_1)A_k'(t), \quad (7)$$

where $k = 1, 2, \ldots, \infty$; matrix elements ϵ_{km}'' are defined by the expression

$$\varepsilon_{km}''(t) = \delta_{km}\left(\frac{1}{\omega_0 T_0} + \Lambda_k\frac{c}{2L\omega_0}\right) - \int_{-a}^{a} U_k(x)\varepsilon''(x,t)U_m(x)\,dx, \quad (8)$$

Λ_k is the magnitude of diffraction losses per pass for a plane mirror resonator, obtained by Vaĭnshteĭn,[10] and δ_{km} is the Kronecker delta.

Initial conditions must be specified for the solution of (5) and (7).

2. INITIAL CONDITIONS

The amplitude and phase of the initial field in every mode are determined by spontaneous emission at the instant the Q is restored. The average number of spontaneous photons in a single mode $\langle n_0\rangle$ is determined by the expression

$$\langle n_0\rangle \approx \frac{1}{8\pi\, T_1\Delta\nu}\left(\frac{\lambda}{a}\right)^2\int N_2\,dv, \quad (9)$$

where $\int N_2 dv$ is the total number of excited atoms in the resonator, $\Delta\nu$ is the linewidth of spontaneous emission of the atoms, T_1 is the lifetime of an excited atom with respect to a spontaneous transition to a lower level, and $(\lambda/a)^2$ is the solid angle subtending the wave vectors of a single mode. The mean square of field intensity of spontaneous emission, $\langle E_0^2\rangle$ in a mode will then equal

$$\langle E_0^2\rangle \approx \frac{\overline{N}_2}{T_1\Delta\nu}\left(\frac{\lambda}{a}\right)^2\hbar\omega_0, \quad \overline{N}_2 = \frac{1}{V}\int N_2\,dv, \quad (10)$$

where $\overline{N}_2$ is the average density of excited atoms in the resonator, and V is the resonator volume.

Spontaneous emission is somewhat enhanced by population inversion. Therefore, the gain per pass of the resonator, k_0, should be added to the expressions for $\langle n_0\rangle$ and $\langle E_0^2\rangle$. In the case of parameters characteristic of Q-switched lasers, $\langle n_0\rangle \gg 1$.

The statistics of spontaneous emission of a medium without feedback should approach the statistics of equilibrium radiation.[11] Consequently, the probability that a single mode will be filled with n photons is given by the Bose-Einstein distribution which, in the classical limit under consideration, $\langle n_0 \rangle \gg 1$, has the form (see, for example, [11-12]):

$$p(n)=\frac{1}{\langle n_0\rangle}\exp\left(-\frac{n}{\langle n_0\rangle}\right). \qquad (11)$$

Consequently, the initial field amplitudes per mode should be selected at random with a considerable dispersion about the mean value. The initial field phases in various modes are totally independent and are uniformly distributed.

The initial spatial distribution of $\epsilon''(x)$ is inhomogeneous. The population inversion is usually highest at the crystal axis and falls off considerably (becomes several times smaller) near the lateral surface.[13-15] The degree of inhomogeneity in the distribution of $\epsilon''(x)$ is determined by the design of the pump, the finish of the lateral crystal surface which affects the focusing of the pumping radiation by the crystal, etc. However, it seems that a 20–30% variation in $\epsilon''(x)$ from the center to the edge of the crystal is unavoidable.[14, 15]

3. SOLUTION OF THE EQUATIONS. PULSE DEVELOPMENT

The solution of the system of equations (5) and (7), together with the boundary conditions, completely describes the space-time development of the giant pulse. However, since it cannot be obtained by analytic means, an electronic computer was used for numerical integration of the equations. The results of numerical integration provide a sufficiently clear picture of laser processes subject to instantaneous Q-switching.

The field distribution generated in the resonator is to a certain degree "smooth" in the transverse dimension. From the physical point of view this means a preponderance of lower order modes. At the instant the Q is restored, all the modes are under the same conditions, since the mean initial amplitude, $\langle E_0^2 \rangle$, does not depend upon the modal order. However, as generation proceeds, only a few modes are effectively excited.

The inhomogeneities of $\epsilon''(x)$ play the principal role in the preferential excitation of lower order modes. Even a 20–30% variation in $\epsilon''(x)$ from the center to the edge of the mirror (or of the crystal end surface) in a filled resonator results in a field distribution characterized by a maximum in the center and a marked drop towards the edge[16] and containing mainly lower order modes. Consequently, the process of solving the infinite number of equations (7) can be limited to a finite number of modes, k_{max}, which is determined in each case

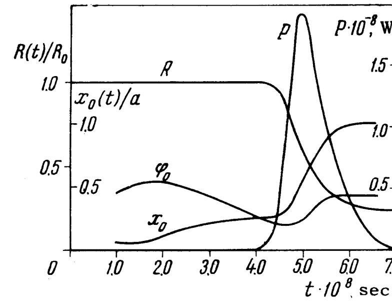

FIG. 1. Development of a giant light pulse. P – power, R – total number of active particles in the resonator, $2x_0$ – half-power width of the generated region ("jet"), φ_0 – divergence of emission, φ_1 – divergence of lowest mode.

by the actual parameters of the laser (crystal diameter, resonator length, and degree of inhomogeneity of the population inversion).

Figure 1 shows the development of a giant pulse for a ruby laser having the following typical parameters: L = 50 cm; 2a = 7 mm; $\lambda = 7 \times 10^{-5}$ cm; $\sigma = 4 \times 10^{-20}$ cm^2; $r_1 r_2 \eta^2 = 0.15$ (r_1, r_2 are reflection coefficients of the mirrors); η is the transparency of the laser crystal and shutter; the gain per pass is $e^{\alpha l} = 12$. The total initial distribution of the population inversion decreased smoothly from the center to the edge by a factor of two. The solution was obtained for the case of $k_{max} = 14$ modes. The truncation of the infinite system to a finite number of equations was always monitored by following the variation of the solution with increasing number of modes. The final solution of the problem was reached when it no longer depended upon k_{max}. The following characteristics were computed: giant pulse power, P_{out}, and the total number of atoms with population inversion, R(t):

$$P_{out}=P\frac{\ln r_1 r_2}{\ln r_1 r_2\eta^2}, \qquad R(t)=\int N(t)\,dv, \qquad (11a)$$

as well as the field distribution over the crystal end surface and in the far field. The far field distribution, $E(t, \varphi)$, which determines the divergence of the giant pulse, was computed according to the equation

$$E(t,\varphi)=\sum_{n=1}^{\infty}\sqrt{\frac{2}{\pi}}\int_0^{\pi}A_n(t)\sin n\xi\cos k\varphi\,\xi\,d\xi$$

$$=\sum_{m=1}^{\infty}\sqrt{\frac{2}{\pi}}\left[\frac{A_{2m}}{m}\,\frac{\sin(\pi k\varphi/2)}{1-(k\varphi/2m)^2}\right.$$

$$\left.+\,i\,\frac{A_{2m-1}}{m+{}^1/_2}\,\frac{\cos(\pi k\varphi/2)}{1-(k\varphi/(2m+1))^2}\right], \qquad (12)$$

where $k = 4a/\lambda$.

The generation process in Fig. 2 is completely described in terms of field intensity distribution, I(x), over the crystal end surface, and the trans-

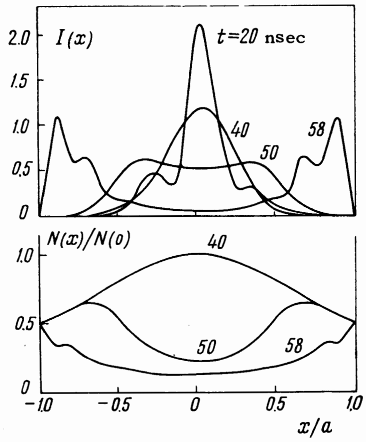

FIG. 2. Instantaneous distributions of emission intensity, I(x), over crystal end surface (top), and population inversion, N(x), (bottom).

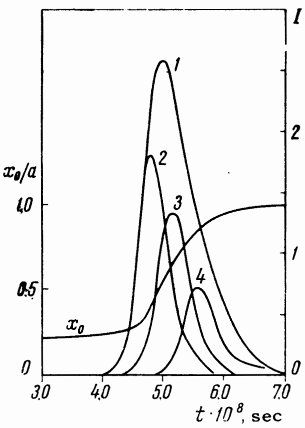

FIG. 4. Fine structure of a giant pulse. Curve 1 – pulse generated by the entire crystal end surface, curves 2, 3, 4 – pulses generated by crystal points x = 0, a/2, 3a/4, respectively, x_0 – half-width of generated region.

verse distribution of population inversion, N(x) in the most typical time instants. Figure 3 shows the distributions of squared mode amplitudes, $|A_k|^2$, for all phases of pulse development, demonstrating that the number of modes used is entirely sufficient for a self-consistent description of the generated field.

The solution given is typical and contains all the essential time instants of the space-time development of a giant pulse. At the instant the Q is restored (t = 0), the mode amplitude and field distribution over the end surface are random, while the population inversion is maximum in the center of the crystal. After a short time (~ 10–20 nsec), the generated field assumes a characteristic pattern (Fig. 2) showing a maximum in the center and a narrow "jet" shape. The "jet" signifies a specific distribution of mode amplitudes (Fig. 3). Further development of generation, accompanied by a practically unchanged number of active particles, R, causes the "jet" to broaden to a quasi-stationary value (t ≈ 40 nsec, Fig. 1), i.e., the natural mode of the filled resonator has been established. If the delay time exceeds the time required for the "jet" to establish itself, generation is independent of the initial values of mode amplitudes. The lowest mode has the maximum amplitude in an established "jet" (t = 40 nsec, Fig. 3). The start of the giant pulse itself is accompanied

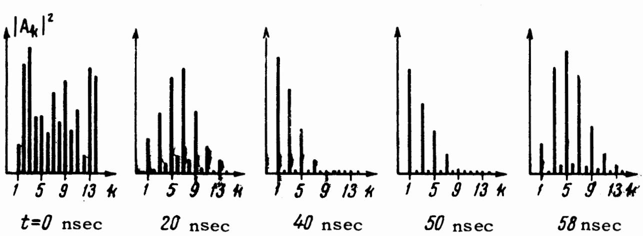

FIG. 3. Instantaneous distributions of mode intensity.

by a sharp broadening of the "jet": the field washes out towards the edges of the crystal. At that time, the divergence decreases at first, only to increase again towards the end of the pulse. The fall of the pulse is characterized by increasing amplitudes of higher modes.

4. FINE STRUCTURE OF THE GIANT PULSE

The above solution shows that the giant pulse is at first emitted by the central region of the crystal; the generation then spreads transversely towards the edges of the crystal in a time interval comparable to the pulse length. Consequently, the length of the pulse emitted by the entire crystal end surface is larger than that emitted by any one point in the end surface. Such phenomena have been experimentally observed.[1)] Figure 4 shows pulses generated by the entire crystal end surface (curve 1), central point x = 0 (curve 2), and points x equal to a/2 and 3a/4 (curves 3, 4) near the edge of the crystal. At the same time, the figure shows broadening of the generation region x_0. The length of pulses generated by the elements of the crystal end surface reaches 5 nsec. The edge regions of the crystal emit with a delay of ~ 10 nsec in comparison with the center of the crystal. The length of the delay depends on the degree of inhomogeneity of population inversion. The present theories of Q-switched lasers fail to take this fact into account and are therefore not capable of arriving at the correct value of the giant pulse length.[2)]

The inhomogeneity of population inversion can substantially change the shape of the giant pulse.

1)Communication from V. S. Zuev to the Nonlinear Optics Symposium, Naroch', 1965.

2)The fact that the usual theoretical value of the giant pulse length is noticeably shorter than the experimental value was pointed out by N. G. Basov.

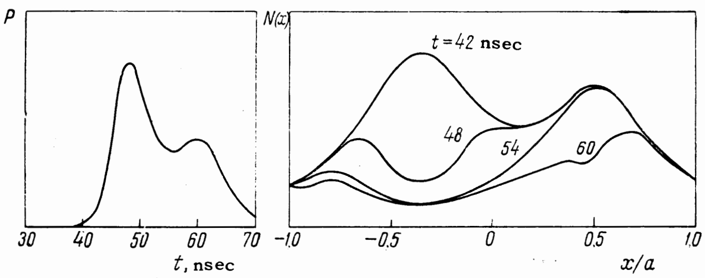

FIG. 5. Pulse shape obtained when initial population inversion had two maxima.

As an example, Fig. 5 shows the shape of a pulse in which the initial distribution of the inversion has two maxima. The generation at first begins in the region of the highest gain; spreading transversely, it then reaches a second inversion maximum. A second intensity peak occurs as the result. Hellwarth[7] reported to the Third Quantum Electronics Conference on giant pulses with a second maximum in the trailing slope of the first pulse, but he failed to explain this phenomenon. The above example indicates that this phenomenon may possibly be related to inhomogeneity of population inversion.

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