DYNAMIC DISSIPATION OF MAGNETIC ENERGY IN THE VICINITY OF A NEUTRAL MAGNETIC FIELD LINE

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Deformation of a magnetic field in a compressible conducting medium in the vicinity of zero field lines is considered. It is demonstrated that displacement of the currents producing the field gives rise to some regions in which an increase of the field gradients is accompanied by a decrease in the density of the medium. Under certain conditions this leads to violation of the freezing-in of the field and to the appearance of strong electric fields that accelerate the charged particles of the medium. The process is essentially a nonstationary one and results in the transformation of the excess magnetic energy into kinetic energy of the fast particles. The mechanism may be the cause of generation of fast particles in cosmic conditions and in laboratory plasma.

THE mechanism of generation of cosmic rays and of fast particles in general under cosmic conditions[1,2], and the causes of appearance of fast particles in some laboratory experiments[3], are still unclear. This problem is especially acute in the case of solar flares, a prolonged study of which has made it possible to accumulate abundant observational material still awaiting a sufficiently exhaustive interpretation[4]. A characteristic feature of these phenomena is the rapid realignment and dissipation of magnetic fields, accompanied by the appearance of particles which are accelerated to high (sometimes ultrarelativistic) energies. One might think that solar flares are only examples of a certain universal mechanism of the transition of the magnetic energy into kinetic energy of accelerated particles.

We discuss below the possibility of relating such a mechanism with certain characteristic features of the dynamics of a conducting medium in the vicinity of a zero field line[1)].

1. MAGNETIC FIELD IN THE VICINITY OF THE ZERO LINE

Let k currents of equal strength I be located on the lines

$$r = R = 1, \quad \varphi_i = 2\pi i / k \quad (i = 0, 1, \ldots, k-1;\ k \geqslant 2)$$

of a cylindrical coordinate system. The magnetic field of such currents can be expressed with the aid of a vector potential $A = \{0, 0, A_0\}$ having only a z component. Namely,

$$\mathbf{H} = [\nabla A_0 \times \mathbf{e}_z] = \left\{\frac{1}{r}\frac{\partial A_0}{\partial \varphi},\ -\frac{\partial A_0}{\partial r}\right\}. \tag{1}$$

Here

$$A_0 = \sum_{i=0}^{k-1} \frac{I}{c}[\ln\{1 - 2r\cos(\varphi - \varphi_i) + r^2\} - C_i]$$
$$= \frac{I}{c}\left[\ln\left\{\prod_{i=0}^{k-1}[1 - 2r\cos(\varphi - \varphi_i) + r^2\right\} - C\right]$$
$$= \frac{I}{c}[\ln\{1 - 2r^k\cos k\varphi + r^{2k}\} - C]. \tag{2}$$

The constants C_i and $C = \Sigma C_i$ (summation from 0 to k − 1) are determined here by the total geometry of the currents, taking into account the closing of their circuits at large distances far from the region of space under consideration.

We are using a Gaussian system of units, with c the velocity of light in vacuum. The line r = 0 is the zero field line (H = 0 when r = 0). In its vicinity, $r \ll 1$, the potential is of the form, accurate to terms of higher order in r^k,

$$A_0(r, \varphi) = -h_0 r^k \cos k\varphi - h_0 C/2, \quad h_0 = 2I/c. \tag{3}$$

Expression (3) is the general equation for the potential of the magnetic field in the vicinity of a zero line of arbitrary order[6]. We note that, in accordance with the definition (1) of the line,

$$A_0(r, \varphi) = \text{const} \tag{4}$$

1)See also[5], in which the two-current case is considered and possible applications are discussed.

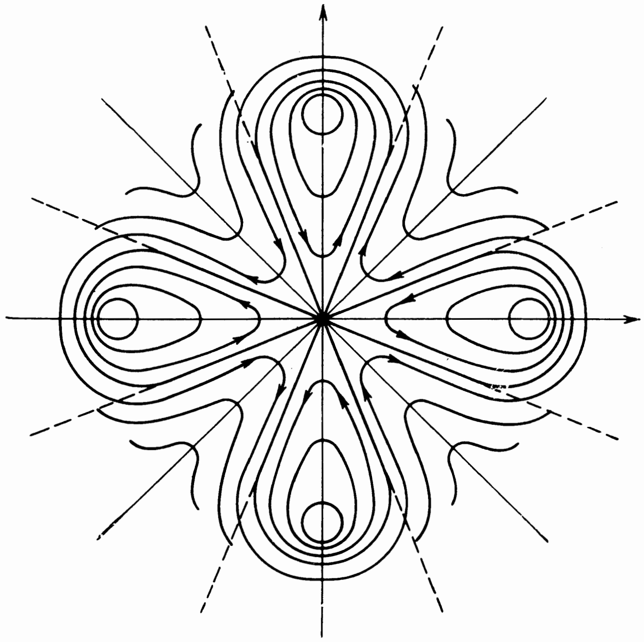

FIG. 1

are the force lines of the magnetic field. The pattern of the force lines in the plane $z = 0$ is indicated in Fig. 1, where we put $k = 4$.

Assume now that the currents in question have changed by an amount ΔI (that is, $I' = (1 + \Delta)I$) and have been displaced radially by an amount δ (that is, $R' = R(1 + \delta)$; the initial distance from the currents to the zero line is assumed to be unity, $R = 1$). As in the derivation of (2) and (3), we can easily verify that the potential $A(R, \varphi)$ of the displacement currents in the vicinity of the zero line is

$$A(r, \varphi) = -\alpha h_0 r^k \cos k\varphi + h_0\beta - h_0 C / 2, \quad (5)$$

and differs from (3) by a constant factor

$$\alpha = (1 + \Delta) / (1 + \delta)^k \approx (1 + \Delta)(1 - k\delta) \quad (6)$$

and by an additive constant $h_0\beta$, where

$$\beta = k(1 + \Delta) \ln (1 + \delta) - {}^1/_2 C\Delta$$
$$\approx k\delta(1 + \Delta)(1 - {}^1/_2\delta) - {}^1/_2 C\Delta. \quad (7)$$

Here the increments Δ and δ are assumed small.

Under conditions when the field is frozen in (see below), the conservation at the additive constant in the expression for the potential has a definite meaning and is equivalent to specifying the boundary conditions in the problem of the displacement of a conducting medium. In fact, during the displacement of the currents the additive constant h_0 in expression (5) is a function of the time and defines in the considered vicinity of the neutral line a homogeneous electric field

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = -\frac{h_0\beta}{c}\mathbf{e}_z.$$

In the presence of a conducting medium, under conditions when the motion of the medium is completely determined by Maxwell's equations and inertia can be neglected, the medium will move under the influence of this field with the "electric-drift" velocity*

$$\mathbf{v} = c\frac{[\mathbf{EH}]}{H^2} = \frac{h_0\beta}{H^2}[\mathbf{He}_z].$$

Thus, the additive constant in (5) plays an essential role in the determination of the character of motion of the medium.

2. TWO-DIMENSIONAL (PLANE) MAGNETOHYDRODYNAMIC FLOW

Assuming that all quantities are independent of the coordinate z, let us consider the flow of a medium with frozen-in magnetic field in the (x, y) plane. The magnetohydrodynamic equations have in this case the form

$$\frac{d\rho}{dt} = -\rho \operatorname{div} \mathbf{v}, \quad \frac{dA}{dt} = 0,$$
$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p - \frac{1}{4\pi\rho}\Delta A\, \nabla A. \quad (8)$$

Here $d/dt = \partial/\partial t + (\mathbf{v}\cdot\nabla)$, and $\mathbf{v}$, ρ, and p are the velocity, density, and gas pressure of the medium; the dissipative terms are assumed to be negligibly small.

We shall henceforth take as the initial equilibrium state, subject to perturbations with small or finite amplitude, a state with constant density and pressure

$$\mathbf{v}_0 = 0, \ p_0 = \text{const}, \ \rho_0 = \text{const}, \ \Delta A_0 = 0. \quad (9)$$

The initial magnetic field $H = |\nabla A_0|$ will be assumed to be sufficiently strong to make the Alfven velocity $V_A = H/\sqrt{4\pi\rho}$ much larger than the adiabatic speed of sound $s = [(\partial p/\partial\rho)_S]^{1/2}$ everywhere except in small vicinity of the zero line. Near the zero line, in accord with (1) and (3), the field is $H = kh_0 r^{k-1}$, and therefore the condition $V_A \gg s$ is equivalent to the condition $r > r_S$, where

$$r_s = [s\sqrt{4\pi\rho} / kh_0]^{1/(k-1)} = [4\pi\gamma n_0 \varkappa T / k^2 h_0^2]^{1/2(k-1)}. \quad (10)$$

On going over to the second expression in (10), the gas is assumed ideal and the adiabatic exponent is assumed equal to γ; n_0 is the concentration of the atoms, T the gas temperature in the initial state, and κ Boltzman's constant. We assume

*$[\mathbf{E}\,\mathbf{H}] \equiv \mathbf{E} \times \mathbf{H}$.

also that the gas is sufficiently ionized so that the electron concentration is $n_e \approx n$.

The assumed condition $V_A \gg s$, together with the condition for the applicability of the approximate expression (3) for the potential A, denotes that we are considering the region of values

$$r_s \ll r \ll 1. \qquad (11)$$

For sufficiently strong currents and a sufficiently low gas concentration, such a region always exists, but with increasing number of currents k, other conditions being equal, the region (11) narrows down and vanishes in final analysis, since it follows from (10) that $r_s \to 1$ as $k \to \infty$. Most convenient in this respect, is the minimum number of currents $k = 2$. Since, however, many-current systems of the type under consideration can arise under certain conditions [7], we shall investigate the general case with $k \geq 2$.

3. SMALL PERTURBATIONS IN THE VICINITY OF THE ZERO LINE

For a small perturbation the values of **v**, p, ρ, and A of the initial equilibrium state (9) of Eq. (8) reduce to the following:

$$\frac{\partial \rho}{\partial t} = -\rho_0 \operatorname{div} \mathbf{v}, \quad \frac{\partial A}{\partial t} = -\mathbf{v}\nabla A_0,$$
$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho_0}\nabla p - \frac{1}{4\pi\rho_0}\nabla A_0 \Delta A. \qquad (12)$$

Hence,

$$\frac{\partial^2 A}{\partial t^2} = \frac{(\nabla A_0)^2}{4\pi\rho_0}\Delta A + \frac{s^2}{\rho_0}\nabla A_0 \nabla \rho,$$
$$\frac{\partial^2 \mathbf{v}}{\partial^2 t} = s^2 \nabla \operatorname{div} \mathbf{v} + \frac{1}{4\pi\rho_0}\nabla A_0 \Delta(\mathbf{v}\nabla A_0). \qquad (13)$$

The condition (11) assumed above, that is, the condition $V_A(r) \gg s$, allows us to discard in (12) and (13) terms that depend on the pressure gradient. If we denote here, the perturbation-induced displacement of the medium by $\xi(r, t)$ and use expression (3) for $A_0(r)$, then (12) and (13) can be reduced to the following system:

$$\frac{\partial^2 A}{\partial t^2} = V_a{}^2 r^{2(k-2)}\left\{r\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) + \frac{\partial^2 A}{\partial \varphi^2}\right\},$$
$$\frac{\partial^2 \xi}{\partial t^2} = \frac{1}{4\pi\rho_0}\nabla A_0 \Delta(\xi \nabla A_0),$$
$$\mathbf{v} = \partial \xi / \partial t, \quad \rho = -\rho_0 \operatorname{div} \xi, \quad A = -(\xi\nabla)A_0. \qquad (14)$$

Here $V_a^2 = (h_0 k)^2/4\pi\rho_0$, and we use the expression for the Laplace operator in cylindrical coordinates. As follows from (14), the perturbation propagates with local Alfven velocity, and the displacement ξ is directed along ∇A_0, that is, transverse to the magnetic force lines. This means that in the region (11) the initial perturbation propagates primarily in the form of fast magnetohydrodynamic waves, and pure hydrodynamic spreading of the gas along the magnetic force lines is much slower. This can be readily verified also if account is taken of the fact that the perturbations under consideration correspond to fast magnetic-sound waves (see [8]), whose phase velocity for $V_A \gg s$ is

$$\omega / k \approx V_A(1 + (s^2 / V_A{}^2)\sin^2\theta) \approx V_A,$$

where θ is the angle between the direction of propagation of the wave and the direction of the magnetic field. In fast magnetic-sound waves, under the condition $V_A \gg s$, the motion of the medium is practically transverse to the magnetic force lines (the ratio of the longitudinal and the transverse components of the velocity of the medium is $v_{\|}/v_{\perp} \approx (s^2/2V_A^2)\sin 2\theta$). Under the same conditions the slow magnetic-sound wave propagates with velocity $s \cos\theta$, and the motion of the medium in such a wave is along the field ($v_{\perp}/v_{\|} \approx (s^2/2V_A^2)\sin 2\theta$).

We now examine the first equation of (14) for the potential A. In the general case of arbitrary $k > 2$ its solution can be obtained in the form of a Fourier-Bessel integral. The solution is particularly easy to obtain in the case $k = 2$, when the equation for A, in variables $x = \ln r$ and φ, reduces to the usual wave equation

$$\frac{\partial^2 A}{\partial t^2} = V_a{}^2\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial \varphi^2}\right). \qquad (15)$$

Let us consider an axially symmetrical (independent of φ) initial perturbation of the potential $A(r, \varphi, 0) = \Phi(r)$, where $\Phi(r)$ is an arbitrary function of the radial variable r. As follows from the expression for the potential of the displaced currents in (5), an axially-symmetrical perturbation of the potential corresponds to a simultaneous displacement and change in the magnitude of the currents such that $(1 + \Delta)/(1 + \delta)^k = 1$. In this case the general solution of (15) will be

$$A(r, t) = \Phi(\exp\{\ln r + V_a t\}). \qquad (16)$$

The plus sign is chosen because we are interested in a wave that converges to the zero line. As seen from (16), this is a converging cylindrical wave whose velocity decreases on approaching the zero line:

$$V(r) \equiv dr / dt = -V_a r = -V_A(r), \qquad (17)$$

Thus, the wave propagates with local Alfven velocity. If the change from $A = 0$ in front of the wave to $A = h_0\beta$ behind the wave (see expressions (5) and (7)) occurs over a distance $\Delta r = l$, then the width of the front of the wave decreases on approaching the zero line like

$$l = l_0 r / R, \tag{18}$$

where l_0 is the width of the front at a distance R from the neutral line. Simultaneously, the field intensity and its gradient increase:

$$H(r) = H(R)\frac{R}{r}, \quad \frac{\partial H}{\partial r}(r) = \frac{\partial H}{\partial r}(R)\left(\frac{R}{r}\right)^2. \tag{19}$$

Thus, a cylindrical wave converging to the zero line leads to a unique cumulative effect with respect to the field and its gradient.

From (14) we can easily determine also the displacement ξ and the change in density ρ. Namely, taking into account that by virtue of the second equation of (14), ξ is directed along ∇A_0, we obtain

$$\xi = -\frac{A}{(\nabla A_0)^2}\nabla A_0. \tag{20}$$

Assuming that A takes on the value $A = 2h_0\delta$ after the passage of the wave (see (7) with $\Delta = 0$ and $\delta \ll 1$), we obtain from this the resultant displacement

$$\xi = \{\xi_r, \xi_\varphi\} = r^{-1}\delta\{\cos 2\varphi, -\sin 2\varphi\}. \tag{21}$$

Thus, the magnitude of the displacement also increases as the wave approaches the zero line of the field. The change in density of the medium is then

$$\rho = -\rho_0 \operatorname{div} \xi = 2r^{-2}\delta\rho_0\cos 2\varphi. \tag{22}$$

It follows from (22) that the density of the medium increases in one pair of opposite quadrants, and decreases in the other.

Thus, even the linear approximation shows that regions in which the field and its gradients increase and the density of the medium decreases are produced near the neutral line when the currents are displaced. Unfortunately, as follows for example from (22), the linear approximation is valid only for $r \gg \sqrt{2\delta}$, and this does not explain the situation arising in the region $r \sim r_S$ if $r_S \ll \sqrt{2\delta}$.

4. DEFORMATION OF THE MAGNETIC FIELD BY A QUASI-ADIABATIC DISPLACEMENT OF THE CURRENTS

We now turn to the exact system of equations (8). For the first two equations we readily obtain the integrals if we go over to Lagrange coordinates

$$\mathbf{r}_0 = \mathbf{r} - \xi(\mathbf{r}, t), \tag{23}$$

where $\mathbf{r}_0$ is the coordinate of the material point prior to displacement, $\mathbf{r}$ its coordinate at the instant of time t, and $\xi(\mathbf{r}, t)$ the displacement vector. Then

$$\rho(\mathbf{r}, t) = \rho_0(\mathbf{r} - \xi(\mathbf{r}, t))\frac{D(\mathbf{r} - \xi(\mathbf{r}, t))}{D(\mathbf{r})}, \tag{24}$$

where $D(\mathbf{r}_0)/D(\mathbf{r})$ is the Jacobian of the transformation from the coordinates $\mathbf{r}_0$ to the coordinates $\mathbf{r}$. The initial density will henceforth be assumed constant: $\rho_0(\mathbf{r}_0) \equiv \rho_0 = \text{const}$.

The integral of the second equation of (8), namely

$$A(\mathbf{r}, t) = A_0(\mathbf{r} - \xi(\mathbf{r}, t)), \tag{25}$$

where $A_0(\mathbf{r}_0)$ is the initial value of the potential, expresses the usual magnetohydrodynamic freezing-in condition: when the medium is displaced the force lines remain bound to the same particles of the medium.

As is clear from (24) and (25), for a known displacement $\xi(\mathbf{r}, t)$, we can uniquely determine the deformation of the magnetic field and the change of the density in each part of space. However, to find $\xi(\mathbf{r}, t)$ in the general case it is necessary to solve the complete system of equations (8) with corresponding initial and boundary conditions. A solution of this nonlinear system is a very complicated problem. We note here that for an incompressible liquid there exists a certain exact particular solution of the system (8) in the vicinity of the second-order zero line ($k = 2$) (see [9]). Since, however, the compressibility plays the fundamental role in what follows, we shall have to use certain simplifications. Namely, we assume that the displacement of the currents is sufficiently rapid compared with the speed of sound, but sufficiently slow compared with the Alfven velocity.

Under these assumptions the boundary conditions of the problem change slowly compared with the velocity of the fast magnetic-sound waves. Therefore, at least during the first stage of the process when the perturbations are carried by fast magnetic-sound waves, there are no grounds for expecting the occurrence of shock waves. Shock waves can be produced in the region $r \sim r_S$, where the Alfven velocity is comparable with the speed of sound, and also during the succeeding stage of the process, when slow magnetic-sound waves which carry matter along the force lines come into play.

If conditions (11) are satisfied, then a region exists in which we can neglect, as before, the pressure and the gasdynamic flow of the medium along the force lines, that is, we can omit from the last equation of (8) the term with the pressure gradient. On the other hand, slow displacement of the currents (compared with the Alfven velocity) allows us to neglect in (8) the acceleration of the medium, that is, to assume that an equilibrium field

$$\Delta A(\mathbf{r}, t) = 0 \tag{26}$$

has time to become established during each stage of the displacement.

The last assumption denotes in fact that the final displacement ξ of interest to us is regarded as an aggregate of successive small perturbations $\delta\xi$ of the type considered in Sec. 3, each of which transforms the system to a neighboring equilibrium state.

Under the assumptions made, the displacement at each instant of time is transverse to the instantaneous force-line pattern, that is, the following conditions should be satisfied (see also the Appendix):

$$\left[\frac{d\xi}{dt} \nabla A(\mathbf{r}, t)\right] = 0. \tag{27}$$

Equations (25)—(27) allow us to determine the displacement $\xi(\mathbf{r}, t)$ for any specified displacement of the field sources.

5. DISPLACEMENT OF THE MEDIUM IN THE VICINITY OF THE NEUTRAL LINE

We shall use (25)—(27) to determine the deformation of the field and the displacement of the medium in the vicinity of a zero line or order k. As follows from the comparison of the expressions for the potentials (3) and (5), the general pattern of the force lines near the neutral line remains the same when the magnitudes and positions of the currents change. Under any such change, the equation for the force lines remains

$$r^k \cos k\varphi = \text{const.}$$

Therefore, by virtue of the condition (27), the displacement of the medium should occur along trajectories orthogonal to these lines. The family of such trajectories is made up of the lines

$$r^k \sin k\varphi = r_0^k \sin k\varphi_0, \tag{28}$$

where r_0 and φ_0 are the initial coordinates of the material plane.

Further, by way of a solution of (26) satisfying the boundary conditions of interest to us (specified change in magnitude and position of the currents), we must choose the potential (5). Together with (25), this yields

$$\alpha r^k \cos k\varphi - \beta = r_0^k \cos k\varphi_0. \tag{29}$$

Equations (28) and (29) express the initial coordinates of the material point in terms of its coordinates after a displacement specified by the parameters (6) and (7). Namely,

$$r_0^{2k} = r^{2k} - (1-\alpha^2) r^{2k} \cos^2 k\varphi - 2\alpha\beta r^k \cos k\varphi + \beta^2,$$

$$\operatorname{ctg} k\varphi_0 = \alpha \operatorname{ctg} k\varphi - \beta / r^k \sin k\varphi. \tag{30}^*$$

To clarify the picture of the displacements of the medium and the deformation of the force lines, we assume for concreteness $\beta > 0$ and denote the force lines by indicating the points where they intersect the rays $\varphi = (2i-1)\pi/k$, $\varphi = 2\pi i/k$, and $\varphi = (2i+1)\pi/k$ (see Fig. 2). Thus, for example, the line $r_{2i+1} = (\beta/\alpha)^{1/k}$ denotes the force line crossing the ray $\varphi = (2\pi+1)\pi/k$ at the point $r = (\beta/\alpha)^{1/k}$.

As can be verified with the aid of (28)—(30), when $\beta > 0$ the picture of the field deformation has the following properties: the neighboring branches of the force lines $r_{2i+-1} = \beta^{1/k}$ go over into the neighboring branches of the force lines $r_{2i} = 0$, that is, into the rays $\varphi = (2i \pm 1/2)\pi/k$; the neighboring branches of the force lines $r_{2i\pm1} = (\beta/(1+\alpha))^{1/k}$ go over into the force line $r_{2i} = (\beta/(1+\alpha))^{1/k}$; the neighboring branches of the force lines $r_{2i} = 0$, that is, the rays $\varphi = (2i \pm 1/2)\pi/k$, go over into the force $r_{2i} = (\beta/\alpha)^{1/k}$. The dashed lines with the arrows in Fig. 2 indicate the character of the displacement.

We note immediately that this picture of the deformation cannot be extended to include the entire range of values $r > 0$. First, by virtue of condition (11), we exclude from consideration the region $r \lesssim r_s$, in which the decisive role is already played by purely gasdynamic flows. We shall assume that

$$r_s \ll r_1 = |\beta|^{1/k}. \tag{31}$$

Under this condition the displacement in the region $r > r_1$ will differ greatly from that described above.

Further, currents not accounted for in the expression (5) for the potential may be produced during the course of the displacement. These currents will distort the picture of the force lines near the neutral line. For a rigorous account of the influence of the resultant currents it is necessary to solve completely the time-dependent problem defined by (8). At the same time, an approximate picture of the deformation of the field in the region $r \gtrsim r_s$ can be established on the basis of

*ctg ≡ cot.

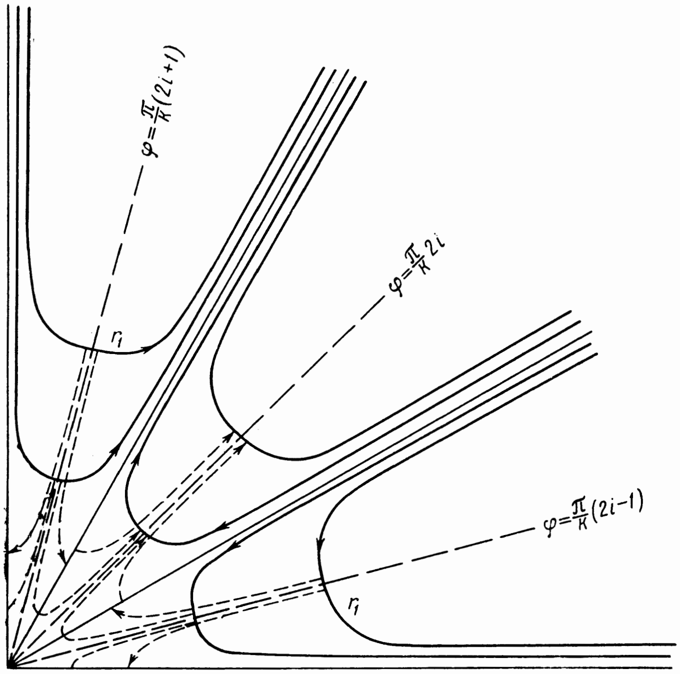

FIG. 2

the frozen-in property and continuity of the force lines and by recognizing that when $r \gtrsim r_1$ the picture of the displacements of the force lines is known.

Finally, it is necessary to recognize that compression regions with high density gradients, where the effect of gas pressure can no longer be neglected, can be produced during the displacement. The existence of such regions can be readily established with the aid of (24). Namely, calculating the Jacobian of the transformation (30) in the cylindrical coordinates, we obtain

$$\rho(r, t) = \rho_0 \frac{r_0}{r} \frac{D(r_0, \varphi_0)}{D(r, \varphi)} = \alpha\rho_0 \left(\frac{r}{r_0}\right)^{2(k-1)}. \quad (32)$$

It follows from this that at the points whose initial coordinates are $r_0 = 0$, the density of the medium becomes infinite. With the aid of (30) we can easily verify that in the deformed field pattern these points are $(i = 0, 1, \ldots, k-1)$

$$r = (\beta/\alpha)^{1/k}, \quad \varphi = 2\pi i/k, \qquad \text{if } \beta > 0,$$

$$r = (|\beta|/\alpha)^{1/k}, \quad \varphi = (2i+1)\pi/k, \quad \text{if } \beta < 0. \quad (33)$$

In the vicinity of such points, the field deformation and the displacement of the medium will be different from those obtained above.

We note that in the regions

$$r > \left[\frac{\beta}{(1+\alpha)\cos k\varphi}\right]^{1/k}, \quad \frac{\pi}{k}\left(2i-\frac{1}{2}\right) < \varphi < \frac{\pi}{k}\left(2i+\frac{1}{2}\right)$$

for $\beta > 0$,

$$r > \left[\frac{|\beta|}{(1+\alpha)\cos k\varphi}\right]^{1/k}, \quad \frac{\pi}{k}\left(2i+\frac{1}{2}\right) < \varphi < \frac{\pi}{k}\left(2i+\frac{3}{2}\right)$$

for $\beta < 0$, (34)

the density is $\rho > \rho_0$, that is, compression of the medium takes place, whereas in the remaining part of space the displacement leads to a decrease in the density. In the region $r \ll |\beta|^{1/k}$ the density of the medium is, in accord with (32),

$$\rho = \rho_0\alpha(r|\beta|^{-1/k})^{2(k-1)}. \quad (35)$$

It follows therefore that in the region $r \ll |\beta|^{1/k}$ a maximum rarefaction of the conducting gas may take place. We note, however, that, in view of the foregoing distortions of the character of the deformation in the region $r < r_1$, formula (35) must be regarded only as a rough estimate of the real rarefaction.

The field-deformation pattern described above in the vicinity of the zero line is shown in Fig. 3. For the region $r \gtrsim r_1$ and far from the point (33), the deformation is described by Eqs. (28)–(33). In the region $r \lesssim r_1$ and in the vicinities of the points (33), only a qualitative picture of the force lines is given, based on their continuity and the condition for the conservation of the frozen-in property during the deformation process.

Figure 3 corresponds to the case $\beta > 0$, that is, as follows from (5), to an increasing distance $(\delta > 0)$ between the currents and the neutral line, or to a decrease in the magnitude of the currents $(\Delta < 0)$. In this case the regions of plasma compression (such a region is shown doubly cross hatched in Fig. 3) and the regions of condensation

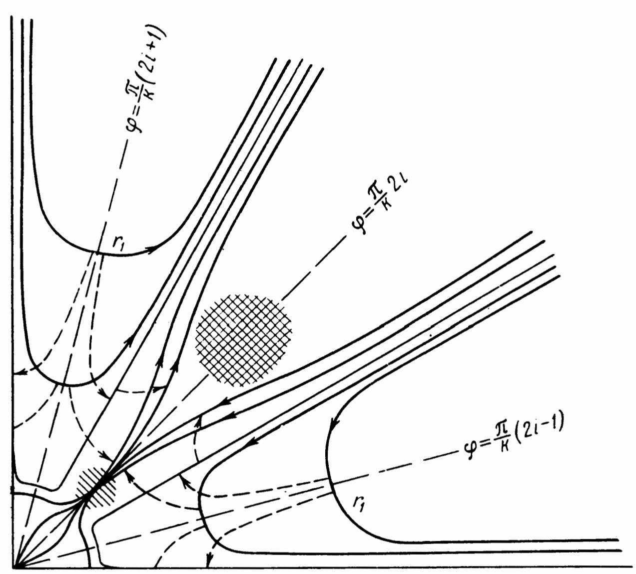

FIG. 3

of the magnetic force lines (single hatching) lie on the rays $\varphi = 2\pi i/k$ joining the neutral point with the points corresponding to the positions of currents.

For the case when the currents come closer or become stronger ($\beta < 0$), as can be seen directly from (28)—(30), all that takes place is rotation of the entire picture through an angle π/k relative to the specified currents. In this case the regions of high compression of the medium $r \sim (|\beta|/\alpha)^{1/k}$ and regions of condensation of the force lines lie on the rays $\varphi = (2i + 1)\pi/k$, which separate the currents.

6. VIOLATION OF THE FROZEN-IN CONDITION AND DYNAMIC DISSIPATION OF THE MAGNETIC FIELD

The foregoing picture of field deformation in the vicinity of the neutral line leads to a very unique situation, consisting in the fact that the concentration of the particles in the medium decreases simultaneously with an increase of the magnetic field intensity and of its gradients. Let us examine this process in greater detail.

The magnetic flux

$$\Phi = \int_0^{r_1} H_\varphi \, dr = A\left(r_1, \frac{\pi}{k}(2i+1)\right) - A(0) = h_0\beta, \quad (36)$$

$$r_1 = |\beta|^{1/k} \approx |k\delta|^{1/k}, \quad (37)$$

which prior to the displacement crossed the ray $\varphi = (2i + 1)\pi/k$ on the segment $0 < r_1$, is concentrated as a result of the displacement within the range of angles $2i\pi/k < \varphi < (2i + \frac{1}{2})\pi/k$, and in region $r \ll |\beta|^{1/k}$ the field direction will be practically radial. This makes it possible to estimate both the magnetic field intensity and its gradient in this region. Mainly, inasmuch as the flux (36) crosses after the displacement an arc of length $r\pi/2k$, the average field intensity in this region is

$$H = 2k\Phi / \pi r = 2kh_0\beta / \pi r. \quad (38)$$

Recognizing that the field reverses sign on going through the ray $\varphi = 2i\pi/k$, the field gradient has an order of magnitude

$$h \approx \frac{2k}{\pi r} H = \left(\frac{2k}{\pi}\right)^2 \frac{h_0\beta}{r^2}. \quad (39)$$

On the other hand, in the region $r \ll |\beta|^{1/k}$, the concentration of matter, in accordance with (35) amounts to

$$n = n_0\alpha r^{2(k-1)}\beta^{-2(k-1)/k}. \quad (40)$$

It follows therefore that the region h/n is

$$\frac{h}{n} = \frac{h_0}{\alpha n_0}\left(\frac{2k}{\pi}\right)^2 \beta^{(3k-2)/k} r^{-2k} \quad (41)$$

and increases rapidly with decreasing r. The maximum value of this ratio is attained for the smallest values of r for which the foregoing picture is still valid ($r \gtrsim r_s$), and its order of magnitude is

$$\frac{h}{n} = \frac{h_0}{\alpha n_0}\left(\frac{2k}{\pi}\right)^2 \beta^{(3k-2)/k} r_s^{-2k}. \quad (42)$$

This ratio plays a fundamental role in the problem under consideration. In fact, the magnetohydrodynamic freezing-in condition implies the possibility of neglecting the displacement currents and assuming that the "truncated" Maxwell equation is valid at each instant of time:

$$\operatorname{curl} \mathbf{H} = 4\pi c^{-1}\mathbf{j}. \quad (43)$$

However, the current $j = neu$, where e is the charge and u the velocity of the charged particles that carry the current, cannot exceed for a given concentration n the value $j = nec$. Therefore, putting $|\operatorname{curl} H| = h$, we arrive at the conclusion that when

$$h / n \gtrsim 4\pi e \quad (44)$$

the freezing-in concept is entirely unsuitable. Under these conditions we must take into account the displacement current in (43), that is, we must take into account the strong electric fields that appear when condition (44) is satisfied. A quantitative analysis of the situation arising when freezing-in is violated will be given in a separate article. Here we shall only dwell briefly on the qualitative aspect of the problem.

Under condition (44), the medium no longer exerts an appreciable influence on the propagation of magnetic perturbations, which assume the character of ordinary electromagnetic waves. In other words, on going into the region of high rarefaction near the neutral line, the magnetohydrodynamic waves are transformed into low-frequency electromagnetic waves. Under the situation considered, in each of the regions of condensation ('cumulation') of the force lines shown in Fig. 3 is singly hatched, two converging magnetohydrodynamic waves form a quasistatic electric-field wave, the intensity of which is equal in order of magnitude to the intensity of the magnetic field on the boundary of the region when the freezing-in is violated. It follows from (38) that the magnetic field in the region of accumulation can exceed by $2\beta/\pi r_s^k$ times the initial field in this region. Therefore the produced electric field may reach quite large values.

We note here that the electric field in the region of cumulation is directed along the z axis in the same direction as the initial currents when $\beta > 0$, and in the opposite direction if $\beta < 0$. The energy of this electric field and of the magnetic field that excites it is consumed in the acceleration of the particles contained in the cumulation region.

Thus, under the conditions considered, the magnetic energy stored in the medium, which can be regarded as continuous, is transported by magnetohydrodynamic waves into the region of higher rarefaction, where it goes over into energy of individual accelerated particles. Here the magnetohydrodynamics of the plasma as a whole merges continuously with the dynamics of the individual particles in the electromagnetic field. It is precisely in this sense that the process of realignment of the magnetic field, accompanied by the appearance of accelerated particles, has been called above dynamic dissipation for short.

Estimates of the number and energy of accelerated particles are contained in [5]. We shall dwell here only on the conditions under which dynamic dissipation is realized.

7. CONDITIONS OF DYNAMIC DISSIPATION

Using (42), we can rewrite the condition (44) in the form

$$(h_0 / n_0)\, k^2 \beta^{(3k-2)/k} r_s^{-2k} \gtrsim \pi^3 e\alpha. \qquad (45)$$

Together with (7), this condition denotes that dynamic dissipation can be connected either with the displacement of the currents or with the change in the magnitude. If the currents change in magnitude, then the resultant induced electric field and consequently also the quantity β (see (5) and (7) depend via the constant C on the total geometry of the currents, that is, on the manner in which the circuits are closed at large distances.

For currents that remain unchanged in magnitude ($\Delta = 0$), the induced electric field $\mathbf{E} = -c^{-1}\mathbf{V} \times \mathbf{H}$ is determined by the displacement velocity $\mathbf{V}$ of the nearest current segments that make the principal contribution to the magnetic field intensity. In this case β depends only on the resultant displacement of the considered parallel sections of the current.

For currents which do not change in magnitude, retaining the principal terms in (6) and (7), we rewrite the condition for dynamic dissipation in the form

$$\frac{h_0}{n_0}(k\delta)^{(3k-2)/k} r_s^{-2k} k^2 \gtrsim \pi^3 e. \qquad (46)$$

It follows therefore that with increasing k the condition (46) becomes less stringent, since it is assumed that $r_s < 1$. We must, however, bear in mind the condition (see (31))

$$r_s = [4\pi\gamma n_0 \varkappa T / h_0^2 k^2]^{1/2(k-1)} \ll \beta^{1/k} \ll 1. \qquad (47)$$

As already noted above, with increasing k the quantity r_s tends to unity and the approximations used in the derivation of the criterion (45) are no longer justified. The limiting case $k \to \infty$ corresponds to a cylindrical current which is homogeneous in φ, and for which there is no magnetic field at all in the region $r < 1$.

We now stop to discuss the temporal characteristics of the process. To be able to neglect the spreading of the medium along the force lines and, in particular, to prevent the rarefaction region with dimensions of the order r_s from becoming filled with the medium from the neighboring regions, we must stipulate that the time τ of the process satisfy the condition

$$\tau \lesssim r_s / s. \qquad (48)$$

This means that the current displacement velocity $V = \delta/\tau$ should be

$$V \gtrsim s\delta / r_s. \qquad (49)$$

The use of an equal sign is still valid here in the sense of order of magnitude, since the amount of matter in the neighboring sections bordering on the maximum-rarefaction region is small.

The dynamic dissipation condition (45) implies acceleration of the particles to relativistic velocities. Actually, dynamic dissipation can be realized also under a less stringent condition, when the dissipation of the magnetic field is connected with the known phenomenon of particle runaway. It is necessary for this purpose that the resultant electric field exceed the critical field E_{cr} corresponding to treble particle runaway [10]. In this case use in the derivation of the condition (45) not the maximum current $j = nec$ but the critical current $j = \sigma E_{cr}$, where σ is the longitudinal conductivity of the plasma.

As a result, the condition for dynamic dissipation accompanied by particle runaway becomes (see also [5])

$$\frac{h_0}{n_0} k^2 \beta^{(3k-2)/k} r_s^{-2k} \gtrsim \pi^3 \alpha \left(\frac{\sigma}{n_0 c} E_{cr}\right). \qquad (50)$$

Under even weaker conditions, when the criterion (50) is not satisfied, the number of accelerating (runaway) particles is small and ordinary Joule (ohmic) dissipation, which leads simply to heating of the medium in the cumulation regions, begins to play the decisive role. In this case the situation apparently corresponds to that considered in [11], the only difference being that two regions of field cumulation and plasma heating are produced.

8. CONCLUSION

Possible applications of the mechanism of dynamic dissipation and certain numerical estimates are considered by the author in[5]. A quantitative analysis of the process of particle acceleration will be presented in a separate article. We point out here only the possible ways of further investigating the magnetohydrodynamic stages of the process. We are referring here to investigations of the time-dependent problem defined by the system of equations (8) under more accurate approximations. Ways of carrying out such investigations may be as follows: 1) rigorous solution of the time-dependent problem, neglecting pressure effects but allowing for the finite velocity of the displaced currents; 2) rigorous analysis of the motions in the region $r \lesssim r_1$, for which only a qualitative treatment was presented in this paper; 3) analysis of the possibility of formation of magnetohydrodynamic shock waves during certain stages of the processes; 4) development of computer programs and analog techniques for the analysis of the exact problem.

The solution of these problems calls for serious efforts. These efforts, however, will be fully justified by the role that may be played by the dynamic dissipation in outer-space physics and in plasma physics in general.

APPENDIX

We introduce the dimensionless variables $\mathbf{v}' = \mathbf{v}/V$, $\rho' = \rho/\rho_0$, $A' = A/a_0$, and $t' = tV_a$, where $V_a^2 = h_0^2/4\pi\rho_0$, V is the characteristic limiting velocity (rate of displacement of the currents), and the unit of distance is taken to be the distance R from the currents to the zero line. In this notation, the last equation of (8) takes the form (we leave out the primes)

$$\varepsilon\frac{\partial \mathbf{v}}{\partial t} + \varepsilon^2(\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\Delta A\,\nabla A. \tag{A.1}$$

Here $\epsilon = V/V_a$; by assumption $\epsilon \ll 1$. We seek the solution of (A.1) in the form

$$A(\mathbf{r}, t) = A^{(0)}(\mathbf{r}, t) + \varepsilon A^{(1)}(\mathbf{r}, t) + \ldots,$$
$$\mathbf{v}(\mathbf{r}, t) = \mathbf{v}^{(0)}(\mathbf{r}, t) + \varepsilon \mathbf{v}^{(1)}(\mathbf{r}, t) + \ldots \tag{A.2}$$

then in the zeroth (adiabatic) approximation we have (see (26))

$$\Delta A^{(0)}(\mathbf{r}, t) = 0. \tag{A.3}$$

A solution of (A.3) satisfying the necessary boundary conditions, is (see (5))

$$A^{(0)}(\mathbf{r}, t) = -\alpha r^k \cos k\varphi + \beta - C/2. \tag{A.4}$$

The solution (A.4) depends on the time only as a parameter, via the boundary values of α and β.

From (A.4) we obtain the next higher order in ϵ:

$$\frac{\partial \mathbf{v}^{(0)}}{\partial t} = -\frac{1}{\rho}\Delta A^{(1)}\,\nabla A^{(0)}. \tag{A.5}$$

Integrating both sides of (A.5) with respect to time, we get (for $\mathbf{v}(\mathbf{r}, 0) = 0$)

$$\mathbf{v}^{(0)} = -\left(\frac{1}{\alpha}\int_0^t \frac{\alpha\Delta A^{(1)}}{\rho}\,dt\right)\nabla A^{(0)}. \tag{A.6}$$

From this, in particular, follows the expression (27).

Further, going over to Lagrangian coordinates and recognizing that $\mathbf{v} = \{dr/dt,\ r\,d\varphi/dt\}$, we obtain the differential equation for the trajectories of the medium:

$$dr/r\,d\varphi = -\cos k\varphi/\sin k\varphi. \tag{A.7}$$

Consequently, the trajectories of motion of the particles of the medium constitute a family of hyperbolas (see (28))

$$r^k \sin k\varphi = \text{const}. \tag{A.8}$$

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