QUASILINEAR ACCELERATION OF PARTICLES BY A TRANSVERSE ELECTROMAGNETIC WAVE

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Submitted to JETP editor November 13, 1965

J. Theoret. Phys. (U.S.S.R.) **50**, 1101–1106 (April, 1966)

A new mechanism of acceleration of charged particles by a plane electromagnetic wave, which does not require the decelerating systems characteristic of linear accelerators, is proposed. Owing to the action of a so-called synchronizing magnetic field with a special spatial distribution, the particle moves along such a trajectory that on the average the electric field does positive work on it. The relations between the particle energy and longitudinal coordinate and the conditions which the synchronizing field must satisfy are determined. The time invariance of the latter facilitates its realization and seems to ensure a high mean intensity of the accelerated particle beam.

THE appreciable increase attained recently in the power of microwave sources, and especially of optical sources, makes it expedient to search for new schemes of interaction between waves and particles, particularly for the purpose of particle acceleration. In addition, as is well known, the inverse problem (slowing down of particles) has a direct bearing on the generation of electromagnetic oscillations in different frequency ranges. In view of the known difficulties of producing slow-wave and waveguide systems for short waves, it is of interest to investigate as fully as possible the possibility of using plane free waves whose phase velocity is equal to the velocity of light c. This entails at least two fundamental difficulties:

a) The high frequency of the wave makes it difficult to realize resonant interaction without resorting to very strong magnetic fields and consequently to large gyromagnetic frequencies. This difficulty can be overcome to a certain degree by using the Doppler effect, when the relativistic particle moves essentially along the wave vector.

b) The already noted directivity of the motion reduces the efficiency of the interaction, since the wave is transverse and consequently only a small component of the electric field participates in the acceleration. This circumstance influences, in particular, the efficiency of the autoresonant mechanism of acceleration, considered by the authors earlier.[1]

In this paper, which is in some respect an extension of [1], we propose a new mechanism for accelerating charged particles in the field of a plane electromagnetic wave. The general idea is to choose the spatial dependence of the time-invariant magnetic field $\mathbf{B}(\mathbf{r})$, which we shall call the synchronizing field, in such a way that the particle will move on a trajectory such that the electric field $\mathbf{E}$ of the wave performs on the average positive work on the particle. To find this $\mathbf{B}(\mathbf{r})$ with allowance for the wave field, it is advantageous to specify the trajectory which is most favorable from the point of view of the gain in work, and then determine the synchronizing field for this trajectory. We must bear in mind here, of course, conditions such as the analyticity of the field in the interaction space, and also the requirement that its spatial period not be too small, an important factor in the case when very short waves are used (for example, light waves).

Let us consider the simplest system, when the particle moves during the acceleration process in one plane (y, z) perpendicular to the synchronizing field $\mathbf{B}_0$, which is directed along the x axis: $B_0 = B_{0x}(y, z)$. The accelerating plane-parallel wave is directed along the z axis in such a way that its magnetic field $\mathbf{B}$ is parallel to $\mathbf{B}_0$, and its electric field is parallel to the y axis:

$$B = B_x = -E_m \sin\varphi, \quad E = E_y = E_m \sin\varphi, \tag{1}$$

$$\varphi = \omega t - kz, \tag{2}$$

ω and $k = \omega/c$ are respectively the frequency and the wave number. Then the phase of the particles, which has a velocity component $v_z = c\beta_z$, can be written in the form

$$\varphi = k\int \frac{1-\beta_z}{\beta}\, dz. \tag{3}$$

In order for the particle to gain energy from the action of the wave in the magnetic field, it is natural to require that the transverse component of the particle velocity $c\beta_y$ change in phase with the electric field; for example,

$$\beta_y = \beta_\perp(\gamma)\sin\varphi = \beta_\perp(\gamma)\sin k\int\frac{1-\beta_z}{\beta_z}dz, \qquad (4)$$

where $\beta_\perp(\gamma)$ is a slowly varying function of the particle energy γ (expressed in fractions of m_0c^2) or of the distance z. Then, using only the kinematic relation $\beta_y^2 + \beta_z^2 = 1 - \gamma^{-2}$ and the particle energy balance, we can readily obtain

$$d\gamma/d\varphi = G\beta_\perp \sin^2\varphi[1-(1-\gamma^{-2}-\beta_\perp^2\sin^2\varphi)^{1/2}]^{-1}, \quad (5)$$

where the independent variable is chosen to be the phase φ, and the dimensionless quantity $G = eE_m/m_0c^2k$ characterizes the efficiency of energy transfer from the wave to the particle. The phase φ is connected with the distance z by the relation (3). If we express γ as a function of φ with the aid of (5), then

$$d\varphi/dz = k[(1-\gamma^{-2}-\beta_\perp^2\sin^2\varphi)^{-1/2}-1]. \qquad (6)$$

Since the term in the round brackets in (6) is always smaller than unity, the phase of the particle does not remain constant, as can happen in ordinary linear accelerators that use slow longitudinal waves. Therefore, approximate solutions of (5) and (6) can be obtained by averaging these equations over the rapidly varying phase φ, which generally speaking is justified when $G \ll 1$, i.e., in the case of sufficiently short waves and not too strong a field. Physically this condition denotes that the particle energy changes little over a distance of the order of the wavelength.

Let us consider now two limiting cases—nonrelativistic motion and strongly relativistic motion, directed essentially along the wave propagation direction.

A. Nonrelativistic Case

In this approximation the term in the round brackets is much smaller than unity, so that after averaging we obtain

$$\frac{d\gamma}{d\varphi}\approx\frac{G\beta_\perp}{2}, \quad \frac{dz}{d\varphi}\approx\frac{2\beta}{\pi k}E(\alpha), \quad \alpha=\frac{\beta_\perp}{\beta}, \qquad (7)$$

where $E(\alpha)$ is a complete elliptic integral of the second kind. The quantity α has the meaning of the sine of the maximum angle of inclination of the particle trajectory to the wave propagation direction, and from the nature of the problem $\alpha < 1$. Integrating the pair of equations (7), we obtain the connection between the energy and the distance

$$Gkz = \frac{4}{\pi}\int_{\gamma_i}^{\gamma}\frac{E(\alpha)}{\alpha}d\gamma, \qquad (8)$$

where γ_i corresponds to the initial particle energy.

In particular, if the amplitude of the oscillations of the transverse velocity constitutes the constant part of the total particle velocity, then α is constant and

$$\gamma = \gamma_i + \pi Gkz\alpha/4E(\alpha). \qquad (9)$$

When $\alpha \ll 1$, i.e., when the motion is very distinctly directed along the wave, we obtain from this

$$\gamma = \gamma_i + Gkz\alpha/2, \qquad (10)$$

so that the increment of the kinetic energy of the particle is

$$\Delta W = eE_m\alpha z/2. \qquad (11)$$

A characteristic feature is that this quantity does not depend on the wavelength. Compared with a linear accelerator having a longitudinal component E_m, the increment decreases by approximately a factor α^{-1}, but the value of E_m itself can in our case be much larger than in ordinary accelerators.

B. Relativistic Case

We now consider the case of relativistic motion directed essentially along the wave, this being a natural limitation when using narrow beams of short-wave radiation. Assuming that $\beta_\perp \ll 1$, we have from (5) and (6) the following averaged equations

$$d\gamma/d\varphi \approx 2G\beta_\perp^{-1}[1-(1+\gamma^2\beta_\perp^2)^{-1/2}], \qquad (12)$$

$$dz/d\varphi \approx 2\gamma^2k^{-1}(1+\gamma^2\beta_\perp^2)^{-1/2}, \qquad (13)$$

from which we get

$$Gkz \approx \int_{\gamma_i}^{\gamma}\frac{\beta_\perp\gamma^2d\gamma}{(\beta_\perp^2\gamma^2+1)^{1/2}-1} \qquad (14)$$

Let us consider first the case of very small transverse velocity oscillations, when we can put $\gamma^2\beta_\perp^2 \ll 1$. It then follows from formula (14) that

$$Gkz \approx 2\int_{\gamma_i}^{\gamma}\beta_\perp^{-1}d\gamma, \qquad (15)$$

or for $\beta_\perp = \text{const} \ll 1$

$$\gamma = \gamma_i + Gkz\beta_\perp/2. \qquad (16)$$

We note that this formula has the same form as the nonrelativistic approximation (10) obtained under the assumption α = const $\ll$ 1, which coincides with the condition $\beta_\perp$ = const $\ll$ 1, since the total velocity of the particle is in practice equal to c. From expression (16), and incidentally also from physical considerations, it follows that the efficiency of interaction increases with increasing $\beta_\perp$, so that it is advantageous to consider also the case $\gamma^2\beta^2 \gg 1$. Then formula (14) yields

$$\gamma = [\gamma_i^2 + Gkz]^{1/2}. \tag{17}$$

Thus, in the range $\gamma^{-1} \ll \beta_\perp \ll 1$ the efficiency of interaction turns out to be independent of $\beta_\perp$.

For the distance over which the energy increases by a factor n it follows from (17) that

$$z_n = (n^2-1)\gamma_i^2/Gk = (n^2-1)\mathscr{E}_i^2/E_m\mathscr{E}_0, \tag{18}$$

where $\mathscr{E}_0$ and $\mathscr{E}_i$ are respectively the rest energy and the total initial energy of the particle. It is interesting to note that the length of acceleration to a given energy (for $\gamma \gg \gamma_i$) is inversely proportional to the mass of the accelerated particle.

Let us consider now those requirements which the synchronizing magnetic field must satisfy. So far we have used only the energy balance, i.e., one of the first integrals of the equation of motion. Using the fact that the magnetic and the electric field of a plane wave are related by

$$\mathbf{B} = k^{-1}[\mathbf{k}\times\mathbf{E}], \tag{19}$$

we can represent the longitudinal component of the equation of motion in the form (see [1])

$$\Omega = c\frac{\beta_z}{\beta_y}\frac{d}{dz}[\gamma(1-\beta_z)], \tag{20}$$

or, after simple transformations

$$\frac{\Omega}{\omega} = \beta_z^{-1}\Big\{\gamma\beta_\perp(1-\beta_z)\cos\varphi - G\sin\varphi\Big[1-\beta_z-\beta_y^2\frac{d\ln\gamma\beta_\perp}{d\ln\gamma}\Big]\Big\}, \tag{21}$$

where $\Omega = eB_0/m_0c$. Inasmuch as φ, γ, β_z and β_y were obtained above as functions of z, Eq. (21) can be considered as a definition of the synchronizing magnetic field specified along the trajectory. In the particular case considered above, $\beta_\perp = \alpha\beta$, $\alpha \ll 1$, expression (21) simplifies somewhat:

$$\frac{\Omega}{\omega} \approx \frac{1-\beta_z}{\beta_z}\{\alpha\gamma\beta\cos\varphi - G(1-\gamma^2\alpha^2\sin^2\varphi)\sin\varphi\}. \tag{22}$$

In the nonrelativistic and relativistic ($\alpha^2\gamma^2 \gg 1$) cases this yields

$$\frac{\Omega}{\omega} \approx \alpha\cos\varphi - G\sin\varphi,$$

$$\varphi = \varphi_i + G^{-1}\alpha^{-1}[(\beta_i^2 + Gkz\alpha)^{1/2} - \beta_i], \quad \beta \ll 1, \tag{23}$$

$$\frac{\Omega}{\omega} \approx \frac{\alpha}{2}(\gamma_i^2+Gkz)^{-1/2}\cos\varphi + G\alpha^2\sin^3\varphi,$$

$$\varphi = \varphi_i + \frac{1}{2G}\ln\Big(1+\frac{Gkz}{\gamma_i^2}\Big).$$

$$\alpha^2\gamma^2 \gg 1. \tag{24}$$

It must be borne in mind that these expressions give the field along the particle trajectory and therefore, in particular, depend on the initial entry phase φ_i. The trajectories themselves can be readily obtained in the cases in question from the equation $dy/dz = \beta_y/\beta_z$, and are of the form

$$k(y-y_i) \approx \alpha\beta_i(\cos\varphi_i - \cos\varphi) + G\alpha^2[\sin\varphi - \sin\varphi_i - (\varphi-\varphi_i)\cos\varphi], \quad \beta \ll 1, \tag{25}$$

$$k(y-y_i) \approx 2\alpha\gamma_i^2[\cos\varphi_i - \cos\varphi\exp(2G(\varphi-\varphi_i))] = 2\alpha[\gamma_i^2\cos\varphi_i - \gamma^2\cos\varphi], \quad \alpha^2\gamma^2 \gg 1. \tag{26}$$

By varying the relation $\Omega(y, z)$ (but maintaining the function $\Omega(z)$ the same along the trajectory), we can choose in optimal fashion the region of initial conditions for which the given field will actually be synchronizing. Thus, for example, if we stipulate that particles with arbitrary y_i enter in the acceleration mode, then the field must be chosen independent of y; however, in this case only particles with initial phase φ_i will enter into synchronism (see (23) and (24)). On the other hand, we can increase to 2π the region of initial phases corresponding to synchronization, but only at the expense of causing the acceleration of only particles with $y|_{z=0} = y_i$. It is necessary then to express φ_i from (25) or (26) as a function of $y - y_i$ and substitute in the expressions (23) and (24) for the field. In this case, for example, for relativistic motion the value of φ in (24) will depend not only on z, but also on y, being a solution of the equation

$$\gamma_i^2\cos\Big\{\varphi - \frac{1}{2G}\ln\Big(1+\frac{Gkz}{\gamma_i^2}\Big)\Big\} = \gamma_i^2\Big(1+\frac{Gkz}{\gamma_i^2}\Big)\cos\varphi + \frac{k(y-y_i)}{2\alpha}. \tag{27}$$

Thus the magnetic field is specified only in those regions of space where the particles can be situated physically.

We note that any relation for $\Omega(y, z)$ agrees with Maxwell's equations and does not call for introduction of additional currents in the interaction space. The magnetic field is in this case purely potential with a symmetry plane (y, z). In this plane there is only an x-component equal to $\partial\Phi/\partial x|_{x=0} = \Omega(y, z)$. This relation can be regarded as a boundary condition of the second boundary-value problem for the Laplace equation, the solution of which always leads to a physically realizable magnetic potential Φ (as is the case in ordinary cyclic accelerators).

It is easy to see that the absolute magnitude of the synchronizing magnetic field is much smaller (when $\alpha \ll 1$) than the field necessary for equality of the gyromagnetic frequency and of the frequency ω of the wave. Thus, in the relativistic case, assuming for an estimate of $\alpha \sim \gamma^{-1}$, we obtain $\Omega/\omega \sim \gamma^{-2} \ll 1$, and in the nonrelativistic case $\Omega/\omega \sim (\alpha^2 + G^2)^{1/2}$. The spatial period of the field along the z axis increases with increasing z, but for relativistic motion it is much greater than the wavelength $2\pi/k$ even for the initial section:

$$\Delta z_i \approx 4\pi\gamma_i / k\alpha. \tag{28}$$

In the nonrelativistic case, naturally, the period of the field should be smaller than the wavelength.

It must be borne in mind that the expressions for the field were obtained above only for a particle which is in synchronism with the wave at all times, i.e., the so-called equilibrium particle. Such rigorous resonance conditions correspond, naturally (within the framework of the averaged equations), to a zero volume in the initial-condition space ($y_i\varphi_i$), just as in ordinary accelerators. Therefore the next step in the investigation of the possibilities of this scheme is to consider particles that differ little from equilibrium and move in a specified magnetic field of the form (23) or (24). We recall that the freedom we have to vary the law governing $\Omega(y, z)$ makes it possible to vary over a wide range the initial conditions corresponding to the exact or approximate resonance. In particular, by suitable choice of the field it is possible to obtain not only phase stability for particles that differ little in their initial conditions from equilibrium, but to increase to a maximum degree the region of initial conditions corresponding to stability, i.e., the acceptance of the system.

In this paper we consider one of the physically simplest variants of acceleration of particles by a wave in a synchronizing magnetic field. The next required step is to investigate the possibility of other more effective variants and to consider in greater detail the question of the stability of the acceleration process. The considerable further increase in the power of the microwave and light beams, on which we can already count, leads us to expect practical applications for the method.

Without touching here on the practical aspects of the matter, we note only that the time-invariance of the synchronizing field greatly facilitates its realization and at the same time makes it possible in principle, to capture the particles in the acceleration mode by injection in each period of the high frequency field of the wave. The latter circumstance makes it possible to expect a large average intensity of the accelerated particle beam.

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[1] A. A. Kolomenskiĭ and A. N. Lebedev, JETP **44**, 261 (1963), Soviet Phys. JETP **17**, 179 (1963)

Translated by J. G. Adashko
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