SCATTERING OF WAVES IN A PLASMA

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It is shown that for a complete description of scattering and transformation of electromagnetic waves in a plasma it is necessary to take into account, besides the electron-number fluctuations, also the eddy-current fluctuations. The scattering of transverse waves with frequencies that are high compared with the Langmuir frequency is considered as an example of the obtained general expressions for the cross section.

INTRODUCTION

MANY recent papers are devoted to scattering of waves in plasmas. Different methods are used, but one might think that they should lead to identical expressions for the cross sections of identical effects. A. G. Sitenko has called our attention to the fact that the scattering cross sections obtained in our papers [1,2] do not coincide with those obtained on the basis of many others.[3-8] We present below a comparison of the results of different approaches and demonstrate the correctness of our results [1,2] with the aid of the method of [3-8].

It must further be noted that the formulas of [3-8], used to calculate concrete scattering cross sections, are in a certain sense similar to those serving as the base of the wave scattering theory in the well known book by Landau and Lifshitz [9]. Namely, the cross section for wave scattering is determined in these papers, as in this book, by the fluctuation of the number of electrons $\langle \delta n_e^2 \rangle$. The discrepancy between the concrete results of our papers and the papers by other authors is connected with the fact that in the plasma, owing to thermal-motion effects (or, what is the same, to the spatial dispersion effect), the wave-scattering cross section is determined not only by the electron-number fluctuation, but also by the eddy-current fluctuation. Only when account is taken of the latter is it possible to obtain all-inclusive expressions for the scattering of longitudinal and transverse waves in a plasma. We point out that a cross section for the scattering of longitudinal waves, such as obtained in [1,2], was confirmed in many papers using other methods [10-12].

The need for the comparison presented below has become especially acute, in our opinion, in connection with the publication of a large number of results on the scattering of waves in a magnetoactive plasma. We believe that a disclosure of the causes of the discrepancy between results of articles using different methods will make it possible to complete more rapidly the work now in progress on the theory of nonlinear interaction of waves in a magnetoactive plasma.

It should be noted that Pogutse [13] obtained for the scattering of transverse waves a general scattering-cross-section formula, in which additional terms besides the fluctuations of the electron-density fluctuations appear. However, no detailed analysis is presented in that paper of the differential scattering cross section, so that the statement made by Pogutse [13] that the formulas of [3-8] are applicable is accurate only for the total cross sections for the scattering of transverse waves.

1. CROSS SECTION FOR THE SCATTERING OF WAVES IN A PLASMA

As is well known [3-5], the cross section for the scattering of an electromagnetic wave with wave vector $\mathbf{k}$ and frequency $\omega = \omega(\mathbf{k})$, resulting in a wave with wave vector $\mathbf{k}'$, is expressed in terms of the current correlator $\mathbf{j}(\omega', \mathbf{k}')$ producing the scattered wave:

$$d\sigma(\mathbf{k}, \mathbf{k}') = -\frac{V}{(2\pi)^3} \frac{d\mathbf{k}'}{W(\mathbf{k})\,|\partial\omega/\partial\mathbf{k}|} \times \operatorname{Im} \int_{-\infty}^{\infty} \frac{d\omega'}{\omega'} A_{ji}(\omega', \mathbf{k}') \langle j_i(\omega', \mathbf{k}') j_j^*(\omega', \mathbf{k}') \rangle, \quad (1)$$

where V is the volume of the scattering system, $W(\mathbf{k})$ the energy density in the incident wave, which in accordance with [5], is equal to

$$W(\mathbf{k}) = \frac{\partial(\omega^2 \varepsilon_{ij}^H(\omega, \mathbf{k}))}{\omega\, \partial\omega} \frac{E_{0i}^* E_{0j}}{16\pi}, \quad (2)$$

$\epsilon_{ij}^{H}(\omega, \mathbf{k})$ is the hermitian part of the dielectric tensor of the medium, and E_0 is the amplitude of the incident wave. The tensor $A_{ij}(\omega, \mathbf{k})$ is connected with the dielectric constant by the relation

$$A_{ij}(\omega, \mathbf{k}) = \left[\varepsilon_{ij}(\omega, \mathbf{k}) - \frac{c^2k^2}{\omega^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\right]^{-1}. \quad (3)$$

The averaging in formula (1) is over the fluctuations.

The current producing the scattered radiation can be determined in the case of a plasma from the kinetic equation that describes the perturbation produced in the distribution function by the interaction between the incident wave and the fluctuations [3,13]:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_a \frac{\partial}{\partial \mathbf{r}}\right) f_a' + \frac{e_a}{m_a}\mathbf{F}_a' \frac{\partial F_a}{\partial \mathbf{v}_a} = -\frac{e_a}{m_a}\left(\mathbf{F}_{0a}\frac{\partial \delta N_a}{\partial \mathbf{v}_a} + \delta\mathbf{F}_a \frac{\partial}{\partial \mathbf{v}_a} f_{a0}\right), \quad (4)$$

where*

$$\mathbf{F}_a' = \mathbf{E}' + c^{-1}[\mathbf{v}_a\mathbf{B}'], \quad \mathbf{F}_{0a} = \mathbf{E}_0 + c^{-1}[\mathbf{v}_a\mathbf{B}_0], \quad \delta\mathbf{F}_a = \delta\mathbf{E} + c^{-1}[\mathbf{v}_a\delta\mathbf{B}],$$

$f_a'(\mathbf{r}, \mathbf{v}_a, t)$ is the perturbation, corresponding to the scattered wave, of the distribution function of the particles of species a, $\mathbf{E}'(\mathbf{r}, t)$ and $\mathbf{B}'(\mathbf{r}, t)$ are the electric and magnetic fields in the scattered waves; $\mathbf{E}_0(\mathbf{r}, t)$ and $\mathbf{B}_0(\mathbf{r}, t)$ are the fields in the incident wave, $f_{a0}(\mathbf{r}, \mathbf{v}_a, t)$ is the plasma perturbation caused by the incident wave; $\delta\mathbf{E}(\mathbf{r}, t)$, $\delta\mathbf{B}(\mathbf{r}, t)$, and $\delta N_a(\mathbf{r}, \mathbf{v}_a, t)$ are respectively the fluctuations of the electric and magnetic fields and the particle-number density of species a in phase space. It is assumed that there is no constant magnetic field.

The field of the incident wave is chosen in the form

$$\mathbf{E}_0(\mathbf{r}, t) = \mathbf{E}_0 e^{-i\omega t + i\mathbf{k}\mathbf{r}}.$$

Substituting this expression in (4) and eliminating the magnetic fields with the aid of Maxwell's equation curl $\mathbf{E} = -\delta\mathbf{B}/c\partial t$, we obtain

$$j_i(\omega', \mathbf{k}') \equiv \sum_a e_a \int d\mathbf{v}_a f_a'(\mathbf{k}', \mathbf{v}_a, \omega') v_{ai} = E_{0j}\alpha_{ji}(\omega, \mathbf{k}, \omega'', \mathbf{k}'', \omega', \mathbf{k}'), \quad (5)$$

where $\omega'' = \omega' - \omega$, $\mathbf{k}'' = \mathbf{k}' - \mathbf{k}$,

$$\alpha_{ji}(\omega, \mathbf{k}, \omega'', \mathbf{k}'', \omega', \mathbf{k}') = -i\frac{\omega'}{4\pi}[\varepsilon_{ilj}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k})\delta E_l(\omega'', \mathbf{k}'') + \delta\varepsilon_{ij}(\omega', \mathbf{k}', \omega, \mathbf{k}, \omega'', \mathbf{k}'')], \quad (6)$$

$$\varepsilon_{ijl}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k}) = -4\pi i \sum_a \frac{e_a^3}{m_a^2}\int d\mathbf{v}_a \frac{v_{ai}}{\omega'} g(\omega', \mathbf{k}', \mathbf{v}_a) \times \Gamma_j(\omega'', \mathbf{k}'', \mathbf{v}_a) g(\omega, \mathbf{k}, \mathbf{v}_a)\Gamma_l(\omega, \mathbf{k}, \mathbf{v}_a)F_a(\mathbf{v}_a), \quad (7)$$

$$\delta\varepsilon_{ij}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k}) = \frac{4\pi}{\omega'}\sum_a \frac{e_a^2}{m_a}\int d\mathbf{v}_a v_{ai} g(\omega', \mathbf{k}', \mathbf{v}_a) \times \Gamma_j(\omega'', \mathbf{k}'', \mathbf{v}_a)\delta N_a(\omega, \mathbf{k}, \mathbf{v}_a), \quad (8)$$

$$g(\omega, \mathbf{k}, \mathbf{v}_a) \equiv (\omega + i0 - \mathbf{k}\mathbf{v}_a)^{-1}, \quad (9)$$

$$\Gamma_j(\omega, \mathbf{k}, \mathbf{v}_a) \equiv \frac{1}{\omega}[\delta_{ij}(\omega - \mathbf{k}\mathbf{v}_a) + k_i v_{aj}]\frac{\partial}{\partial v_{ai}} \equiv \gamma_{ij}(\omega, \mathbf{k}, \mathbf{v}_a)\frac{\partial}{\partial v_{ai}}. \quad (10)$$

We note that ϵ_{ijl} is the coefficient of the nonlinear dielectric constant arising when account of a current quadratic in the field is taken [1], and $(\delta\epsilon_{ij} + \delta_{ij}\delta(\omega)\delta(\mathbf{k}))$ coincides with the dielectric constant of an inhomogeneous plasma, where the quantity $\delta N_a(\omega, \mathbf{k}, \mathbf{v}_a)$ serves as the distribution function.

Substituting (6)—(8) in (1) we obtain the following expression for the scattering cross section of the waves in the plasma¹⁾:

$$\begin{aligned} d\sigma(\mathbf{k}, \mathbf{k}') = &-\frac{V}{(2\pi)^3}\frac{d\mathbf{k}'}{16\pi^2 W(\mathbf{k})\,|\partial\omega/\partial\mathbf{k}|}\,\mathrm{Im}\Big\{E_{0n}E_{0m}^*\int_{-\infty}^{\infty} d\omega'\omega' \\ &\times A_{ji}(\omega', \mathbf{k}')\Big[\varepsilon_{iln}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k}) \\ &\times \varepsilon^*_{jpm}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k})\cdot\langle\delta E_l\,\delta E_p\rangle_{\omega'', \mathbf{k}''} \\ &- \sum_a \frac{4\pi e_a^2}{m_a}\int d\mathbf{v}_a\gamma_{qi}(\omega', \mathbf{k}', \mathbf{v}_a)g^2(\omega', \mathbf{k}', \mathbf{v}_a)\gamma_{qn}(\omega, \mathbf{k}, \mathbf{v}_a) \\ &\times\langle\delta N_a\,\delta E_p\rangle_{\omega'', \mathbf{k}''}\varepsilon^*_{jpm}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k}) \\ &- \sum_a \frac{4\pi e_a^2}{m_a}\int d\mathbf{v}_a\gamma_{pj}(\omega', \mathbf{k}', \mathbf{v}_a)g^{*2}(\omega', \mathbf{k}', \mathbf{v}_a)\gamma_{pm}(\omega, \mathbf{k}, \mathbf{v}_a) \\ &\times\langle\delta E_l\,\delta N_a\rangle_{\omega'', \mathbf{k}''}\varepsilon_{iln}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k}) \\ &+ \sum_{ab}\frac{(4\pi e_a e_b)^2}{m_a m_b}\int d\mathbf{v}_a\,d\mathbf{v}_b\,\gamma_{pi}(\omega', \mathbf{k}', \mathbf{v}_a)\gamma_{qj}(\omega', \mathbf{k}', \mathbf{v}_b) \\ &\times\gamma_{pn}(\omega, \mathbf{k}, \mathbf{v}_a)\gamma_{qm}(\omega, \mathbf{k}, \mathbf{v}_b)g^2(\omega', \mathbf{k}', \mathbf{v}_a) \\ &\times g^{*2}(\omega', \mathbf{k}', \mathbf{v}_b)\langle\delta N_a\,\delta N_b\rangle_{\omega'', \mathbf{k}''}\Big]\Big\}. \end{aligned} \quad (11)$$

We shall show that the expression obtained in this manner for the cross section coincides with the expression obtained in [1,2] from the equations of nonlinear electrodynamics of a plasma. To this

*$[\mathbf{v}_a\mathbf{B}'] \equiv \mathbf{v}_a \times \mathbf{B}'$.

¹⁾An expression similar to (12) was obtained by Bass and Blank[14], but an analysis of this expression was made there only for the case of combination of scattering (decay and coalescence of waves).

end we substitute in (11) the expressions for correlators

$$\langle \delta E_i \delta E_j \rangle_{\omega, \mathbf{k}}, \quad \langle \delta N_a \delta E_i \rangle_{\omega, \mathbf{k}}, \quad \langle \delta N_a \delta N_b \rangle_{\omega, \mathbf{k}},$$

which are contained, for example, in[15], and we consider a plasma which is in thermodynamic equilibrium at a temperature T. In this case the cross section can be written in the form

$$d\sigma(\mathbf{k}, \mathbf{k}') = \frac{V}{(2\pi)^3} \frac{d\mathbf{k}'}{2\pi W(\mathbf{k}) |\partial\omega/\partial\mathbf{k}|} T \,\mathrm{Im} \Big\{ E_{0n} E_{0m}^* \int_{-\infty}^{\infty} d\omega' \frac{\omega'}{\omega''}$$

$$\times A_{ji}(\omega', \mathbf{k}') \,\mathrm{Im}\, [A_{lp}(\omega'', \mathbf{k}'') S_{pjm}(\omega'', \mathbf{k}'', -\omega, -\mathbf{k}, \omega', \mathbf{k}')$$

$$\times S_{iln}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k}) - V_{ijnm}(\omega', \mathbf{k}', \omega, \mathbf{k})] \Big\}, \quad (12)$$

where

$$\mathrm{Im}\, V_{ijnm}(\omega', \mathbf{k}', \omega, \mathbf{k}) = \frac{\omega''}{T} \sum_a \frac{4\pi^2 e_a^4}{m_a^2} \int d\mathbf{v}_a \,\gamma_{pi}(\omega', \mathbf{k}', \mathbf{v}_a)$$

$$\times \gamma_{qj}(\omega', \mathbf{k}', \mathbf{v}_a) \gamma_{pn}(\omega, \mathbf{k}, \mathbf{v}_a) \gamma_{qm}(\omega, \mathbf{k}, \mathbf{v}_a) |g(\omega', \mathbf{k}', \mathbf{v}_a)|^4$$

$$\times \delta(\omega' - \omega - \mathbf{k}'\mathbf{v}_a + \mathbf{k}\mathbf{v}_a) F_a(\mathbf{v}_a), \quad (13)$$

$$S_{ilj}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k}) = \varepsilon_{ijl}(\omega', \mathbf{k}', \omega'', \mathbf{k}'', \omega, \mathbf{k})$$

$$+ \varepsilon_{ilj}(\omega', \mathbf{k}', \omega, \mathbf{k}, \omega'', \mathbf{k}''). \quad (14)$$

This form of the cross section corresponds precisely to the general expression used by us in[1,2].

2. SCATTERING OF WAVES WHOSE PHASE VELOCITIES ARE LARGE COMPARED WITH THE PARTICLE THERMAL VELOCITIES

Let us consider a case when the incident and scattered waves have phase velocities greatly exceeding the thermal velocities of both the electrons and the ions. We expand only quantities that depend on ω, $\mathbf{k}$ and ω', $\mathbf{k}'$ in formula (11), assuming that $(\mathbf{k}\cdot\mathbf{v}/\omega) \ll 1$ and $(\mathbf{k}'\cdot\mathbf{v}/\omega') \ll 1$. We integrate with respect to the velocities by using for the correlators the expressions contained, for example, in[15]. An analysis of the expressions obtained in this case shows that the contribution from the first of three terms in (11) can be neglected if one disregards the fluctuations of the transverse field[2)]. The main contribution of the cross section is made by the last term in (11). In this case the first nonvanishing term of the expansion in powers of $\mathbf{k}\cdot\mathbf{v}/\omega$ is anomalously small in many cases. Under these conditions it becomes necessary to take into account also higher powers in the expansion of the last term in (11) in powers of $\mathbf{k}\cdot\mathbf{v}/\omega$. The expression for the cross section takes the form

$$d\sigma(\mathbf{k}, \mathbf{k}') = -\frac{V}{(2\pi)^3} \frac{d\mathbf{k}'}{W(\mathbf{k}) |\partial\omega/\partial\mathbf{k}|}$$

$$\times \frac{e^4}{m^2} \mathrm{Im} \Big\{ E_{0n} E_{0m}^* \int_{-\infty}^{\infty} \frac{d\omega'}{\omega'^3} A_{ji}(\omega', \mathbf{k}')$$

$$\times \Big[\delta_{in} \delta_{jm} \langle \delta n_e^2 \rangle_{\omega'', \mathbf{k}''} + \frac{1}{\omega^2} (k_n' k_m' \langle \delta v_i \,\delta v_j \rangle^{tr}_{\omega'', \mathbf{k}''}$$

$$+ k_i k_m' \langle \delta v_n \,\delta v_j \rangle^{tr}_{\omega'', \mathbf{k}''} + k_n' k_j \langle \delta v_i \,\delta v_m \rangle^{tr}_{\omega'', \mathbf{k}''}$$

$$+ k_i k_j \langle \delta v_n \,\delta v_m \rangle^{tr}_{\omega'', \mathbf{k}''}) \Big] \Big\}, \quad (15)$$

where

$$\langle \delta n_e^2 \rangle_{\omega, \mathbf{k}} \equiv \int d\mathbf{v}\, d\mathbf{v}' \langle \delta N_e(\mathbf{v}) \delta N_e(\mathbf{v}') \rangle_{\omega, \mathbf{k}}$$

$$= \frac{k^2}{2\pi e^2 \omega} \frac{1}{|\varepsilon^l|^2} \{\varepsilon_e'' T_e |1 + \varepsilon_i|^2 + \varepsilon_i'' T_i |\varepsilon_e|^2\}, \quad (16)$$

$$\langle \delta v_i \,\delta v_j \rangle^{tr}_{\omega, \mathbf{k}} \equiv \langle \delta v_{e,i} \,\delta v_{e,j} \rangle^{tr}_{\omega, \mathbf{k}}$$

$$= \frac{T_e}{m_e} \Big(\delta_{ij} - \frac{k_i k_j}{k^2} \Big) \varepsilon_e''(\omega, k) \frac{k^2 T_e}{2\pi e^2 \omega}, \quad (17)$$

$$\varepsilon_a(\omega, k) = (k r_{Da})^{-2} (1 - J_+(\beta_a)), \quad J_+(\beta) = \beta e^{-\beta^2/2} \int_{+i\infty}^{\beta} e^{\tau^2/2} d\tau,$$

$$\varepsilon_a'' = \mathrm{Im}\, \varepsilon_a = (k r_{Da})^{-2} (\pi/2)^{1/2} \beta_a \exp(-\beta_a^2/2),$$

$$\varepsilon_a' = \mathrm{Re}\, \varepsilon_a, \quad \varepsilon^l = 1 + \varepsilon_e + \varepsilon_i,$$

$$r_{Da}^{-2} = 4\pi N_a e_a^2 / T_a, \quad \beta_a = \omega / k v_{Ta}, \quad v_{Ta} = \sqrt{T_a / m_a}, \quad (18)$$

m_e and e are the mass and charge of the electrons; T_e and T_i are the electron and ion temperatures.

Allowance for the terms with $\langle \delta v_p \delta v_q \rangle^{tr}_{\omega,\mathbf{k}}$ for the scattering of longitudinal and transverse waves is the feature distinguishing the results of our paper[2] from the results by other authors[3–8]. These terms change qualitatively the expressions, for example, for the scattering cross sections of longitudinal waves under conditions when the waves are scattered by electrons and the contributions corresponding to different "diagrams" cancel one another[10]. To the contrary, under conditions when the scattering is due to ions, the fluctuations of the transverse velocities are of little effect and formula (10.45) of the book by Sitenko is[4] convenient for the plasma-wave scattering cross section.

3. SCATTERING OF TRANSVERSE WAVES

In this section we illustrate the use of formula (15) by considering the scattering of an unpolarized

[2)]Certain cases when account of the fluctuations of the transverse field may be necessary, are considered in[1,16].

transverse wave with formation of a wave which is also transverse. We discuss here the case of high-frequency waves, when we can assume with high degree of accuracy that $\omega = kc$ and $\omega' = k'c$. It is obvious that this is possible when the frequencies are high compared with the electron Langmuir frequency.

In (15) we must substitute the expressions

$$E_{0n}E_{0m}^* = |E_0|^2(\delta_{nm} - k_n k_m / k^2), \quad W(\mathbf{k}) = |E_0|^2 / 4\pi,$$
$$\mathrm{Im}\, A_{ji}(\omega', \mathbf{k}') = -\pi(\delta_{ij} - k_i' k_j' / k'^2)\,\delta[1 - k'^2c^2/\omega'^2].$$

We then obtain

$$d\sigma^{(t\to t')}(\mathbf{k},\mathbf{k}') = \left(\frac{e^2}{mc^2}\right)^2 Vc \frac{d\mathbf{k}'}{4\pi k'^2}\Big[(1+\cos^2\vartheta)\langle\delta n_e^2\rangle_{\omega'',\mathbf{k}''} + 4\sin^2\vartheta(2-\cos\vartheta)\frac{N_e(k''r_{De})^2}{(k-k')c}\varepsilon_e''(\omega'',k'')\frac{v_{Te}^2}{c^2}\Big], \quad (19)$$

where ϑ is the angle between $\mathbf{k}$ and $\mathbf{k}'$.

In [3-8], in the scattering of transverse waves, we take into account only the first term.

In order for the scattering by the fluctuations of the vortical velocity of the electrons to become decisive, it is necessary that the angle ϑ be ~ 1 and that the following inequality be satisfied

$$v_{Te}/c \gg m/m_i. \quad (20)$$

In the case of a hydrogen plasma this is satisfied for temperatures higher than 2000°. In addition, the following inequalities must be satisfied: either

$$\frac{\omega_{Le}^2}{\omega^2} \gg \sqrt{\frac{v_{Te}}{c}\frac{\omega_{Le}}{\omega}} \gg \frac{\omega''}{\omega} \gg \frac{\omega_{Li}}{\omega},\ \frac{v_{Ti}}{c}\left[\ln\frac{c^2}{v_{Te}v_{Ti}}\frac{T_e}{T_i}\right]^{1/2}, \quad (21)$$

or

$$\frac{\omega_{Li}}{\omega} \gg \frac{\omega''}{\omega} \gg \sqrt{\frac{mv_{Te}}{m_i c}},\ \frac{v_{Ti}}{c}\left[\ln\frac{c^2}{v_{Te}v_{Ti}}\frac{T_e}{T_i}\right]^{1/2}. \quad (22)$$

It follows hence, in particular, that even though the frequency of the scattered waves may exceed greatly the electron Langmuir frequency, it nevertheless cannot be too large. Thus, the frequency of the scattered wave should be smaller than $\omega_{Le}\sqrt{c/v_{Te}}$.

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