EFFECTIVE POTENTIAL FOR PARTICLE MOTION IN A HIGH-FREQUENCY FIELD

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A general expression is derived for the effective potential associated with the motion of a charged particle in a combined quasistatic magnetic field and a high-frequency electromagnetic field. The amplitude of the high-frequency field is assumed to be small.

THE average forces exerted on a charged particle by inhomogeneous high-frequency (rf) field and a magnetostatic field have been considered for a number of particular cases. Recently a paper has been published by Teichmann,[1] who derived rather general expressions for these forces in the case in which the hydrodynamic approximation applies. In developing a kinetic theory it is desirable to have as many possible integrals of motion of the particle as possible. Below we present certain results which pertain to this problem.

It is assumed that there are slowly varying (in space and time) fields $\mathbf{E}_0$ and $\mathbf{B}_0$ and rf fields $\mathbf{E}$ and $\mathbf{B}$ with slowly varying complex amplitudes and frequency $\omega(t)$. The problem is solved to second order in the small parameter

$$\delta = \max\{\tau_\sim/\tau_0,\ v\tau_\sim/L\},\quad \delta \ll 1,$$

where $v = |\mathbf{v}|$ is the magnitude of the particle velocity, τ_0 is the smallest characteristic time for a change in the complex amplitudes of $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{E}_0$, $\mathbf{B}_0$, $L = \min(\lambda, L_1)$, λ is the wavelength, L_1 is the smallest characteristic length over which the amplitudes of $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{E}_0$, $\mathbf{B}_0$ vary:

$$\tau_\sim = \max\left\{\frac{2\pi}{\omega},\ \frac{2\pi}{|\Omega|},\ \left|\frac{2\pi}{\omega-|\Omega|}\right|\right\},\quad \Omega = \frac{eB_0}{mc},\quad B \ll B_0.$$

The smoothed motion of a particle[2] of mass m and charge e along the lines of force of $\mathbf{B}_0$ is described by the equation[3]

$$\frac{dP_\parallel}{dt} = \mathbf{P}_\perp\frac{d\mathbf{h}}{dt} + eE_{0\parallel}(\mathbf{R}) - \mu\frac{\partial B_0}{\partial s} - \frac{\partial U_\sim}{\partial s}$$

$$\mathbf{P} = m\dot{\mathbf{R}},\quad \mathbf{h} = \frac{\mathbf{B}_0(\mathbf{R})}{B_0(\mathbf{R})},\quad \mu = \frac{mu^2}{2B_0} = \text{const},$$

where $\mathbf{R}$ is the radius vector of the smoothed motion and the subscripts $\parallel$ and $\perp$ denote vector components parallel and perpendicular to $\mathbf{B}_0(\mathbf{R})$, s is the length measured along the field $\mathbf{B}_0$, u_c is the velocity of the cyclotron rotation of the particle and $U_\sim$ is the effective rf potential:

$$U_\sim = \frac{e^2}{4m\omega^2}E_\parallel E_\parallel^* + \frac{e^2}{4m(\omega^2-\Omega^2)}\times\left(E_nE_n^* + E_\nu E_\nu^* - 2\frac{\Omega}{\omega}\operatorname{Re}\{iE_nE_\nu^*\}\right),$$

where $E_\parallel$, E_n, and E_ν are the projections of the rf electric field on the unit vector along the tangent $\mathbf{h}$, the normal, and the binormal to the line of force $\mathbf{B}_0(\mathbf{R})$. In the derivation of these equations it is assumed that the effects of multiple and fractional cyclotron resonances can be neglected. In many of the cases (for example for small curvature of the lines of force of $\mathbf{B}_0$ or drift $\mathbf{P}_\perp/m$ or for suitable symmetry) the term $\mathbf{P}_\perp d\mathbf{h}/dt$ can be neglected.

In the stationary state, $E_{0\parallel} = -\partial V/\partial s$, the smoothed motion along the lines of force of $\mathbf{B}_0$ is described by the Hamiltonian

$$\mathcal{H} = P_\parallel^2/2m + eV(s) + \mu B_0(s) + U_\sim(s), \qquad (1)$$

$$d\mathcal{H}/dt = 0.$$

We note that the conservation of total energy of the particle, which holds when $\omega \gg |\Omega|$, does not hold in the general case. For example, when $\omega \ll |\Omega|$ and with linear polarization of $\mathbf{E}_\perp$, it is found that $U_\sim = W_\parallel - W_\perp$ where $W_\parallel$ and $W_\perp$ are the energy associated with the longitudinal and transverse oscillatory motion in the rf field. Consequently, in certain cases the particle will be pulled into the region of maximum rf electric field.

Using the integral of motion (1), we find, for example, stationary smoothed distribution functions for a collisionless plasma.[3] In this case we must take account of the self-consistent electrostatic field V(s) which arises because of charge separation. Let the distribution function for the motion of the electrons and ions (singly charged) at a point $s = s_0$ be written in the form

$$g_{e,i} \sim \exp\left(-\frac{\mu B_0(s_0)}{T_{\perp e,i}}\right)\exp\left(-\frac{P_\parallel^2}{2m_{e,i}T_{\parallel e,i}}\right),$$

It follows that the density $n_e \approx n_i \approx n$ along the lines of force of $\mathbf{B}_0$ is given by the formula

$$n(s) = n(s_0)\frac{B_0(s)}{B_0(s_0)}\frac{T_{\perp e}'}{T_{\perp e}}$$

$$\times \exp\left\{\frac{T_{\|i}}{T_{\|e}+T_{\|i}}\ln\frac{T_{\perp i}'T_{\perp e}}{T_{\perp i}T_{\perp e}'} - \frac{U(s)-U(s_0)}{T_{\|e}+T_{\|i}}\right\},$$

$$T_\perp' = T_\perp\left(1+\frac{T_\perp}{T_\|}\frac{B_0(s)-B_0(s_0)}{B_0(s_0)}\right)^{-1}, \quad U = U_{\sim e} + U_{\sim i}.$$

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Note added in Proof (February 22, 1966). The results obtained in this paper are in agreement with certain results of solutions of specialized cases.[4] An investigation of the problem of uniqueness will be published in the Czechoslovakian Journal of Physics.

[1] I. Teichmann, Nuclear Fusion 5, 107 (1965).

[2] D. Sivukhin, Voprosy teorii plazmy (Problems of Plasma Physics), Vol. 1, Gosatomizdat 1963, Consultants Bureau, New York, 1965.

[3] R. Klima, Czech. J. Phys. (in press).

[4] Ya. L. Al'pert, A. V. Gurevich and L. P. Pitaevskiĭ, Iskusstvennye sputniki v razrezhennoi plazme (Artificial Satellites in a Rarefied Plasma), Nauka, 1964; A. P. Popryadukhin, ZhTF 35, 927 (1965), Soviet Phys. Tech. Phys. 10, 714 (1965).

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