MULTIPLE SCATTERING OF NITROGEN AND OXYGEN IONS IN ALUMINUM

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Submitted to JETP editor October 22, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **50**, 589–594 (March, 1966)

We have used a system of proportional counters to measure the angular distributions of N^{14} and O^{16} ions with initial energy ~0.3 MeV/nucleon after multiple scattering in aluminum foils. The data obtained have been analyzed on the basis of Molière's theory. The experimental angular distributions agree satisfactorily with the theoretical distributions if we use for the charge of the moving ion the mean-square charge of ions in a beam of equilibrium charge composition.

Up to the present time a considerable number of studies have been made of the multiple scattering of charged particles in matter. However, the overwhelming majority of experiments have been carried out with electrons and mesons, and specific experiments with heavy particles practically all have been in the region of rather high energies, $E \gtrsim 1$ MeV/nucleon. One of the characteristic features of the scattering processes at these energies is that the theoretical analysis of the results is greatly simplified by the complete stripping of the nucleus of the incident particle. In particular, this simplification is contained in the multiple-scattering theory of Molière,[1] which has been very widely used as the result of its generality and extremely refined mathematical treatment.

Recently studies have been initiated at Moscow State University on multiple scattering of various particles in matter at comparatively low energies. Studies of the scattering of 75–200 keV protons in carbon, aluminum, and copper[2] showed that in these cases the principal regularities of the phenomenon are preserved and a somewhat modified Molière theory describes the experimental results fairly well; here the effects associated with charge exchange of the ion in passing through the material have practically no effect on the nature of the angular distribution of the scattered particles. The process becomes considerably more complicated in the case of scattering of heavier ions of roughly the same energy. Thus, the results of a study of the multiple scattering of He^{4+} ions in C and Al[3] indicates, in particular, that the effect of partial screening of the field of the moving nucleus by atomic electrons has an important effect in this case.

In the present paper we report the results of a study of multiple scattering of N^{14} and O^{16} ions in aluminum foils for an initial energy of ~ 0.3 MeV/nucleon. The measurements were made with the 72-cm cyclotron with the basic elements of the technique previously developed for a study of equilibrium distributions of charges in ion beams.[4] The experimental arrangement is shown in Fig. 1. Beams of triply charged N and O ions, separated by the magnetic mass analyzer 2, passed through a diaphragm 3 1 mm in diameter and struck the target-scatterer 4 with an angular spread of ~0.005 rad. The scattered ions were detected by the proportional counters 9 located be-

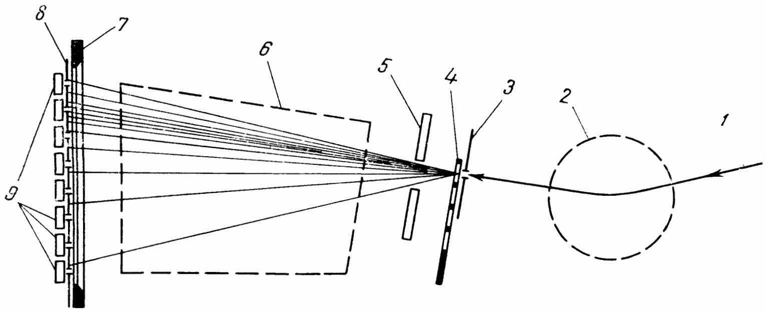

FIG. 1. Experimental arrangement: 1 – ion beam from cyclotron, 2 – magnetic mass analyzer, 3 – diaphragm 1 mm in diameter, 4 – target-scatterer, 5 – sliding channel, 6 – magnetic analyzer, 7 – slit 0.08 × 110 mm, 8 – moveable 2-mm slits, 9 – proportional counters.

Table I

Ions	t, μg/cm^2	E_0, MeV	ΔE, MeV	$F_{i\infty} = N_i / \Sigma N_i$							$\bar{i}^2$
				$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	
O^{16}	125	5.01	0.75	~$4\cdot10^{-4}$	0.0036±8%	0.051±3%	0.273±1.5%	0.445±1.5%	0.173±2%	~0.04	25.0
N^{14}	125	4.32	0.71	~$6.5\cdot10^{-4}$	0.0105±6%	0.136±2%	0.488±1.5%	0.337±1.5%	0.0245±4%	~0.0025	18.5
	256	4.32	1.45	~$12\cdot10^{-4}$	0.020±11%	0.211±4%	0.511±3%	0.240±4%	0.016±12%	~0.001	16.8

hind a slit 0.08 mm high by 110 mm long in the plane of the beam at a distance of ~ 0.5 m from the target; the place of entry of the ions into the counters was determined by the position of a system of moveable slits 8. Between the target and the counters there were a sliding channel 5 and a magnetic analyzer 6. The targets were free aluminum foils obtained by vacuum evaporation of the metal.

Figures 2 and 3 show the angular distributions $f(\theta)$ of N^{14} and O^{16} ions multiply scattered in targets of thickness $t = 125 \pm 1.6$ and $256 \pm 2.3\,\mu g/cm^2$. The initial energy of the N^{14} ions was 4.32 ± 0.15 MeV ($v = 7.7 \times 10^8$ cm/sec), and of the O^{16} ions 5.01 ± 0.15 MeV ($v = 7.75 \times 10^8$ cm/sec). The statistical errors shown are ~2% near the peak of the distribution, 2–4% at half height, and 5–10% in the region 0.1 f(0°); the angles were determined with an accuracy of ~5′.

In addition to the angular distributions, we measured the charge composition of the beam of ions scattered at angles up to ±1.5°. These measurements were made with the magnetic analyzer 6 and the sliding channel 5 narrowed to 1 mm. The results obtained are shown in Table I.

The angular distributions obtained were analyzed on the basis of the Molière-Bethe theory,[1, 5] developed for scattering of fast charged particles by atoms described by a statistical model. The results of the calculations are given in Table II and in Figs. 2 and 3. The expression for the screening angle χ_a, which characterizes the interaction of the particle with the screened field of the target atom, was obtained from this theory by approximating the Thomas-Fermi function by the sum of three exponentials and has the form

$$\chi_a = (\lambda\!\!\!^{-} / a)\sqrt{1.13 + 3.76\alpha^2}, \qquad (1)$$

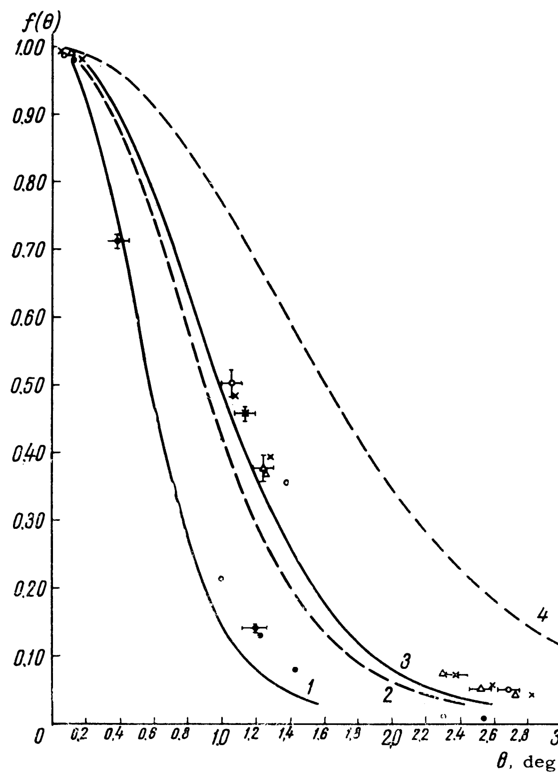

FIG. 2. Angular distributions for scattering of N^{14} ions in Al. E_0 = 4.32 MeV (7.70 × 10^8 cm/sec); t = 125 μg/cm^2: ● – experimental points, curve 1 – theory with $Z_1^2 = 20.0 = \bar{i}^2$, 2 – theory with Z_1^2 = 49; t = 256 μg/cm^2: o, x, △ – different series of measurements, curve 3 – theory with $Z_1^2 = 18.5 = \bar{i}^2$, 4 – theory with Z_1^2 = 49.

Table II

Ions and E_0, MeV	t, μg/cm^2	$\bar{E}$, Mev	$\bar{\alpha}$	$\bar{\chi}_0$, rad	Q_b	B	Z_1^2	$\bar{\chi}_c$, rad	$\bar{\chi}_a$, rad	$\chi_c\sqrt{B}$, rad – deg
N^{14}, 4.32	125	3.98	27.0	$0.307\cdot10^{-4}$	36.9	5.05	49	$9.71\cdot10^{-3}$	$1.60\cdot10^{-3}$	$2.18\cdot10^{-2}$=1.25°
							20.0	$6.20\cdot10^{-3}$	$1.03\cdot10^{-3}$	$1.39\cdot10^{-2}$=0.80°
	256	3.58	28.4	$0.323\cdot10^{-4}$	75.5	5.95	49	$15.5\cdot10^{-3}$	$1.78\cdot10^{-3}$	$3.78\cdot10^{-2}$=2.16°
							18.5	$9.52\cdot10^{-3}$	$1.09\cdot10^{-3}$	$2.32\cdot10^{-2}$=1.33°
O^{16}, 5.01	125	4.64	30.5	$0.266\cdot10^{-4}$	36.9	5.05	64	$9.55\cdot10^{-3}$	$1.57\cdot10^{-3}$	$2.14\cdot10^{-2}$=1.23°
							26.5	$6.15\cdot10^{-3}$	$1.01\cdot10^{-3}$	$1.38\cdot10^{-2}$=0.79°
	256	4.24	32.0	$0.278\cdot10^{-4}$	75.5	5.95	64	$14.95\cdot10^{-3}$	$1.72\cdot10^{-3}$	$3.66\cdot10^{-2}$=2.10°
							25.0	$9.35\cdot10^{-3}$	$1.07\cdot10^{-3}$	$2.28\cdot10^{-2}$=1.31°

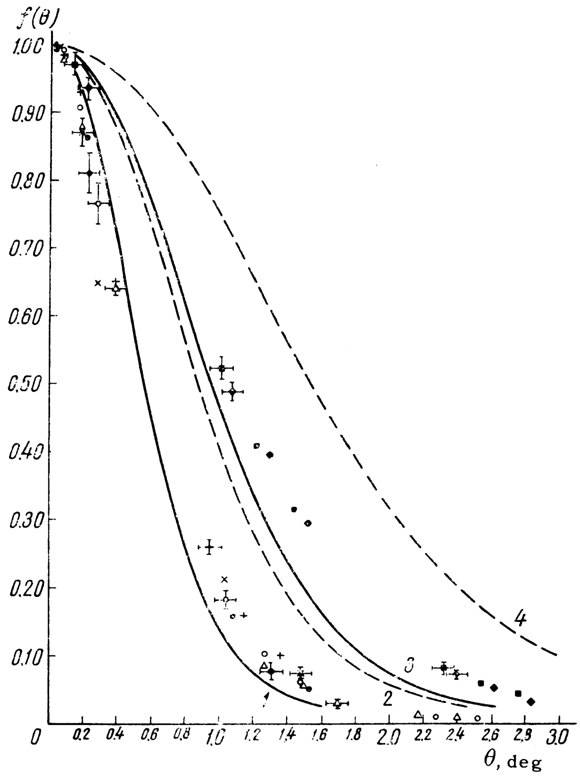

FIG. 3. Angular distributions of O^{16} ions scattered in Al. E_0 = 5.01 MeV (7.75×10^8 cm/sec); t = 125 μg/cm²: ●, o, ×, +, △ – different series of measurements, curve 1 – theory with $Z_1^2 = 26.5 = \overline{i^2}$, 2 – theory with $Z_1^2 = 64$; t = 256 μg/cm²: ♦, ■ – experimental points, curve 3 – theory with $Z_1^2 = 25.0 = \overline{i^2}$, 4 – theory with $Z_1^2 = 64$.

where

$$\lambda\!\!\!^{-} = \hbar / p, \quad a = 0.885 a_0 Z_2^{-1/3} \quad (a_0 = e^2/\hbar c),$$

$$\alpha = Z_1 Z_2 e^2 / \hbar v = Z_1 Z_2 / 137\beta;$$

p, v, and Z_1 are the momentum, velocity, and charge of the incident particle, and Z_2 is the atomic number of the target atoms. Here the shape of the angular distribution of scattered particles is determined by the single parameter B, which depends only on the "effective number of collisions"

$$Q_b = \chi_c^2 / \chi_a^2, \tag{2}$$

where

$$\chi_c = \sqrt{4\pi N t}\,(Z_1 Z_2 e^2 / pv) \tag{3}$$

(Nt is the number of scattering centers per cm² of target), and the half-width of the distribution at a height $e^{-1}f(0°)$ is given by the relation[6]

$$\Theta_{1/e} \approx \chi_c \sqrt{B - 1.2}. \tag{4}$$

Under our conditions the relation $\alpha \gg 1$ is satisfied (see Table II) and therefore, according to (1)–(3), Q_b is practically independent of the velocity of the scattered particle:

$$Q_b = \chi_c^2 / \chi_a^2 \approx 4\pi N t a^2 / 3.76. \tag{5}$$

The absence in expression (5) of a dependence on the charge of the particle is a defect of the theory; it originates from the fact that in calculation of χ_a the field of the incident particle is assumed to be purely Coulomb. Therefore the theoretical calculations on the basis of formulas (3)–(5) in the energy region of interest to us should lead to exaggeration of the widths of the angular distributions of the scattered particles. For our cases this exaggeration amounts to ~50% (the dashed curves in Figs. 2 and 3). In this connection we attempted to "correct" the theory by introduction of an effective charge of the ion, $Z_{eff} < Z_1$. For Z_{eff} we took the mean-squared ionic charge $\overline{i^2}$ in a beam with equilibrium charge composition. The results of the calculations using $\overline{i^2}$ are shown in Figs. 2 and 3 by the solid lines. In each case the calculations were made for an average ion energy

$$\bar{E} = E_0 - \Delta E / 2;$$

the values of ΔE were determined on the basis of the known data on specific energy losses of N and O ions.[7] Values of $\overline{i^2}$ for $\bar{E}$ were found by extrapolation of values listed in Table I, on the basis of the known dependence of $\overline{i^2}$ on energy for N^{14} ions in celluloid.[8]

We can see from the figures that this very simple approximation leads to satisfactory agreement of the half-widths of the theoretical and experimental angular distributions.

Comparison of the shapes of the experimental and theoretical distributions f(θ) shows that for large angles Z_{eff}^2 apparently should be greater than $\overline{i^2}$, and for small angles, on the other hand, $Z_{eff}^2 < \overline{i^2}$. This fact is not surprising for, according to (4) and (5), the choice of Z_{eff} does not affect the value of Q_b, which determines (through the parameter B) the shape of the distribution, but changes only the angle scale for χ_c. Therefore, to take into account more accurately the screening of the nuclear field of the scattered particle and its change during the charge-exchange process, it is necessary to consider the dependence of this screening angle χ_a (and consequently Q_b) on the degree of ionization of the incident ion.

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Translated by C. S. Robinson
76