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THEORY OF ABSORPTION OF SOUND IN CURRENT-CARRYING SUPERCONDUCTORS

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Sound absorption in impurity superconductors in which a current is flowing is studied by quantum field theory methods developed by Abrikosov and Gor'kov. The dependence of the absorption coefficient on the temperature and on the quantity which determines the current in the sample, $s = p_0 v_s$ (p_0 is the Fermi momentum, v_s is the velocity of the pairs) is found.

At the present time, a significant number of researches[1-3] have been devoted to the study of superconductors in the presence of a current in them. The interest in this question stems from the fact that the current flowing through a specimen creates a number of features which are associated with the tendency of the current to disrupt the superconductivity. By increasing the momentum of the pair, and thus changing the current in the superconductors, it is possible under corresponding conditions (see [4]) to create a situation whereby the superconductivity in the specimen vanishes. In this sense, the effect of the current on the superconductors is analogous to the effect of paramagnetic impurities,[5] where the exchange interaction of the electron spins with the spins of the impurity atoms leads to the elimination of the energy gap in the spectrum of elementary excitations (the so-called gapless superconductivity), and also to the disruption of the superconductivity.

In the present work, the effect of the current on sound absorption in superconducting alloys is investigated. Here, it is assumed that the frequency of the incident sound $\omega_0 \ll 1/\tau$, where τ is the relaxation time of the electrons in the normal metal.

1. The interaction of the sound wave with the electrons can be expressed in the following fashion:

$$H_{int} = \int \hat{\lambda}_{\alpha\beta} u_{\alpha\beta}(x) \psi^+(x) \psi(x)\, d\mathbf{r}, \tag{1}$$

where $u_{\alpha\beta}(x)$ is the deformation tensor of the body, $\hat{\lambda}_{\alpha\beta}$ is a tensor whose components are equal, in order of magnitude, to the Fermi energy and in momentum space are functions of the momentum of the electron, $\psi(x)$ is the operator of the electron field.

By using the formula of the thermodynamics of irreversible processes, which determines the density of the force which a sound wave exerts on a system of electrons,

$$\sigma_{\alpha\beta} = \delta\langle H_{int}\rangle / \delta u_{\alpha\beta} \tag{2}$$

(the angle brackets denote averaging over the grand canonical ensemble), we obtain the energy absorption per unit volume

$$W = \omega_0 \operatorname{Im}\left\{\frac{\delta\langle H_{int}\rangle}{\delta u_{\alpha\beta}} u_{\alpha\beta}^*\right\}. \tag{3}$$

Here ω_0 is the frequency of sound and

$$\langle H_{int}\rangle = \int \varphi(x) G(xx)\, d\mathbf{r}, \tag{4}$$

where $G(xx')$ is the temperature Green's function of the electrons [$G(xx)$ is its value for $\mathbf{r} = \mathbf{r}'$, $\tau = \tau' - 0$], and $\varphi(x) = \hat{\lambda}_{\alpha\beta} u_{\alpha\beta}(x)$.

With account of Eq. (1), we write down the system of equations for the functions $G(xx')$ and $F(xx')$:

$$\left(-\frac{\partial}{\partial\tau} + \frac{\nabla^2}{2m} + \mu\right) G(xx') + \Delta(x) F^+(xx') - \varphi(x) G(xx') = \delta(x - x'),$$

$$\left(\frac{\partial}{\partial\tau} + \frac{\nabla^2}{2m} + \mu\right) F^+(xx') + \Delta^+(x) G(xx') - \varphi(x) F^+(xx') = 0. \tag{5}$$

We represent the Green's functions $G(xx')$, $F(xx')$, and $F^+(xx')$ in the form

$$G(xx') = G^{(0)}(xx') + G^{(1)}(xx'),$$
$$F(xx') = F^{(0)}(xx') + F^{(1)}(xx'),$$
$$F^+(xx') = F^{(0)+}(xx') + F^{(1)+}(xx'),$$

where $G^{(0)}(xx')$ and $F^{(0)}(xx')$ are the Green's functions in the absence of the field, while $G^{(1)}(xx')$ and $F^{(1)}(xx')$ are the additions, which are linear in the deformation tensor. By linearizing Eq. (5), we get

$$\left(-\frac{\partial}{\partial\tau} + \frac{\nabla^2}{2m} + \mu\right) G^{(1)}(xx') + \Delta^{(0)} F^{(1)}(xx')$$

$$= \varphi(x) G^{(0)}(xx') - \Delta^{(1)}(x) F^{(0)+}(xx'),$$

$$\left(\frac{\partial}{\partial\tau} + \frac{\nabla^2}{2m} + \mu\right) F^{(1)+}(xx') - \Delta^{(0)} G^{(1)}(xx')$$
$$= \varphi(x) F^{(0)+}(xx') + \Delta^{(1)+}(x) G^{(0)}(xx'). \quad (6)$$

From these equations, it is possible to express $G^{(1)}(xx')$ in terms of the quantities on the right hand side of (6). For this purpose, it is convenient to write down this expression in operator form, and then to find the operator which is inverse to the operator of the left side of Eq. (6). By proceeding in such fashion, we obtain the following expression for the contribution $G^{(1)}(xx')$ to the Green's function, which is linear in $u_{\alpha\beta}(x)$:

$$G^{(1)}(xx') = \int \varphi(x'')[G(x''x')G(xx'') + F(x''x')F^{+}(xx'')]\, d^4x''$$
$$+ \int [\Delta^{(1)}(x'') G(x''x') F(xx'') - \Delta^{(1)+}(x'') G(x''x')$$
$$\times F^{+}(xx'')]\, d^4x''. \quad (7)$$

Here and below, we omit the symbol (0) in the functions $G(xx')$ and $F(xx')$. The expression (7) must be averaged over the locations of randomly distributed impurities. If we use the well-known formulas

$$\Delta(x) = \lim_{\substack{r' \to r \\ \tau' \to \tau + 0}} |g| F(xx'), \qquad \Delta^{+}(x) = \lim_{\substack{r' \to r \\ \tau' \to \tau + 0}} |g| F^{+}(xx'),$$

then it is easy to see that the second integral in (7), after averaging, gives a correction of the next order relative to the fundamental term.

By substituting (7) in Eqs. (4) and (3), we get the following formula for the quantity W:

$$W = \frac{\omega_0 T}{(2\pi)^3}$$
$$\times \sum_{\omega} \mathrm{Im} \int d\mathbf{p}\, \overline{[|\varphi(\mathbf{pq})|^2 G(p_+) G(p_-) + |\varphi(\mathbf{pq})|^2 F(p_+) F^{+}(p_-)]}. \quad (8)$$

Here the bar denotes averaging over the locations of the impurities,

$$p_{+,(-)} = (\mathbf{p} \pm q/2,\ \omega \pm \omega_0/2).$$

By introducing the notation

$$\Pi_1(\mathbf{pq}\omega_0) = \overline{|\varphi(\mathbf{pq})|^2 G(p_+) G(p_-)} + \overline{|\varphi(\mathbf{pq})|^2 F(p_+) F^{+}(p_-)} \quad (9)$$

and using the method of summation of ladder diagrams developed by Abrikosov and Gor'kov,[6] we easily obtain an integral equation for the quantity $\Pi_1(\mathbf{pq}\omega_0)$:

$$\Pi_1(\mathbf{pq}\omega_0) = |\varphi(\mathbf{pq})|^2 [G(p_+)G(p_-) + F(p_+)F^{+}(p_-)]$$
$$+ G(p_+)G(p_-)\Lambda_1 - F(p_+)G(p_-)\Lambda_2 - F(p_+)F^{+}(p_-)\Lambda_3$$
$$- G(p_+)F(p_-)\Lambda_4, \quad (10)$$

$$\Lambda_i = \frac{n}{(2\pi)^3} \int |U(\mathbf{p} - \mathbf{p}')|^2 \Pi_i(\mathbf{p}'\mathbf{q}\omega_0)\, d\mathbf{p}' \quad (11)$$

(n is the concentration of the impurities, U is the interaction potential of the electron with the impurity).

We shall not write out the equations for Π_2, Π_3, and Π_4, inasmuch as only $\Pi_1(\mathbf{pq}\omega_0)$ enters in (8). We only remark that the equations are greatly simplified if we do not take into account the spatial dispersion of the sound wave, that is, if we put $\mathbf{q} \to 0$ in (10). In this case (see [6]) a set of two integral equations is used which has the form

$$\Pi_1(\mathbf{p}\omega_0) = |\varphi(\mathbf{p})|^2 [G(p_+)G(p_-) + F(p_+)F^{+}(p_-)]$$
$$+ [G(p_+)G(p_-) + F(p_+)F^{+}(p_-)]\Lambda_1 - [F^{+}(p_+)G(p_-)$$
$$+ G(p_+)F(p_-)]\Lambda_2,$$
$$\Pi_2(\mathbf{p}\omega_0) = |\varphi(\mathbf{p})|^2 [F(p_+)G(p_-) + G(-p_+)F^{+}(p_-)]$$
$$+ [F(p_+)G(p_-) - G(-p_+)F^{+}(p_-)]\Lambda_1 + [G(-p_+)G(p_-)$$
$$- F^{+}(p_+)F^{+}(p_-)]\Lambda_2. \quad (12)$$

If we introduce the notation

$$\Lambda_i(\mathbf{p}\omega\omega_0) = |\varphi(\mathbf{p})|^2 \lambda_i(\omega\omega_0),$$

then Eqs. (12) can easily be put in algebraic form (in the following formulas, the bar denotes averaging over the directions of **p**)

$$\overline{|\varphi(\mathbf{p})|^2}\lambda_1 = [\overline{|\varphi(\mathbf{p})|^2 G(p_+)G(p_-)}$$
$$+ \overline{|\varphi(\mathbf{p})|^2 F(p_+)F(p_-)}](1 + \lambda_1)$$
$$- 2\overline{|\varphi(\mathbf{p})|^2 F(p_+)G(p_-)}\lambda_2,$$
$$\overline{|\varphi(\mathbf{p})|^2}\lambda_2 = [\overline{|\varphi(\mathbf{p})|^2 F(p_+)G(p_-)}$$
$$- \overline{|\varphi(\mathbf{p})|^2 G(-p_+)F(p_-)}](1 + \lambda_1)$$
$$- [\overline{|\varphi(\mathbf{p})|^2 F(p_+)F(p_-)} - \overline{|\varphi(\mathbf{p})|^2 G(-p_+)G(p_-)}]\lambda_2, \quad (13)$$

where, for example,

$$\frac{\overline{|\varphi(\mathbf{p})|^2 G(p_+)G(p_-)}}{\overline{|\varphi|^2}}$$
$$= \frac{n}{(2\pi)^3 \overline{|\varphi|^2}} \int |U(\mathbf{p} - \mathbf{p}')|^2 |\varphi|^2 G(p_+')G(p_-')\, d\mathbf{p}'$$
$$= \frac{\Delta^2}{4\tau_0 \eta (\omega^2 + \Delta^2)^{3/2}},$$

$$\frac{1}{\tau_0} = \frac{n m p_0}{(2\pi)^2 \overline{|\varphi|^2}} \int |U(\theta)|^2 |\varphi|^2 d\Omega,$$

$$\overline{|\varphi|^2} = \frac{1}{4\pi} \int |\varphi(p)|^2 d\Omega_p,$$

$$\frac{1}{\tau}=\frac{nmp_0}{(2\pi)^2}\int |U(\theta)|^2 d\Omega, \quad \frac{1}{\tau'}=\frac{1}{\tau}-\frac{1}{\tau_0}.$$

Here m is the mass of the electron, p_0 is the limiting Fermi momentum.

By calculating the other terms of Eq. (13) in similar fashion, we get

$$\lambda_1=\frac{1}{2\tau_0(\eta\nu+\eta_-\nu_-)}\left(1-\frac{\omega\omega_--\Delta^2}{\nu\nu_-}\right)(1+\lambda_1)+\frac{i(\omega_-\Delta+\omega\Delta_-)/2}{\tau_0\nu\nu_-(\eta\nu+\eta_-\nu_-)}\lambda_2,$$

$$\lambda_2=\frac{i(\omega_-\Delta+\omega\Delta_-)/2}{\tau\nu\nu_-(\eta\nu+\eta_-\nu_-)}(1+\lambda_1)+\frac{1}{2\tau_0(\eta\nu+\eta_-\nu_-)}\left(1-\frac{\Delta^2-\omega\omega_-}{\nu\nu_-}\right)\lambda_2, \tag{14}$$

where

$$\eta=1+\frac{1}{2\tau\nu}, \quad \eta_-=1+\frac{1}{2\tau\nu_-},$$

$$\nu=(\omega^2+\Delta^2)^{1/2}, \ \nu_-=(\omega_-^2+\Delta^2)^{1/2}, \quad \omega_-=\omega-\omega_0. \tag{15}$$

Solving the set (14), we find

$$\lambda_1=\frac{1}{2}\left(1-\frac{\omega\omega_--\Delta^2}{\nu\nu_-}\right)\frac{1}{\tau_0(\eta\nu+\eta_-\nu_-)-1},$$

$$\lambda_2=\frac{i(\omega_-\Delta+\omega\Delta_-)/2}{\nu\nu_-}\frac{1}{\tau_0(\eta\nu+\eta_-\nu_-)-1}. \tag{16}$$

Together with (12), (9), and (8), this gives

$$W=\frac{\omega_0 T p_0 m|\varphi|^2}{2\pi}\sum\frac{\tau_0}{\tau_0(\eta\nu+\eta_-\nu_-)-1}\frac{\nu\nu_--\omega\omega_--\Delta^2}{\nu\nu_-}. \tag{17}$$

We have thus obtained an expression for the density of absorbed energy in the temperature technique. It is well known that all the physical quantities, including the density of absorbed energy, must be expressed in terms of a retarded Green's function, which is an analytic function in the upper complex half plane ω. The latter is obtained if we use Eq. (1), and write down the coupling of the operator $\sigma_{\alpha\beta}$ in the Heisenberg representation with an operator in the interaction representation:

$$\tilde{\sigma}_{\alpha\beta}(x)=S^{-1}(t)\sigma_{\alpha\beta}(x)S(t), \tag{18}$$

where

$$S(t)=T\exp\left\{i\int_{-\infty}^{t}\sigma_{\alpha\beta}(x)u_{\alpha\beta}(x)d^4x\right\}. \tag{19}$$

Now, recalling Eq. (17), we can find the quantity W by means of analytic continuation.[7] Using the condition $\omega_0\tau \ll 1$, we get

$$W=\frac{\omega_0^2\tau' p_0 m|\varphi|^2}{\pi^2 T}\int_{\Delta}^{\infty}\mathrm{ch}^{-2}\frac{\omega'}{2T}d\omega'. \tag{20}*$$

Carrying out the integration, we get the following expression for the absorption coefficient α_s:

$$\alpha_s=\frac{2\omega_0^2\tau' m p_0}{\pi^2\rho s^2(\mathbf{n})}\overline{|\lambda_{\alpha\beta}(\mathbf{p})n_\alpha n_\beta'|^2}\,2n(\Delta), \tag{21}$$

where ρ is the density of the material, $s(\mathbf{n})$ is the sound velocity, $\mathbf{n}=\mathbf{q}/|\mathbf{q}|$, $\mathbf{n}'$ is the unit vector which determines the polarization of the sound wave and n(x) is the distribution function of the electrons in the metal.

2. We now consider the superconductor in the presence of a current. The presence of a current in the superconductor corresponds to a definite momentum of the electron pairs. The Green's function of the impurity superconductors was found by Abrikosov and Gor'kov.[6] Later, Maki[2] generalized this function to include the case of a pair momentum differing from zero. In this case, they have the form

$$G(p)=\frac{\tilde{\omega}-sz+\xi}{(\tilde{\omega}-sz)^2-\xi^2-\tilde{\Delta}^2}, \quad F^+(p)=\frac{i\tilde{\Delta}}{(\tilde{\omega}-sz)^2-\xi^2-\tilde{\Delta}^2}. \tag{22}$$

Here, in the case in which $\tau\Delta \ll 1$, the following relation holds:

$$\frac{\omega}{\Delta}=u\left(1-\frac{\zeta}{\sqrt{1-u^2}}\right), \quad u=\frac{\tilde{\omega}}{\tilde{\Delta}}, \quad \zeta=\frac{\tau\alpha}{\Delta}, \tag{23}$$

where z is the cosine of the angle between the momentum of the electron on the Fermi surface and the direction of the pair momentum, while $\alpha = 2s^2/3$ and $s = p_0 v_s$.

In Eqs. (22) and (23) we have neglected the effect of the magnetic field of the current. This is valid if the pair velocity v_s is very large in comparison with v_j, the Larmor velocity of the pair in the magnetic field of the current. This latter is easily estimated. We obtain

$$v_j/v_s \sim e^2/mc^2 d \ll 1,$$

where d is the thickness of the film.

We replace in (8) the functions G(p) and F(p) by the Green's function of the superconductor in the presence of a current, and carry out the procedure of averaging the entire expression over the locations of the impurities. Without making detailed calculations, which are completely analogous to those set forth above, we note that if we use Eqs. (22) and (23), then, after simple calculations, we get

*ch = cosh.

$$\alpha_s = \frac{\omega_0{}^2\tau' p_0 m}{\pi^2 T\rho s^2(\mathbf{n})}\,\overline{|\lambda_{\alpha\beta}(\mathbf{p})n_\alpha n_\beta'|^2}$$

$$\times\int\limits_{\omega_0}^{\infty} d\omega\,\mathrm{ch}^{-2}\frac{\omega}{2T}\left\{\frac{1}{\sqrt{1-u^2}}\,\mathrm{Im}\frac{1}{\sqrt{1-u^2}}\right.$$

$$\left.-\frac{u}{\sqrt{1-u^2}}\,\mathrm{Im}\frac{u}{\sqrt{1-u^2}}\right\} \qquad (24)$$

The quantity $\omega_0 = \Delta(1-\xi^{2/3})^{3/2}$ in the last formula is the gap in the energy spectrum of the superconductor in the presence of the current. The expression in the curly brackets can easily be computed if we use the first formula of (23) and recall that the quantities $\mathrm{Im}\,(1/\sqrt{1-u^2})$ and $\mathrm{Im}\,(u/\sqrt{1-u^2})$ are different from zero for $\omega > \omega_0$. Expanding (23) in a series in $u - u_0$ we get, after integration,

$$\frac{\alpha_s}{\alpha_n} = \frac{4}{3}\left(\frac{\Delta}{\tau\alpha}\right)^{2/3}\left[1-\left(\frac{\tau\alpha}{\Delta}\right)^{2/3}\right]^{-1/2}\frac{T}{\Delta}$$

$$\times\exp\left\{-\frac{\Delta}{T}\left[1-\left(\frac{\tau\alpha}{\Delta}\right)^{2/3}\right]\right\},\quad \frac{\tau\alpha}{\Delta}<1,$$

$$\frac{\alpha_s}{\alpha_n} = 3\cdot 2^{-1/3}(1-2^{-1/3})\Gamma\left(\frac{5}{3}\right)\zeta\left(\frac{2}{3}\right)\left(\frac{T}{\Delta}\right)^{2/3}$$

$$-3\cdot 2^{-5/3}(1-2^{-1/3})\;\Gamma\left(\frac{7}{3}\right)\zeta\left(\frac{4}{3}\right)\left(\frac{T}{\Delta}\right)^{4/3},\quad \frac{\tau\alpha}{\Delta}=1,$$

$$\frac{\alpha_s}{\alpha_n} = 1-\left(\frac{\Delta}{\tau\alpha}\right)^2+\Gamma(3)\zeta(2)\left(\frac{\Delta}{\tau\alpha}\right)^4\left[1-\left(\frac{\Delta}{\tau\alpha}\right)^2\right]$$

$$\times\left[2+\left(\frac{\Delta}{\tau\alpha}\right)^2\right]\left(\frac{T}{\Delta}\right)^2,\quad \frac{\tau\alpha}{\Delta}>1. \qquad (25)$$

In these formulas the value of Δ remains undetermined. The dependence of Δ on the temperature and on the quantity s is easily obtained if we use the equation

$$\Delta = \frac{|g|T}{(2\pi)^3}\sum\int d\mathbf{p}F(p),$$

where $F(p)$ is the thermodynamic Green's function in the presence of the current. Proceeding in standard fashion, we have for temperature $T \ll \Delta$,[5, 2]

$$\ln\frac{\Delta}{\Delta_{00}} = -\frac{\pi}{4}\frac{\tau\alpha}{\Delta}-2\left(\frac{2}{3}\right)^{1/2}\left(\frac{\Delta}{\tau\alpha}\right)^{2/3}\left[1-\left(\frac{\tau\alpha}{\Delta}\right)^{2/3}\right]$$

$$\times\Gamma\left(\frac{3}{2}\right)\left(\frac{T}{\Delta}\right)^{3/2}e^{-\omega_0/T},\quad \frac{\tau\alpha}{\Delta}\leqslant 1,$$

$$\ln\frac{\Delta}{\Delta_{00}} = -\operatorname{arcch}\frac{\tau\alpha}{\Delta}-\frac{1}{2}\left[\frac{\tau\alpha}{\Delta}\arcsin\frac{\Delta}{\tau\alpha}-\left(1-\left(\frac{\Delta}{\tau\alpha}\right)^2\right)^{1/2}\right.$$

$$\left.-\frac{1}{6}\left(\frac{\pi T}{\Delta}\right)^2\left(\frac{\Delta}{\tau\alpha}\right)^2\left[1-\left(\frac{\Delta}{\tau\alpha}\right)^2\right]^{-1/2},\quad \frac{\tau\alpha}{\Delta}>1, \qquad (26)$$

where Δ_{00} is the gap for $T = 0$, $s = 0$.

For $\tau\alpha = \Delta$, the quantity $\omega_0 = 0$, that is, the gap in the energy spectrum of single-frequency excitations disappears. If we take into account that here $\ln(\Delta/\Delta_{00}) = -\pi/4$, we get s_0, the value of s for which the gap disappears. For $T = 0$, we have

$$s_0 = \sqrt{\frac{3\Delta_{00}}{2\tau}}\,e^{-\pi/8}. \qquad (27)$$

At this value of s, as is easily seen from the second formula of (25), the sound absorption coefficient depends in power-law fashion on the temperature and is equal to

$$\frac{\alpha_s}{\alpha_n} = D\left(\frac{5}{3}\right)\left(\frac{T}{\Delta_{00}}\right)^{2/3}-2^{-4/3}D\left(\frac{7}{3}\right)\left(\frac{T}{\Delta_{00}}\right)^{4/3}, \qquad (28)$$

where

$$D(\nu) = 3\cdot 2^{2-\nu}e^{\pi(\nu-1)/4}(1-2^{2-\nu})\Gamma(\nu)\zeta(\nu-1).$$

For $s > s_0$, we have, from the second equation of (26),

$$\ln\frac{2\tau\alpha}{\Delta_{00}} = -\frac{1}{12}\left[\left(\frac{\Delta}{\tau\alpha}\right)^2+2\left(\frac{\pi T}{\tau\alpha}\right)^2\right].$$

From this equality, setting Δ equal to zero, we get the critical value s_{c0} for the quantity s. For $T = 0$, we have

$$s_{c0} = \frac{1}{2}\sqrt{\frac{3\Delta_{00}}{\tau}}. \qquad (29)$$

Equating (29) with (27), we have

$$s_0 = 0.96\,s_{c0}$$

Thus, we see that in the superconducting phase there exists a region $s_0 \le s \le s_{c0}$ where the gap in the energy spectrum is missing. We note that there exists a similar analysis of gapless superconductivity.[2, 3] The dependence of the value of Δ on s and T in this region is:

$$\Delta \cong \sqrt{3}\,\Delta_{00}\left(\frac{s}{s_{c0}}\right)^2\left[1-\left(\frac{s}{s_{c0}}\right)^2-\frac{2}{3}\left(\frac{s_{c0}}{s}\right)^4\left(\frac{\pi T}{\Delta_{00}}\right)^2\right]^{1/2}.$$

Using the last of the formulas (25), we see that in this region the value of α_s/α_n depends to some degree on the temperature:

$$\frac{\alpha_s}{\alpha_n} \cong 1-12\left[1-\frac{s^2}{s_{c0}{}^2}-\frac{2}{3}\frac{s_{c0}{}^4}{s^4}\left(\frac{\pi T}{\Delta_{00}}\right)^2\right]$$

$$\times\left[1-4\Gamma(3)\zeta(2)\frac{s_{c0}{}^4}{s^4}\left(\frac{T}{\Delta_{00}}\right)^2\right].$$

We consider, finally, the region $s < s_0$. For sufficiently low values of s, we have

$$\Delta \cong \Delta_{00}\left(1-\frac{\pi}{8}\frac{s^2}{s_{c0}{}^2}\right),$$

and from the first formula of (25), we get

$$\frac{\alpha_s}{\alpha_n} \cong \frac{8}{3\cdot 2^{1/3}} \frac{T}{\Delta_{00}} \left(\frac{s_{c0}}{s}\right)^{4/3} \exp\left\{-\frac{\Delta_{00}}{T}\left[1-\frac{3}{2^{5/3}}\left(\frac{s}{s_{c0}}\right)^{4/3}\right]\right\}. \quad (30)$$

We note that this formula in the limit $s \to 0$ does not transform to the formula for the superconductor in the absence of the current. This is connected with the fact that the possibility of the expansion of Eq. (24) in powers of $u - u_0$ imposes some limitation on the region of application of Eq. (30). The latter can be obtained if we estimate the next term of such an expansion. This gives

$$s \gg (3T/\tau)^{1/2}.$$

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